Mathematics 1

Mathematics Department
Shady Side Academy
Pittsburgh, PA
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SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS
MATH I

MISSION/HISTORY: As part of an on-going curriculum review the Mathematics Department of Shady Side Academy sent two members of the senior school math faculty to visit Phillips Exeter Academy [PEA] in 2008 to observe their classes. After this observation and much reflection, the department decided to adopt this problem-based curriculum. The materials used in the Mathematics I and II courses are taken directly from PEA. We thank the teachers at PEA for the use of their materials.

RATIONALE: The Shady Side Academy Mathematics Department Goals are as follows:
S*tudents will develop the habit of using mathematical reasoning based on logical thinking. Students will develop adequate skills necessary to solve problems mathematically.*
Students will recognize that the structure and order of mathematics can be discovered in the world around us.
Students will recognize the connections of mathematics to other disciplines.
Students will express themselves clearly in mathematical discourse.
Students will be familiar with and proficient in appropriate technology.
Students will achieve their highest mathematical goals.
Students will gain an appreciation for the study of mathematics.

In addition, the teachers in the Department of Mathematics want you to be an articulate student of mathematics. We want you to be able to speak and write mathematics well. We want you to be a fearless problem solver so that you approach problems with curiosity and not trepidation. The Mathematics II classroom is student-centered. The curriculum is problem- based with an integrated design. You will continually learn new material while reviewing prior topics.

EXPECTATIONS: In order for you to be successful in this course, the Mathematics Department has the following suggestions and expectations. First, we expect you to attempt every problem. More than merely writing the problem number, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. Next, seek help wherever you can find it. We expect you to cooperate with your peers and teachers. The Mathematics Department is a team of teachers striving to help all students reach their potential. You are encouraged to ask any teacher for help if your own is not available. Finally, as stated in the Student Handbook on page 13: "Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets." We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

To the Student

Contents: Members of the PEA Mathematics Department have written the material in this book. As you work through it, you will discover that algebra and geometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a section on tangents to circles. The curriculum is problem-centered, rather than topic-centered. Techniques and theorems will become apparent as you work through the problems, and you will need to keep appropriate notes for your records — there are no boxes containing important theorems. You will begin the course with this binder of problems, graph paper, and a protractor. All of your solutions are to be kept in this binder. It will be periodically collected and will factor into your term grade. There is no index in your binder but the reference section at the end should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

Comments on problem-solving: You should approach each problem as an exploration. Reading each question carefully is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. It is important to make accurate diagrams whenever appropriate. Useful strategies to keep in mind are: create an easier problem, guess and check, work backwards, and recall a similar problem. It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day. Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher. If you have taken the time to think about a problem, you should bring to class a written record of your efforts, not just a blank space in your notebook. The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. Proper spelling is essential for clear written communication.

About technology: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind: write before you calculate, so that you will have a clear record of what you have done; store intermediate answers in your calculator for later use in your solution; pay attention to the degree of accuracy requested; refer to your calculator's manual when needed; and be prepared to explain your method to your classmates. Also, if you are asked to "graph $y = (2x - 3)/(x + 1)$", for instance, the expectation is that, although you might use your calculator to generate a picture of the curve, you should sketch that picture in your notebook or on the board, with correctly scaled axes.

Shady Side Academy
Introductory Math Guide for Students

Homework

First, we expect you to attempt every problem. More than merely writing the problem number in your notebook, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. As stated in the Student Handbook: "Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets." We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

Going to the Board

It is very important to go to the board to put up homework problems. Usually, every homework problem is put on the board at the beginning of class, presented, and then discussed in class. By doing this, you will develop your written and oral presentation skills.

Plagiarism

You can get help from almost anywhere, but make sure that you cite your help, and that all work shown or turned in is your own, even if someone else showed you how to do it. Never copy work from others. Teachers do occasionally give problems/quizzes/tests to be completed at home. You may not receive help on these assessments, unless instructed to by your teacher; it is imperative that all the work is yours.

Math Extra-Help

Getting help is an integral part of staying on top of the math program here at Shady Side Academy. It can be rather frustrating to be lost and stuck on a problem. Teachers, peer tutors, study groups, the internet, your resource book and classmates are all helpful sources.

Teachers and Meetings

The very first place to turn for help should be your teacher. Teachers at SSA are always eager to help you succeed. The Math Department office is located on the 3rd floor of Rowe Hall. Individual meetings can be arranged with teachers during study halls, free periods, or after school. You can always ask or email any teacher in the department for help. Getting help from your teacher is the first and most reliable source to turn to for extra help.

SSA Student Quotes

"This program really helped me learn and understand the concepts of Algebra II. It helped us as a group because we covered materials together. We all said our own ideas and accepted when they were wrong. It gave each individual confidence in their understanding of the material. Some days we did not check over every problem like I would have liked to, but this allowed me to be a frequent visitor in the math office. It was a different approach that ran very smoothly in this class."
--Betsy Vuchinich, '12

"I loved Math II this year because the curriculum was completely different from anything I've previously encountered in math. We didn't use a book for the majority of the year, instead we focused on more complicated word problems and worked together in small groups to solve these difficult problems. This forced us to think through the problems and think about "Why?" more so than "How?" and this was a much different look for a math class. Working with your peers in a setting that promoted group work was refreshing, and I enjoyed it very much. I hope the Math Department continues to use this curriculum."
--Jonathan Laufe, '12

"I came into the year unsure of what to think about this approach to mathematics. I had criticism and positive words about the packet, and I didn't know what to expect. Though sometimes I was confused, in the end, everything worked out."
--Erin Gorse, '12

"The curriculum definitely took some getting used to but once you figure it out, it has a balance of being challenging and easy at the same time."
--Elijah Williams, '13

"I thought the word problems were unnecessary at first, then I found my mind starting to expand."
--Guy Philips, '13

"Math I is a great way to learn and if I had to describe it in one word it would be 'Utopian': The classroom environment motivates me to do better and it teaches you to either accept your method or to abandon it for a better one. The fact that the teachers act as moderators in the classroom makes it a better way to learn because it really gets you to think. I loved Math One and look forward to doing more Exeter problems in Math II."
--Adam D'Angelo, '14

SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

Policy on Plagiarism and Cheating

At the beginning of each course, each teacher in the Mathematics Department will explain to the class what is expected with regard to the daily completion of homework, the taking of in-class tests, make-up tests, and take-home tests. Students will be told whether or not they may use books and/or other people when completing in-class or out-of class assignments/tests. The consequences listed below will take effect if a teacher suspects that a student is in violation of the instructions given for a particular assignment or test.

PLAGIARISM

Plagiarism is the act of representing something as one's own without crediting the source. This may be manifest in the mathematics classroom in the form of copying assignments, fabricating data, asking for or giving answers on a test, and using a "cheat sheet" on an exam.

CHEATING

If, during an in-class test, the teacher in that room considers that a student has violated the teacher's instructions for the test, the teacher will instruct the student that there is a suspicion of cheating and the teacher will initiate the consequences below. If a student is taking a make-up test out of class and any teacher considers that the student is, or has been, cheating the teacher will bring the issue to the notice of the Department Chair, and initiate the consequences below. Sharing the content of a particular test with an individual who has not taken the test is considered by the department to be cheating by both parties.

CONSEQUENCES

When a teacher suspects plagiarism or academic dishonesty, the teacher and Department Chair will speak with the student. The Department Chair, in conjunction with the Dean of Student Life, will determine whether plagiarism or academic dishonesty has occurred. If plagiarism or academic dishonesty is determined, the Dean of Student Life and the Department Chair make the decision about the appropriate response to the situation, which will likely include referral to the Discipline Committee. The Department Chair will contact the family to discuss the infraction and consequence. If a Discipline Committee referral is made, the Dean of Student Life will follow up with the family as well.

In any case of plagiarism or cheating, the student concerned will likely receive a failing grade for that piece of work, as well as any other appropriate steps deemed necessary by the Department Chair and the Dean of Academic Life.

Math I Guided Notes

The following pages are a place for you to organize the concepts, topics, formulas and ideas you learn this year. You can use them in any way you wish. It is suggested that when you come upon an important finding or result in class or on your own, that you write it in these notes so that it is easily accessible when it comes time to study for an exam or review material. These notes are not a substitute for taking notes in other ways, and the Mathematics Department encourages you to use a notebook to have a record of your work, corrections and any notes you get in class. We hope this is useful to you, and we welcome any feedback.

--SSA Senior School Math Department

Area/Perimeter/Volume:

Sketch	Area	Perimeter
Square with sides *s*		
Rectangle with sides *w* and 1		
Triangle with base *b* and height *h*		

Two Characteristics of Direct Variation are:

1. _____

2. _____

Lines:

The *slope-intercept form* of a line is:

where b _____ and m _____

The *point-slope form* of a line is:

where h _____ k _____ and m = _____

The general form of a line is:

Linear Systems of Equations

The two methods for solving a system of linear equations are:

1. _____ and
2. _____

Absolute Value

Absolute value has to do with _____

Notes on solving absolute value equations:	Notes on solving absolute value inequalities:

Quadratic Equations:

The four ways we have of solving quadratic equations are:

- _____
- _____
- _____
- _____

Radicals

"Like radicals" are radicals with the same:

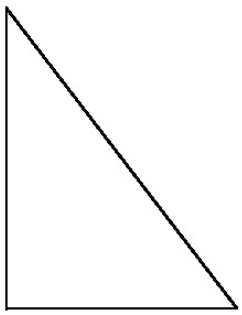
1. _____
2. _____

"Rationalizing a denominator" means

The Pythagorean Theorem states: In a right triangle,_____

_____

Label the right triangle below to match your definition above.



Distance

You find the distance between two points by:

Triangles

Special right triangles:

Type	**Ratio/Notes/Drawing**
45-45-90	
30-60-90	

Distance, Work, Rate, Time

Sample chart: What kinds of things do you put in each box and how do they relate to each other?

Exponents

Exponent rule	Explanation/Proof/Reason	Exponent rule	Explanation/Proof/Reason

Mathematics 1

1. Assuming that light travels at about 186 thousand miles per second, and the Sun is about 93 million miles from the Earth, how much time does light take to reach the Earth from the Sun?

2. Assuming that it takes 1.25 seconds for light to travel from the Moon to the Earth, how many miles away is the Moon?

3. Many major-league baseball pitchers can throw the ball at 90 miles per hour. At that speed, how long does it take a pitch to travel from the pitcher's mound to home plate, a distance of 60 feet 6 inches? Give your answer to the nearest hundredth of a second. There are 5280 feet in a mile and 12 inches in a foot.

4. You have perhaps heard the saying, "A journey of 1000 miles begins with a single step." How many steps would you take to finish a journey of 1000 miles? What information do you need in order to answer this question? Find a reasonable answer. What would your answer be if the journey were 1000 kilometers?

5. In an offshore pipeline, a cylindrical mechanism called a "pig" is run through the pipes periodically to clean them. These pigs travel at 2 feet per second. What is this speed, expressed in miles per hour?

6. A class sponsors a benefit concert and prices the tickets at \$8 each. Jordan sells 12 tickets, Andy 16, Morgan 17, and Pat 13. Compute the total *revenue* brought in by these four people. Notice that there are two ways to do the calculation.

7. Kelly telephoned Brook about a homework problem. Kelly said, "Four plus three times two is 14, isn't it?" Brook replied, "No, it's 10." Did someone make a mistake? Can you explain where these two answers came from?

8. It is customary in algebra to omit multiplication symbols whenever possible. For example, $11x$ means the same thing as $11 \cdot x$. If the multiplication dot were simply removed, which of the following expressions would continue to have the same meaning?

(a) $4 \cdot \frac{1}{3}$ **(b)** $1.08 \cdot p$ **(c)** $24 \cdot 52$ **(d)** $5 \cdot (2 + x)$

9. Wes bought some school supplies at an outlet store in Maine, a state that in 2016 had a 6.5% sales tax. Including the sales tax, how much did Wes pay for 2 jackets priced at \$49.95 and 3 pair of pants priced at \$17.50?

10. (Continuation) A familiar feature of arithmetic is that multiplication *distributes* over addition. Written in algebraic code, this property looks like $a(b + c) = ab + ac$. Because of this property, there are two equivalent methods that can be used to compute the answer in the problem above. Explain, using words and complete sentences.

11. Woolworth's had a going-out-of-business sale. The price of a telephone before the sale was \$39.98. What was the price of the telephone after a 30% discount? If the sale price of the same telephone had been \$23.99, what would the (percentage) discount have been?

12. Kai took a trip from Stratford to Paris in 2013, and needed to exchange 500 British pounds for euros. The exchange rate was 1 pound to 1.23 euros. How many euros did Kai receive in this exchange?

13. On a road map of Uganda, the scale is $1 : 1\,500\,000$. The distance on the map from Kampala to Ft. Portal is 17 cm. What is the real world distance in km between these two cities?

14. Choose any number. Double it. Subtract six and add the original number. Now divide by three. Repeat this process with other numbers, until a pattern develops. By using a *variable* such as x in place of your number, show that the pattern does not depend on which number you choose initially.

15. Compute each of the following. For some of these, there are two ways to compute the result. Explain.

(a) $3(2+3+5)$ **(b)** $\frac{1}{3}(9+6-3)$ **(c)** $(9+6-3)\div 3$ **(d)** $3(2\cdot 3\cdot 5)$ **(e)** $3\div(9+6-3)$

16. A blueprint of a building gives a scale of 1 inch = 8 feet. If the blueprint shows the building sitting on a rectangle with dimensions 16 inches by 25 inches, what is the actual area of the rectangle on which the building sits? Express your answer in square feet.

17. Simplify $x+2+x+2+x+2+x+2+x+2+x+2+x+2+x+2+x+2$.

18. Without resorting to decimals, find equivalences among the following nine expressions:

$$\frac{2\cdot 3}{5} \quad \left(\frac{3}{5}\right)\cdot 2 \quad 3\cdot\left(\frac{2}{5}\right) \quad \left(\frac{2}{5}\right)\left(\frac{3}{3}\right) \quad \left(\frac{5}{3}\right)2 \quad 2\div\frac{5}{3} \quad \frac{2}{5} \quad \frac{5}{3}\div\frac{1}{2} \quad \frac{3}{5/2}$$

19. What is the value of $3 + (-3)$? What is the value of $(-10.4) + 10.4$? These pairs of numbers are called *opposites*. What is the sum of a number and its opposite? Does every number have an opposite? State the opposite of:

(a) -2.341 **(b)** $1/3$ **(c)** x **(d)** $x+2$ **(e)** $x-2$

20. As shown on the *number line* below, k represents an unknown number between 2 and 3. Plot each of the following, extending the line if necessary:

(a) $k+3$ **(b)** $k-2$ **(c)** $-k$ **(d)** $6-k$

21. On a map of South Asia, Nepal looks approximately like a rectangle measuring 8.3 cm by 2.0 cm. The map scale is listed as $1 : 9\,485\,000$. What is the approximate real world area of Nepal in square kilometers?

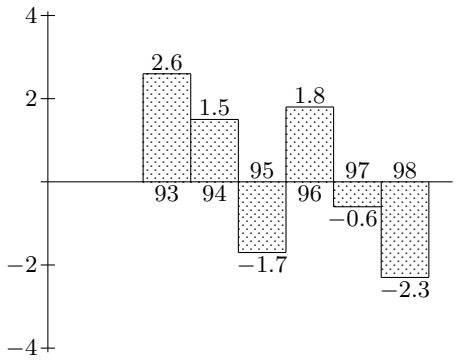
22. The area of the surface of a sphere is described by the formula $S = 4\pi r^2$, where r is the radius of the sphere. The Earth has a radius of 3960 miles and dry land forms approximately 29.2% of the Earth's surface. What is the area of the dry land on Earth? What is the surface area of the Earth's water?

23. At 186 282 miles per second, how far does light travel in a year? Give your answer in miles, but use *scientific notation*, which expresses a number like 93 400 000 as 9.34×10^7 (which might appear on your calculator as 9.34 E7 instead). A year is approximately 365.25 days. The answer to this question is called a *light-year* by astronomers, who use it to measure huge distances. Other than the Sun, the star nearest the Earth is Proxima Centauri, a mere 4.2 light-years away.

24. Before you are able to take a bite of your new chocolate bar, a friend comes along and takes 1/4 of the bar. Then another friend comes along and you give this person 1/3 of what you have left. Make a diagram that shows the part of the bar left for you to eat.

25. Later you have another chocolate bar. This time, after you give away 1/3 of the bar, a friend breaks off 3/4 of the remaining piece. What part of the original chocolate bar do you have left? Answer this question by drawing a diagram.

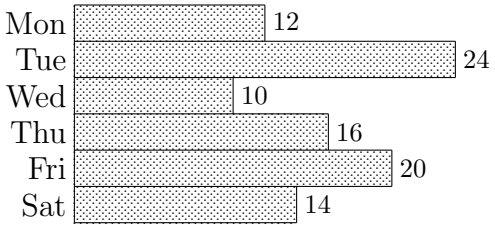
26. *Profits* for the Whirligig Sports Equipment Com-pany for six fiscal years, from 1993 through 1998, are graphed at right. The vertical scale is in millions of dol-lars.
Describe the change in profit from
(a) 1993 to 1994;
(b) 1994 to 1995;
(c) 1997 to 1998.
During these six years, did the company make an overall profit or sustain an overall *loss*? What was the net change?



27. The temperature outside is dropping at 3 degrees per hour. Given that the temperature at noon was 0°, what was the temperature at 1 pm? at 2 pm? at 3 pm? at 6 pm? What was the temperature t hours after noon?

28. One year, there were 528 students at the Academy, 63 of whom lived in Morewood Hall. To the nearest tenth of a percent, what part of the student population lived in Morewood that year?

29. Jess and Taylor go into the cookie-making business. The chart shows how many dozens of cookies were sold (at $3.50 per dozen) during the first six days of business.
(a) What was their total *income* during those six days?
(b) Which had more income, the first three days or the last three days?



(c) What was the percentage decrease in sales from Tuesday to Wednesday? What was the percentage increase in sales from Wednesday to Thursday?
(d) Thursday's sales were what percent of the total sales?
(e) On average, how many dozens of cookies did Jess and Taylor sell each day?

30. Here is another number puzzle: Pick a number, add 5 and multiply the result by 4. Add another 5 and multiply the result by 4 again. Subtract 100 from your result and divide your answer by 8. How does your answer compare to the original number? You may need to do a couple of examples like this until you see the pattern. Use a variable for the chosen number and show how the pattern holds for any number.

31. Jess takes a board that is 50 inches long and cuts it into two pieces, one of which is 16 inches longer than the other. How long is each piece?

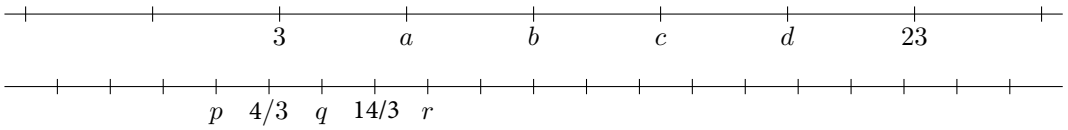
32. Consider the sequence of numbers $2, 5, 8, 11, 14, \ldots$, in which each number is three more than its predecessor.
(a) Find the next three numbers in the sequence.
(b) Find the 100^{th} number in the sequence.
(c) Using the variable n to represent the position of a number in the sequence, write an expression that allows you to calculate the n^{th} number. The 200^{th} number in the sequence is 599. Verify that your expression works by evaluating it with n equal to 200.

33. A group of ten people were planning to contribute equal amounts of money to buy some pizza. After the pizza was ordered, one person left. Each of the other nine people had to pay 60 cents extra as a result. How much was the total bill?

34. Let k represent some unknown non-integer number greater than 1. Mark your choice on a number line. Then locate each of the following:
(a) $-k$ **(b)** $k+2$ **(c)** $k-3$ **(d)** $\sqrt{k}$ **(e)** k^2

35. For each of the following, find the value of x that makes the *equation* true. The usual way of wording this instruction is *solve for* x:
(a) $2x = 12$ **(b)** $-3x = 12$ **(c)** $ax = b$

36. On each of the following number lines, all of the labeled points are evenly spaced. Find *coordinates* for the seven points designated by the letters.



37. Let k represent some unknown positive non-integer number greater than 1. Mark your choice on a number line. Then locate each of the following:
(a) $-k$ **(b)** $k+2$ **(c)** $k-3$ **(d)** $\sqrt{k}$ **(e)** k^2

38. (Continuation) What changed for $\sqrt{k}$ and k^2 when you chose k between 0 and 1 compared to $k > 1$?

39. Write each of the following as a product of x and another quantity:
(a) $16x + 7x$ **(b)** $12x - 6x$ **(c)** $ax + bx$ **(d)** $px - qx$

40. Solve each of the following equations for x:
(a) $16x + 7x = 46$ **(b)** $12x - 6x = 3$ **(c)** $ax + bx = 10$ **(d)** $px - qx = r$

41. The volume of a pyramid is one third its height times the area of its base. The Louvre pyramid has a height of 20.6 meters and a square base with sides of 35 meters. Find its volume, rounded to the nearest tenth. Include units in your answer.

42. You have seen that multiplication distributes over addition. Does multiplication distribute over subtraction? Does multiplication distribute over multiplication? Does multiplication distribute over division? Use examples to illustrate your answers.

43. Consider the sequence of numbers $50, 48, 46, 44, \ldots$
(a) What are the next three numbers in this sequence?
(b) Using n to represent the position of a number in this sequence, write an expression that allows you to calculate the n^{th} number.
(c) Use your expression to find the 100^{th} number in this sequence.

44. Simplify each of the following:
(a) the sum of $6x + 2$ and $-8x + 5$;
(b) the result of subtracting $5x - 17$ from $8x + 12$;
(c) the product of $7x$ and $4x - 9$.

45. Solve $\frac{2}{3}(3x + 14) = 7x + 6$, by first multiplying both sides of the equation by 3, before applying the distributive property.

46. In each of the following, use appropriate algebraic operations to remove the parentheses and combine *like terms*. Leave your answers in a simple form.
(a) $x(2x) + 2(x + 5)$ **(b)** $2x(5x - 2) + 3(6x + 7)$ **(c)** $5m(3m - 2n) + 4n(3m - 2n)$

47. Find *whole numbers* m and n that fit the equation $3m + 6n = 87$. Is it possible to find whole numbers m and n that fit the equation $3m + 6n = 95$? If so, find an example. If not, explain why not.

48. If m and n stand for integers, then $2m$ and $2n$ stand for even integers. Explain. Use the distributive property to show that the sum of any two even numbers is even.

49. (Continuation) Show that the sum of any two odd numbers is even.

50. Solve $9x + 2 = \frac{3}{4}(2x + 11)$.

51. Simplify the expression $k - 2(k - (2 - k)) - 2$ as much as possible. Your final answer should not use parentheses.

52. Solve the following equation for *h*: $d=ht^2+h$

53. Last year the price of a gizmo was \$240.
(a) This year the price increased to \$260. By what percent did the price increase?
(b) If the price next year were 5% more than this year's price, what would that price be?
(c) If the price dropped 5% the year after that, show that the price would not return to \$260. Explain the apparent paradox.

54. Alex is driving at a constant rate of 50 miles per hour. How far will he travel in:
(a) 2 hours (b) 4.3 hours (c) 20 minutes

55. Which number is closer to zero, $\frac{4}{5}$ or $-\frac{5}{4}$? Which is smaller?

56. Several Form III students were meeting in a room. After 45 of them left, the room was 5/8 as full as it was initially. How many preps were in the room at the start of the meeting?

57. Suppose you are a detective assigned to a robbery case. In this case, the robbery occurred at 2:30 PM. You have a witness who saw the suspect at a gas station 12 miles from the robbery site at 2:48 PM. The suspect claims innocence, arguing that it would have been impossible to get to the gas station in that amount of time. Do you agree? Support your answer.

58. Often it is necessary to rearrange an equation so that one variable is expressed in terms of others. For example, the equation $D = 3t$ expresses D in terms of t. To express t in terms of D, divide both sides of this equation by 3 to obtain $D/3 = t$.
(a) Solve the equation $C = 2\pi r$ for r in terms of C.
(b) Solve the equation $p = 2w + 2h$ for w in terms of p and h.
(c) Solve the equation $3x - 2y = 6$ for y in terms of x.

59. On a number line, what number is halfway between **(a)** -4 and 11? **(b)** m and n?

60. Coffee beans lose 12.5% of their weight during roasting. In order to obtain 252 kg of roasted coffee beans, how many kg of unroasted beans must be used?

61. The product of two negative numbers is always a positive number. How would you explain this rule to a classmate who does not understand why the product of two negative numbers must be positive?

62. Solve the following equation for h: $S = lw + 2lh + 2wh$

63. Temperature is measured in both Celsius and Fahrenheit degrees. These two systems are related: the *Fahrenheit* temperature is obtained by adding 32 to 9/5 of the *Celsius* temperature. In the following questions, let C represent the Celsius temperature and F the Fahrenheit temperature.
(a) Write an equation that expresses F in terms of C.
(b) Use this equation to find the value of F that corresponds to $C = 20$.
(c) On the Celsius scale, water freezes at 0° and boils at 100°. Use your formula to find the corresponding temperatures on the Fahrenheit scale. Do you recognize your answers?
(d) A quick way to get an approximate Fahrenheit temperature from a Celsius temperature is to double the Celsius temperature and add 30. Explain why this is a good approximation. Convert 23° Celsius the quick way. What was the difference between your answer and the correct value? For what Celsius temperature does the quick way give the correct value?

64. You measure your stride and find it to be 27 inches. If you were to walk to Millvale, a town 4.5 miles north of Pittsburgh, how many steps would you have to take? Remember that there are 12 inches in a foot, and 5280 feet in a mile.

65. The Millers make a 70-mile Thanksgiving trip to visit their grandparents. Pat Miller believes in driving at a steady rate of 50 miles per hour.
(a) How much time will it take Pat to make the trip?
(b) How many miles will the Millers travel in 18 minutes?
(c) Write an expression for the number of miles they will cover in t minutes of driving.**(d)** After t minutes of driving, how many miles remain to be covered?

66. The length of a certain rectangle exceeds its width by exactly 8 cm, and the *perimeter* of the rectangle is 66 cm. What is the width of the rectangle? Although you may be able to solve this problem using a method of your own, try the following approach, which starts by guessing the width of the rectangle. Study the first row of the table below, which is based on a 10-cm guess for the width. Then make your own guess and use it to fill in the next row of the table. If you have not guessed the correct width, use another row of the table and try again.

guess	*length*	*perimeter*	*target*	*check?*
10	$10 + 8 = 18$	$2(10) + 2(18) = 56$	66	no

Now use the experience gained by filling in the table to write an equation for the problem: Write w in the *guess* column, fill in the length and perimeter entries in terms of w, and set your expression for the perimeter equal to the target perimeter. Solve the resulting equation. This approach to creating equations is called the *guess-and-check* method.

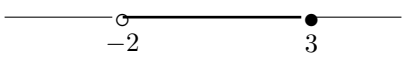
67. Solve for x: **(a)** $3x - 4 = 11$ **(b)** $-2x + 5 = -1$ **(c)** $7x + 4 = 12$ **(d)** $ax + b = c$

68. Recall the rule of thumb for estimating the conversion of degrees Celsius to degrees Fahrenheit. Approximately how many degrees Fahrenheit is 30 degrees Celsius?

69. *Number-line graphs.* Observe the following conventions, which may already be familiar:

- To indicate an *interval* on the number line, thicken that part of the number line.
- To indicate that an endpoint of an interval is included, place a solid dot on the number.
- To indicate that an endpoint is not included, place an open circle on the number.

For example, the diagram illustrates those numbers that are greater than -2 and less than or equal to 3.



Draw a number line for each of the following and indicate the numbers described:
(a) All numbers that are exactly two units from 5.
(b) All numbers that are more than two units from 5.
(c) All numbers that are greater than -1 and less than or equal to 7.
(d) All numbers that are less than four units from zero.

70. Percent practice: **(a)** 25% of 200 is what number? **(b)** 200 is 25% of what number? **(c)** Express 2/25 as a decimal; as a percent. **(d)** Express 24% as a decimal; as a fraction.

71. It is sometimes necessary to write fractions with variables in the denominator. Rewrite by hand each of the following as a single fraction. This is called *combining over a common denominator*.
(a) $\frac{3}{a}+\frac{7}{a}$ **(b)** $\frac{3}{a}+\frac{7}{2a}$ **(c)** $\frac{3}{a}+\frac{7}{b}$ **(d)** $3+\frac{7}{b}$

72. Ryan earns x dollars every seven days. Write an expression for how much Ryan earns in one day. Ryan's spouse Lee is paid twice as much as Ryan. Write an expression for how much Lee earns in one day. Write an expression for their combined daily earnings.

73. Solve for x: **(a)** $2(x-3)=4$ **(b)** $-3(2x+1)=5$ **(c)** $a(bx+c)=d$

74. Boarding student Avery just bought 10 gallons of gasoline, the amount of fuel used for the last 355 miles of driving. Being a curious sort, Avery wondered how much fuel had been used in city driving (which takes one gallon for every 25 miles) and how much had been used in freeway driving (which takes one gallon for each 40 miles). Avery started by guessing 6 gallons for the city driving, then completed the first row of the guess-and-check table below. Notice the failed check. Make your own guess and use it to fill in the next row of the table.

city g	*freeway g*	*city mi*	*freeway mi*	*total mi*	*target*	*check*
6	$10-6=4$	$6(25)=150$	$4(40)=160$	$150+160=310$	355	no

Now write c in the city-gallon column, fill in the remaining entries in terms of c, and set your expression for the total mileage equal to the target mileage. Solve the resulting equation.

75. On a number line, graph all numbers that are closer to 5 than they are to 8.

76. Remy walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was at only 3 mph, however. Express as a fraction
(a) the time Remy spent walking home;
(b) the total time Remy spent walking.

77. The sum of four *consecutive integers* is 2174. What are the integers?

78. (Continuation) The smallest of four consecutive integers is n. What expression represents the next larger integer? Write an expression for the sum of four consecutive integers, the smallest of which is n. Write an equation that states that the sum of four consecutive integers is s. Solve the equation for n in terms of s. Check that your answer to the previous question satisfies this equation by considering the case $s=2174$.

79. Solve for x: **(a)** $2(x-1)=3(x+2)$ **(b)** $-4(2x-2)=3(x+1)$

80. Write an expression that represents the number that

(a) is 7 more than x; **(b)** is 7 less than x; **(c)** is x more than 7;
(d) exceeds x by 7; **(e)** is x less than 7; **(f)** exceeds 7 by x.

81. There are 396 people in a theater. If the *ratio* of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the theater?

82. On a number line, graph a number that is twice as far from 5 as it is from 8. How many such numbers are there?

83. Intervals on a number line are often described using the symbols $<$ ("less than"), $>$ ("greater than"), $\leq$ ("less than or equal to"), and $\geq$ ("greater than or equal to"). As you graph the following inequalities, remember the *endpoint convention* regarding the use of the dot $\bullet$ and the circle $\circ$ for included and excluded endpoints, respectively:
(a) $x < 5$ **(b)** $x \geq -6$ **(c)** $-12 \geq x$ **(d)** $4 < x < 8$ **(e)** $x < -3$ or $7 \leq x$

84. Solve the equation $A = P + Prt$ for r. Solve the equation $A = P + Prt$ for P.

85. Using a number line, describe the location of $\dfrac{x+y}{2}$ in relation to the locations of x and y. Is your answer affected by knowing whether x and y are positive or not?

86. Draw a number line for each of the following and indicate the numbers described (if any):
(a) The numbers that are less than 2 or greater than 4.
(b) The numbers that are less than 2 and greater than 4.

87. Find the smallest positive integer divisible by every positive integer less than or equal to 5.

88. The indicator on the oil tank in my home indicated that the tank was one-eighth full. After a truck delivered 240 gallons of oil, the indicator showed that the tank was half full. What is the capacity of the oil tank, in gallons?

89. A team has started its season badly, winning 1 game, losing 6, and tying none. The team will play a total of 25 games this season.
(a) What percentage of the seven games played so far have been wins?
(b) Starting with its current record of 1 win and 6 losses, what will the cumulative winning percentage be if the team wins the next 4 games in a row?
(c) Starting with its current record of 1 win and 6 losses, how many games in a row must the team win in order for its cumulative winning percentage to reach at least 60%?
(d) Suppose that the team wins ten of its remaining 18 games. What is its final winning percentage?
(e) How many of the remaining 18 games does the team need to win so that its final winning percentage is at least 60%? Is it possible for the team to have a final winning percentage of 80%? Explain.

90. Graph on a number line the intervals described below:
(a) All numbers that are greater than 1 or less than -3.
(b) All numbers that are greater than -5 and less than or equal to 4.
(c) All numbers whose squares are greater than or equal to 1.

91. Use *inequality* notation to represent the intervals described below.
(a) All numbers that are greater than 1 or less than -3.
(b) All numbers that are greater than -5 and less than or equal to 4.
(c) Every number whose square is greater than or equal to 1.

92. *Combine over a common denominator* each of the expressions below. Express each of your answers in *lowest terms*.

(a) $\frac{27}{5} + \frac{3y}{4}$ **(b)** $\frac{4m}{5} - \frac{2}{3}$ **(c)** $2 + \frac{x}{3}$ **(d)** $\frac{x}{2} + \frac{2x}{3} - \frac{3x}{4}$

93. Solve the following for x:

(a) $4 - (x + 3) = 8 - 5(2x - 3)$ **(b)** $x - 2(3 - x) = 2x + 3(1 - x)$

94. *Guessing birthdays*. Pat is working a number trick on Kim, whose birthday is the 29^{th} of February. The table below shows the sequence of questions that Pat asks, as well as the calculations that Kim makes in response. Another column is provided for the algebra you are going to do to solve the trick. Use the letters m and d for month and day.

Instruction	*Kim*	*Algebra*
Write the number of your birthmonth	2	m
Multiply by 5	10	
Add 7	17	
Multiply by 4	68	
Add 13	81	
Multiply by 5	405	
Add the day of the month of your birthday	434	

After hearing the result of the last calculation, Pat can do a simple mental calculation and then state Kim's birthday. Explain how. To test your understanding of this trick, try it on someone whose birthday is unknown to you.

96. Do each of the following:
(a) On a number line, graph $x < 2$.
(b) On the same line, graph $x - 5 < 2$; how does this new graph relate to the graph of $x < 2$?
(c) On the same line, graph $x + 3 < 2$; how does this new graph relate to the graph of $x < 2$?

97. Solve the following for x: **(a)** $\frac{x}{2} + \frac{x}{5} = 6$ **(b)** $\frac{x}{3} + \frac{x+1}{6} = 4$

98. Using the variable x to represent a certain number, write an algebraic expression to represent each of the following:
(a) Eleven more than one third of the number.
(b) Three times the difference between the number and twelve.
(c) Two times the number, decreased by the sum of the number squared and two.

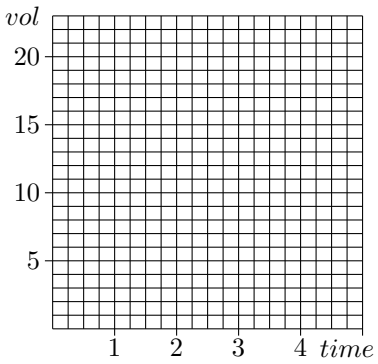
99. By hand, combine the following over a common denominator.

(a) $\frac{1}{4} + \frac{1}{5}$ **(b)** $\frac{1}{10} + \frac{1}{11}$ **(c)** $\frac{1}{x} + \frac{1}{x+1}$

What happens to the expression in (c) when $x = 4$ and when $x = 10$? Hopefully you have the same expressions as in parts (a) and (b), respectively.

100. It takes one minute to fill a four-gallon container at the Exeter spring. How long does it take to fill a six-gallon container? Fill in the missing entries in the table below, and plot points on the grid at right.

time	1			2		3		4		5
volume	4	5	6		11		14		19	



Notice that it makes sense to connect the dots you plotted (thereby forming a continuous pattern).

101. Ryan took 25 minutes to type the final draft of a 1200-word English paper. How much time should Ryan expect to spend typing the final draft of a 4000-word History paper?

102. Which of the following seven expressions does not belong in the list?

$a-b+c$ $\quad c-b+a$ $\quad c-(b-a)$ $\quad -b+a+c$ $\quad a-(b-c)$ $\quad b-(c-a)$ $\quad a+c-b$

103. Last week, Chris bought a movie for \$10.80 during a 25%-off sale. The sale is now over. How much would the same movie cost today?

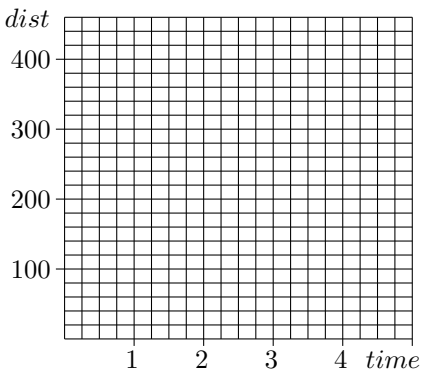
104. Forrest is illegally *texting* while driving along the freeway at 70 miles per hour. How many feet does the car travel during the 3-second interval when Forrest's eyes are not on the road?

105. The statement "x is between 13 and 23" defines an interval using two simultaneous inequalities: $13 < x$ *and* $x < 23$. The statement "x is not between 13 and 23" also uses two inequalities, but they are *non*-simultaneous: $x \le 13$ *or* $23 \le x$. Graph these two examples on a number line. Notice that there is a compact form $13 < x < 23$ for only one of them.

106. Tickets to a school play cost \$2 if bought in advance and \$3 at the door. By selling all 400 tickets, \$1030 was collected. Let x represent the number of tickets sold in advance. **(a)** In terms of x, how many tickets were sold at the door?
(b) In terms of x, how much money was taken in by the tickets sold at the door?

107. (Continuation) Write and solve an equation to find out how many tickets were sold in advance.

108. Crossing a long stretch of the Canadian plains, passenger trains maintain a steady speed of 80 mph. At that speed, what distance is covered in half an hour? How much time is needed to cover 200 miles? Fill in the missing entries in the table below, and plot points on the grid at right.



time	0	1/2		1	2		3		4	t
distance			60			200		300		

109. Problems Canadian plains describe *proportional relationships*, in which one quantity can be expressed as a fixed constant time another quantity. This constant is called a *proportionality constant*.
(a) Show that the volume and time in #100 are proportional by expressing the volume as a constant multiplied by the time;
(b) Show that the distance and time in #108 are proportional by finding a proportionality constant.

110. (Continuation) Which of the following describe quantities that are proportional to one another?
(a) The gallons of water in a tub and the number of minutes since the tap was opened.
(b) The height of a ball and the number of seconds since it was thrown.
(c) The length of a side of a square and the perimeter of the square.
(d) The length of a side of a square and the area of the square.
(e) The temperature outside dropping at 3 degrees per hour and the number of hours since noon, when it was 0 degrees.

111. (Continuation) Sketch graphs for each of the situations described above. Be sure to include meaningful descriptions and scales for each axis.

112. Remy walked to a friend's house, m miles away, at an average rate of 4 mph. The m-mile walk home was at only 3 mph. Remy spent 2 hours walking in all. Find the value of m.

113. The sides of a rectangle in the coordinate plane are parallel to the axes. Two of the vertices of the rectangle are $(3, -2)$ and $(-4, -7)$. Find coordinates for the other two vertices. Find the area of the rectangle.

114. The rectangle shown at right has been broken into four smaller rectangles. The areas of three of the smaller rectangles are shown in the diagram. Find the area of the fourth one.

234	312
270	

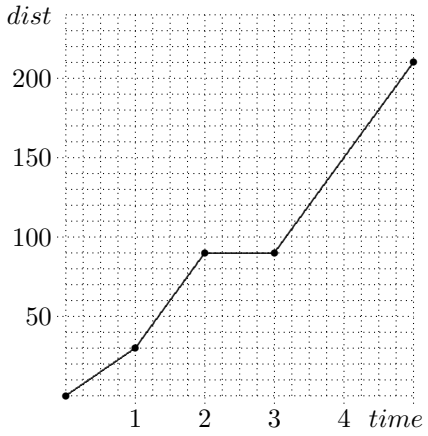
115. Tory goes shopping and buys pencils and notebooks. Pencils cost 29 cents each and notebooks cost \$2.59 each. Tory bought a total of 8 items. Let p represent the number of pencils bought and write expressions for each of the following in terms of p:
(a) the number of notebooks Tory buys,
(b) the amount Tory spends on pencils,
(c) the amount Tory spends on notebooks, and
(d) how much Tory spends all together.

116. (Continuation) If Tory's bill is \$9.22, how many pencils does Tory buy?

117. Chandler was given \$75 for a birthday present. This present, along with earnings from a summer job, is being set aside for a mountain bike. The job pays \$6 per hour, and the bike costs \$345. To be able to buy the bike, how many hours does Chandler need to work?

118. (Continuation) Let h be the number of hours that Chandler works. What quantity is represented by the expression $6h$? What quantity is represented by the expression $6h + 75$?
(a) Graph the solutions to the inequality $6h + 75 \geq 345$ on a number line.
(b) Graph the solutions to the inequality $6h + 75 < 345$ on a number line.
(c) What do the solutions to the inequality $6h + 75 \geq 345$ signify?

119. Sandy recently made a 210-mile car trip, starting from home at noon. The graph at right shows how Sandy's distance from home (measured in miles) *depends* on the number of hours after noon. Make up a story that accounts for the four distinct parts of the graph. In particular, identify the speed at which Sandy spent most of the afternoon driving.



120. If you bike 10 miles from *PEA* to the beach in 40 minutes, you will most likely not be traveling at a constant speed. But if you did, what speed would it be? This value is your *average speed* for the trip.

121. (Continuation) On the return trip from the beach, you pedal hard for the first ten minutes and cover 4 miles. Tired, you slow down and cover the last 6 miles in 36 minutes. What is your *average speed* for the return trip?

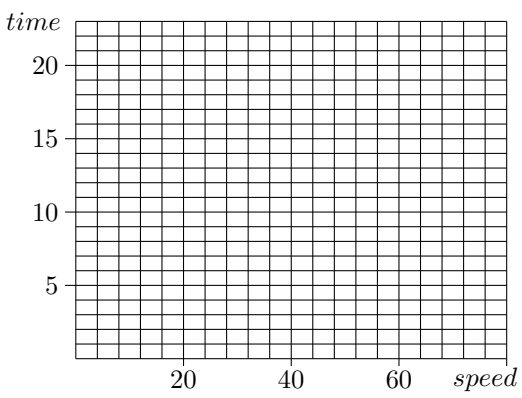
122. Solve the inequality $3 - x > 5$ using only the operations of addition and subtraction. Is $x = 0$ a solution to the inequality?

123. Alden paid to have some programs printed for the football game last weekend. The printing cost per program was 54 cents, and the plan was to sell them for 75 cents each. Poor weather kept many fans away from the game, however, so unlucky Alden was left with 100 unsold copies, and lost \$12 on the venture. How many programs did Alden have printed?

124. The Mount Major hike starts in Alton Bay, 716 feet above sea level. The summit is 1796 feet above sea level, and it takes about 45 minutes for a typical hiker to make the climb. Find the rate at which this hiker gains altitude, in feet per minute.

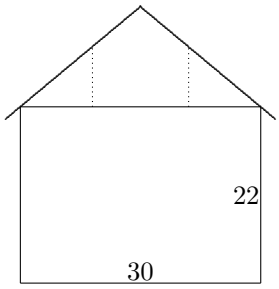
125. To do a college visit, Wes must make a 240-mile trip by car. The time in hours required to complete the trip depends on the speed at which Wes drives, of course, as the table below shows. Fill in the missing entries, and plot points on the grid provided. Are the quantities time and speed proportional? It makes sense to connect your plotted points with a *continuous* graph. Explain why.

speed	15	20	25			48		60		r
time		12		8	6		4.8		3	



126. Pat bought several pens at Walgreen's, for 60 cents each. Spending the same amount of money at the Bookstore, Pat then bought some pens that cost 80 cents each. In all, 42 pens were bought. How many pens did Pat buy at the bookstore?

127. Pittsburgh building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal run.



(a) Given the other dimensions in the diagram, will the builder be allowed to carry out this plan?

(b) Two vertical supports (shown dotted in the diagram) are to be placed 6 feet from the center of the building. How long should they be?

128. Suppose that n represents an integer. What expression represents the next larger integer? the previous integer? the sum of these three consecutive integers?

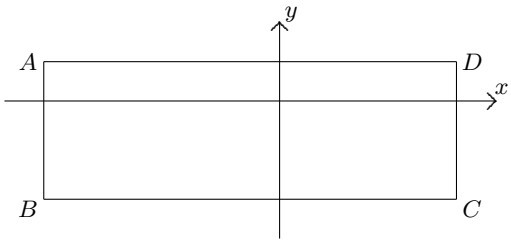
129. A small pool is 20 feet long, 12 feet wide and 4 feet deep. There are 7.5 gallons of water in every cubic foot. At the rate of 5 gallons per minute, how long will it take to fill this pool?

130. Jan walks 2 miles at a constant speed of 3 miles per hour and then runs 1 mile at a constant speed of 8 miles per hour.

(a) What is Jan's *average speed* for the entire trip?

(b) Is the average speed in part (a) equal to the average of Jan's two speeds?

131. The rectangle $ABCD$ shown at right has sides that are parallel to the coordinate axes. Side AD is three times the length of side AB and the perimeter of $ABCD$ is 56 units.



(a) Find the dimensions of $ABCD$.
(b) Given the information $D = (9, 2)$, find the coordinates for points A, B, and C.

132. A ladder is leaning against the side of a building. Each time I step from one rung to the next, my foot moves 6 inches closer to the building and 8 inches further from the ground. The base of the ladder is 9 ft from the wall. How far up the wall does the ladder reach?

133. Each step of the stairs leading from room 9 to room 107 in the Academy Building has a vertical *rise* of 7 inches and a horizontal *run* of 12 inches. Each step of the marble staircase leading to the Assembly Hall has a vertical rise of 5.5 inches and a horizontal run of 13 inches.
(a) Which flight of stairs do you think is steeper? Why?
(b) Calculate the ratio *rise/run* for each flight of stairs, and verify that the greater ratio belongs to the flight you thought to be steeper.

134. (Continuation) The *slope* of a line is a measure of how steep the line is. It is calculated by dividing the change in y-coordinates by the corresponding change in x-coordinates between two points on the line: $\text{slope} = \dfrac{\text{change in } y}{\text{change in } x}$. Calculate the slope of the line that goes through the two points $(1, 3)$ and $(7, 6)$. Calculate the slope of the line that goes through the two points $(0, 0)$ and $(9, 6)$. Which line is steeper?

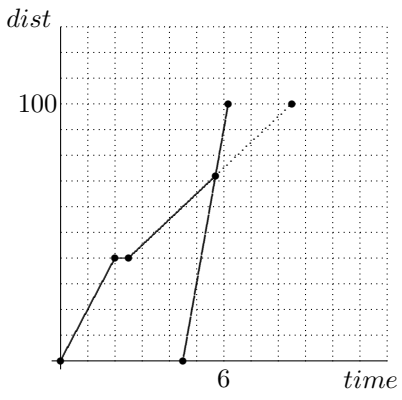
135. At noon one day, the Exeter River peaked at 11 feet above flood stage. It then began to recede, its depth dropping at 4 inches per hour.
(a) At 3:30 that afternoon, how many inches above flood stage was the river?
(b) Let t stand for the number of hours since noon, and h stand for the corresponding number of inches that the river was above flood stage. Make a table of values, and write an equation that expresses h in terms of t.
(c) Plot h *versus* t, putting t on the horizontal axis.
(d) For how many hours past noon was the river at least 36 inches above flood stage?

136. Solve the following for x: **(a)** $\dfrac{x}{5} - \dfrac{x+2}{10} = 1$ **(b)** $\dfrac{x}{6} + \dfrac{x-1}{4} = \dfrac{1}{2}$

137. (Continuation) Using rate, time, and distance as the context, write a problem that could be solved using the equation in part (b) of the previous problem.

138. Cass decided to sell game programs for the SSA-Fox Chapel field hockey game. The printing cost was 20 cents per program, with a selling price of 50 cents each. Cass sold all but 50 of the programs, and made a profit of \$65. How many programs were printed?Letting p represent the number of programs printed, set up an equation that describes this situation. Then solve your equation for p.

139. At noon one day, Allie left home to make a long bike ride to the family camp on Mud Lake, a distance of 100 km. Later in the day, the rest of the family packed some things into their van and drove to the lake along Allie's bike route. They overtook Allie after driving for 1.2 hrs, stopped long enough to put Allie and bicycle in the van, and continued to the camp. Refer to the graph as you answer the following questions about the day's events:



(a) Allie pedaled at two different rates during the biking part of the trip. What were they?
(b) After biking for a while, Allie stopped to take a rest. How far from home was Allie then? How long did Allie rest?
(c) How far from home was Allie when the family caught up?
(d) At what time did the family arrive at the camp?
(e) At what time would Allie have arrived, if left to bicycle all the way?
(f) What distance separated Allie and the rest of the family at 5 pm?

140. The perimeter of a rectangle is 100 and its length is x. What expression represents the width of the rectangle?

141. When a third of a number is subtracted from a half of the same number, 60 is the result. Find the number.

142. Draw the segment from (3, 1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

143. Eugene and Wes are solving the inequality $132 - 4x \leq 36$. Each begins by subtracting 132 from both sides to get $-4x \leq -96$, and then each divides both sides by -4. Eugene gets $x \leq 24$ and Wes gets $x \geq 24$, however. Show who is correct by substituting $x = 0$ into both the original inequality and the answer.

144. (Continuation) Cameron now suggests that the problem could have been done by solving the equation $132 - 4x = 36$ first. Complete the reasoning behind this strategy.

145. (Continuation) Deniz remarks, "A tricky thing about inequalities is when you try to multiply them or divide them by negative numbers, but you can avoid this altogether. Cameron just told us one way to avoid it, and there is another way, too." Explain.

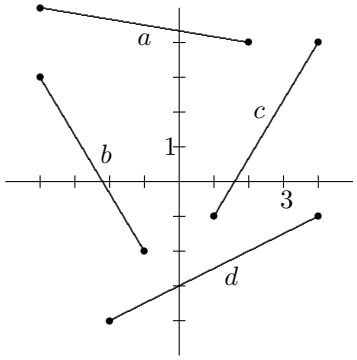
146. Solve the following inequality for x: $2(1 - 3x) - (x - 5) > 1$

147. Golf balls costs $0.90 each at Yinz's Club, which as an annual $25 membership fee. At Gene's Sporting Goods, the price is $1.35 per ball for the same brand. What factor determines were you shop?

148. Each beat of your heart pumps approximately 0.06 liter of blood.
(a) If your heart beats 50 times, how much blood is pumped?
(b) How many beats does it take for your heart to pump 0.48 liters?

149. (Continuation) Proportional relationships can be written in the form $y = kx$, and it is customary to say that *y depends on x*.
(a) Find an equation that shows how the volume pumped, V, depends on the number of beats, n.
(b) Graph this equation, using an appropriate scale, and calculate its slope.
(c) What does the slope represent in this context?
(d) Is this a continuous graph?

150. Estimate the slopes of all the segments in the diagram. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.



151. Find the slope of the line containing the points (4, 7) and (6, 11). Find coordinates for another point that lies on the same line and be prepared to discuss the method you used to find them.

152. To earn Hall of Fame distinction at SSA, a girl on the cross-country team must run the 5-km course in less than 20 minutes. What is the average speed of a 20-minute runner, in km per hour? in meters per second? Express your answers to two decimal places.

153. (Continuation) The *proportion* $\frac{5}{20} = \frac{x}{60}$ is helpful for the previous question. Explain this proportion, and assign units to all four of its members.

154. Which is greater, 73 percent of 87, or 87 percent of 73?

155. Corey deposits \$300 in a bank that pays 4% annual interest. How much interest does Corey earn in one year? What would the interest be if the rate were 6%?

156. The stretch of a spring is proportional to the weight attached to the spring. If a weight of 20 ounces stretches a spring 5 cm, what weight would stretch the spring 8 cm?

157. (Continuation) Recall that when two values are proportional, their relationship can be modeled by an equation in the form y = kx, and it is customary to say that y depends on x. Find an equation that shows how the stretch of the spring, d, depends on the weight, w. Graph your equation. What does the slope of the line represent in this context?

158. Alex was hired to unpack and clean 576 very small items of glassware, at five cents per piece successfully unpacked. For every item broken during the process, however, Alex had to pay \$1.98. At the end of the job, Alex received \$22.71. How many items did Alex break?

159. Each of the data sets at right represents points on a line. In which table is one variable proportional to the other? Why does the other table not represent a proportional relationship? Fill in the missing entry in each table.

x	y
0	4
4	10
10	19
16	

x	y
0	0
4	6
10	15
16	

160. (Continuation) Plot the data from the tables on the same set of axes and use a ruler to draw a line through each set of points. By looking at the graph, how could you recognize the proportional relationship? What similarities and differences are there between the two lines drawn?

161. Suppose that n represents a positive even integer. What expression represents the next even integer? Find three consecutive even integers whose sum is 204. Find two ways to do this.

162. A car and a small truck started out from Oakmont at 8:00 am. Their distances, in miles, from Oakmont, recorded at hourly intervals, are recorded in the tables at right. Plot this information on the same set of axes and draw two lines connecting the points in each set of data. What is the slope of each line? What is the meaning of these slopes in the context of this problem?

time	*car*	*truck*
8 : 00	0	0
9 : 00	52	46
10 : 00	104	92
11 : 00	156	138
12 : 00	208	184

163. (Continuation) Let t be the number of hours each vehicle has been traveling since 8:00 am (thus $t = 0$ means 8:00 am), and let d be the number of miles traveled after t hours. For each vehicle, write an equation relating d and t.

164. Day student Chris does a lot of babysitting. When parents drop off their children and Chris can supervise at home, the hourly rate is \$3. If Chris has to travel to the child's home, there is a fixed charge of \$5 for transportation in addition to the \$3 hourly rate.
(a) Graph $y = 3x$ and $y = 3x + 5$. What do these lines have to do with the babysitting context? What feature do they have in common? How do they differ?
(b) What does the graph of $y = 3x + 6$ look like? What change in the babysitting context does this line suggest?
(c) What does the graph of $y = 10x + 5$ look like? What change in the babysitting context does this line suggest?

165. If k stands for an integer, then is it possible for k2 + k to stand for an odd integer?Explain.

166. Can you think of a number k for which $k^2 < k$ is true? Graph all such numbers on a number line. Also describe them using words and using algebraic notation.

167. Solve $\frac{x}{4} + \frac{x+1}{3} \leq \frac{1}{2}$ and shade the solution interval on a number line.

168. By hand, find coordinates for the points where the line $3x + 2y = 12$ intersects the x-axis and the y-axis. These points are called the *x-intercept* and *y-intercept*, respectively. Use these points to make a quick sketch of the line.

169. How much time does it take for a jet to go 119 miles, if its speed is 420 mph? Be sure to specify the units for your answer.

170. Using a graphing tool, with the window set as $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$, graph the line $y = 0.5x + 3$. Notice that you can see both axis intercepts. Now graph $y = 0.1x + 18$ using the same window settings. What happens? Why? Calculate by hand the axis intercepts and adjust your window so that they are visible.

171. Drivers in distress near Blawnox have two towing services to choose from: Brook's Body Shop charges \$3 per mile for the towing, and a fixed \$25 charge regardless of the length of the tow. Morgan Motors charges a flat \$5 per mile. On the same system of axes, represent each of these choices by a *linear* graph that plots the cost of the tow versus the length of the tow. If you needed to be towed, which service would you call, and why?

172. Compare the graph of $y = 2x + 5$ with the graph of $y = 3x + 5$.
(a) Describe a context from which the equations might emerge.
(b) Linear equations that look like $y = mx+b$ are said to be in *slope-intercept form.* Explain. The terminology refers to which of the two intercepts?

173. Driving from Boston to New York one day, Sasha covered the 250 miles in five hours. Because of heavy traffic, the 250-mile return took six hours and fifteen minutes. Calculate average speeds for the trip *to* New York, the trip *from* New York, and the round trip. Explain why the terminology *average speed* is a bit misleading.

174. Find the value of x that makes $0.1x + 0.25(102 - x) = 17.10$ true.

175. So that it will be handy for paying tolls and parking meters, Lee puts pocket change (dimes and quarters only) into a cup attached to the dashboard. There are currently 102 coins in the cup, and their monetary value is \$17.10. How many of the coins are dimes?

176. Find all the values of x that make $0.1x + 0.25(102 - x) < 17.10$ true.

177. Without using parentheses, write an expression equivalent to $3(4(3x - 6) - 2(2x + 1))$.

178. Day student Morgan left home at 7:00 one morning, determined to make the ten-mile trip to PEA on bicycle for a change. Soon thereafter, Morgan's parent noticed forgotten math homework on the kitchen table, got into the family car, and tried to catch up with the forgetful child. Morgan had a fifteen-minute head start, and was pedaling at 12 mph, while the parent pursued at 30 mph. Was Morgan reunited with the homework before reaching PEA that day? If so, where? If not, at what time during first period (math, which starts at 8:00) was the homework delivered?

179. Farmer MacGregor needs to put a fence around a rectangular carrot patch that is one and a half times as long as it is wide. The project uses 110 feet of fencing. How wide is the garden? 2/3a

180. Combine over a common denominator: $\frac{1}{a} + \frac{2}{3a} + 3$

181. If 6% of x is the same as 5% of 120, then what is x?

182. Find the solution sets and graph them on a number line.
(a) $46 - 3(x + 10) = 5x + 20$
(b) $46 - 3(x + 10) < 5x + 20$
(c) $46 - 3(x + 10) > 5x + 20$

183. At 1 pm, you start out on your bike at 12 mph to meet a friend who lives 8 miles away. At the same time, the friend starts walking toward you at 4 mph. At what time will you meet your friend? How far will your friend have to walk?

184. The population of a small town increased by 25% two years ago and then decreased by by 25% last year. The population is now 4500 people. What was the population before the two changes?

185. The volume of a circular cylinder is given by the formula $V = \pi r^2 h$.
(a) To the nearest tenth of a cubic cm, find the volume of a cylinder that has a 15-cm radius and is 12-cm high.
(b) Solve the volume formula for h. Then, if the volume is 1000 cc and the radius is 10 cm, find h to the nearest tenth of a cm.

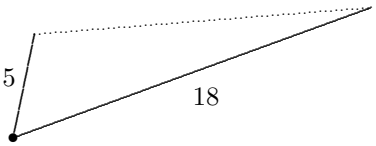
186. Combine over a common denominator:

(a) (3x/5) + (2x/3) (b) (4/3b) - (7/6b) + 2 (c) (6/5a) + (3/10b) - 1

187. Which of the following pairs of quantities vary directly?
(a) the circumference of a circle and the diameter of the circle;
(b) the distance traveled in two hours and the (average) rate of travel;
(c) the number of gallons of gasoline bought and the cost of the purchase;
(d) the area of a circle and the radius of the circle.

188. A jet, cruising at 26 400 feet, begins its descent into Logan Airport, which is 96 miles away. Another jet, cruising at 31 680 feet, is 120 miles from Logan when it begins its descent. Which of these two paths of descent is steeper? Is a linear *model* reasonable to use in this situation? Explain.

189. The diagram shows two steel rods hinged at one end. The other end is connected by a bungee cord (the dotted segment), whose unstretched length is 10 inches. The rods are 5 inches and 18 inches long. Use inequality symbols to describe all the possible lengths for the bungee cord, which stays attached at both ends while it is being stretched.



190. Find all values of x that make $-2(x-3) < 4$ true.

191. Solve the following inequalities and shade their solution intervals on a number line.

(a) $\frac{2x}{3} + \frac{3x+5}{2} \leq 5$ **(b)** $\frac{1}{2}(x-1) + 3 > \frac{1}{3}(2x+1) - 1$

192. At noon, my odometer read 6852 miles. At 3:30 pm, it read 7034 miles.
(a) What was my average rate of change during these three and a half hours?
(b) Let t represent the number of hours I have been driving since noon and y represent my odometer reading. Write an equation that relates y and t. Assume constant speed.
(c) Graph your equation.
(d) Show that the point (5,7112) is on your line, and then interpret this point in the context of this problem.

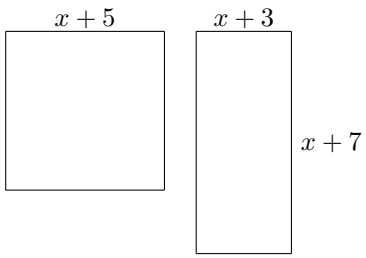
193. What is the slope between $(3, 7)$ and $(5, 4)$? $(5, 4)$ and $(3, 7)$? (a, b) and (c, d)? (c, d) and (a, b)?

194. On top of a fixed monthly charge, Avery's cellphone company adds a fee for each text message sent. Avery's November bill was \$50.79, which covered 104 text messages. The bill for October, which covered 83 text messages, was only \$46.59.
(a) What is the price of a text message?
(b) What is the fixed monthly charge?
(c) What would Avery be charged for a month that included 200 text messages?
(d) What would Avery be charged for a month that included m text messages?

195. (Continuation) A friend suggested that I change my cellphone company. This new company has a fixed monthly charge of \$39.99, but it charges only 12 cents for each text message. Is this a better deal than the one described in the previous problem? Give evidence.

196. How far apart on a number line are **(a)** 12 and 18? **(b)** 12 and -7? **(c)** -11 and -4?

197. For what values of x will the square and the rectangle shown at right have the same perimeter?



198. The point $(3, 2)$ is on the line $y = 2x + b$. Find the value of b. Graph the line.

199. Are $(2, 9)$ and $(-3, -6)$ both on the line $y = 4x + 6$? If not, find an equation for the line that does pass through both points.

200. After you graph the line $y = 4x + 6$, find
(a) the y-coordinate of the point on the line whose x-coordinate is 2;
(b) the x-coordinate of the point on the line whose y-coordinate is 2.

201. The *absolute value* of a non-zero number can be defined by $|a| = a$ or $-a$, whichever is positive. Note: $|0|$ is defined to be 0. Evaluate each of the following:
(a) $|4|$ **(b)** $|-3|$ **(c)** $|5-8|$ **(d)** $|-3-1|$ **(e)** $|-5|-|12|$

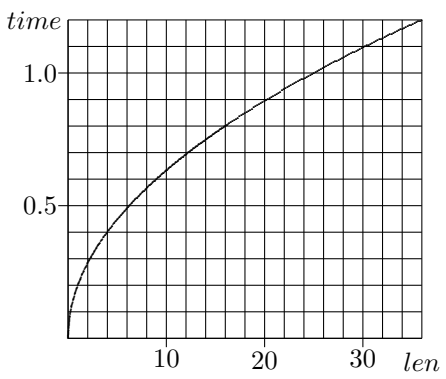
202. In each of the following, calculate the slope of the line determined by the given pair of points. Assuming that the first coordinate represents time measured in minutes and the second coordinate represents distance above sea level measured in feet, interpret your answers in a a rate-time-distance context.
(a) $(2, 8)$ and $(5, 17)$ **(b)** $(3.4, 6.8)$ and $(7.2, 8.7)$ **(c)** $\left(\frac{3}{2}, -\frac{3}{4}\right)$ and $\left(\frac{1}{4}, 2\right)$

203. If you double all the sides of a square, a larger square results. By what percentage has the perimeter increased? By what percentage has the area increased?

204. Combine the following fractions: $\dfrac{2}{3a} + \dfrac{1}{3} - \dfrac{4}{a}$.

205. Find the x-intercept and the y-intercept of the equation $y = -3/2\,x + 6$. Graph.

206. The graph shows how the length (measured in cm) of a pendulum is related to the time (measured in sec) needed for the pendulum to make one complete back-and-forth movement (which is called the *period*). Find the length of a pendulum that swings twice as often as a 30-cm pendulum.



207. A toy manufacturer is going to produce a new toy car. Each one costs \$3 to make, and the company will also have to spend \$200 to set up the machinery to make them. **(a)** What will it cost to produce the first hundred cars? the first n cars?
(b) The company sells the cars for \$4 each. Thus the company takes in \$400 by selling one hundred cars. How much money does the company take in by selling n cars?
(c) How many cars does the company need to make and sell in order to make a profit?

208. A cyclist rides 30 km at an average speed of 9 km/hr. At what rate must the cyclist cover the next 10 km in order to bring the overall average speed up to 10 km/hr.?

209. On a number line, what is the distance between 6 and -6? between 24 and 17? between 17 and 24? between t and 4? This last question is harder to answer because it depends on whether t is smaller than or greater than 4. Is the answer $t - 4$ or $4 - t$? This is an absolute value calculation: use absolute value signs to express the distance between t and 4. What is the distance between the numbers a and b on the number line? What is the relationship between $|p - q|$ and $|q - p|$?

210. Let $P = (x, y)$ and $Q = (1, 5)$. Write an equation that states that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y - 5 = 3(x - 1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

211. (Continuation) What do the lines $y = 3(x-1)+5$, $y = 2(x-1)+5$, and $y = -\frac{1}{2}(x-1)+5$ all have in common? How do they differ from each other?

212. Given that $48 \le n \le 1296$ and $24 \le d \le 36$, what are the largest and smallest values that the expression $\frac{n}{d}$ can possibly have? Write your answer $smallest \le \frac{n}{d} \le largest$.

213. Jess has 60 ounces of an alloy that is 40% gold. How many ounces of pure gold must be added to this alloy to create a new alloy that is 75% gold?

214. The table at right shows data that Morgan collected during a 10-mile bike ride that took 50 minutes. The cumulative distance (measured in miles) is tabled at ten-minute intervals.
(a) Make a *scatter plot* of this data. Why might you expect the data points to line up? Why do they not line up?
(b) Morgan's next bike ride lasted for 90 minutes. Estimate its length (in miles), and explain your method. What if the bike ride had lasted t minutes; what would its length be, in miles?

time	*dist*
0.0	0.0
10.0	2.3
20.0	4.4
30.0	5.7
40.0	8.2
50.0	10.0

215. Write an equation for the line that goes through the point (1,5) and that has slope $\frac{2}{3}$.

216. The equation $5x - 8y = 20$ expresses a linear relationship between x and y. The point (15, 7) is either on the graph of this line, above it, or below it. Which? How do you know?

217. On a number line, how far is each of the following numbers from zero?

(a) 45 **(b)** -7 **(c)** x **(d)** 0

218. Solve: **(a)** $A = \frac{1}{2}bh$ for b; **(b)** $A = 2\pi rh + \pi r^2$ for h.

219. On a number line, how far is each of the following numbers from 5?

(a) 17 **(b)** -4 **(c)** x

220. When asked to find the distance between two numbers on a number line, Jamie responded with the following answers. What two numbers do you think Jamie was talking about?

(a) $|9 - 4|$ **(b)** $|9 + 4|$ **(c)** $|x - 7|$ **(d)** $|3 - x|$ **(e)** $|x + 5|$ **(f)** $|x|$

221. To graph linear equations such as $3x + 5y = 30$, one can put the equation into slope-intercept form, but (unless the slope is needed) it is easier to find the x- and y-intercepts and use them to sketch the graph. Find the axis intercepts of each of the following and use them to draw the given line. An equation $ax + by = c$ is said to be in *standard form*.

(a) $20x + 50y = 1000$ **(b)** $4x - 3y = 72$

222. Find an equation for the line containing the points $(-3, 0)$ and $(0, 4)$.

223. Write an equation in point-slope form for
(a) the line that goes through $(2, 5)$ and $(6, -3)$;
(b) the line that goes through point (h, k) and that has slope m.

224. Casey goes for a bike ride from Arnold to Plum, while an odometer keeps a cumulative record of the number of miles traveled. The equation $m = 12t + 37$ describes the odometer reading m after t hours of riding. What is the meaning of 12 and 37 in the context of this trip?

225. Find an equation for the line that passes through the points $(4.1, 3.2)$ and $(2.3, 1.6)$.

226. Rearrange the eight words "between", "4", "the", "17", "is", "and", "x", and "distance" to form a sentence that is equivalent to the equation $|x-17| = 4$. By working with a number line, find the values of x that fit the equation.

227. As you know, temperatures can be measured by either Celsius or Fahrenheit units; $30°C$ is equivalent to $86°F$, $5°C$ is equivalent to $41°F$, and $-10°C$ is equivalent to $14°F$.
(a) Plot this data with C on the horizontal axis and F on the vertical axis.
(b) Verify that these three data points are *collinear*.
(c) Find a linear equation that relates C and F.
(d) Graph F versus C. In other words, graph the linear equation you just found.
(e) Graph C versus F. You will need to re-plot the data, with C on the vertical axis.
(f) On New Year's Day, I heard a weather report that said the temperature was a balmy $24°C$. Could this have happened? What is the corresponding Fahrenheit temperature?
(g) Water boils at $212°F$ and freezes at $32°F$ at sea level. Find the corresponding Celsius temperatures.
(h) Is it ever the case that the temperature in degrees Fahrenheit is the same as the temperature in degrees Celsius?

228. A recent poll about crime in schools claimed that 67% of Americans approved of a bill being debated in Congress. The poll also reported a 3% margin of error.
(a) Make a number-line graph of the possible approval ratings in this report.
(b) Explain why $|x - 0.67| \leq 0.03$ describes your graph.

229. Translate the sentence "the distance between x and 12 is 20" into an equation using algebraic symbols. What are the values of x being described?

230. The solution of $|x| = 6$ consists of the points 6 and -6. Show how to use a test point on the number line to solve and graph the inequality $|x| \leq 6$. Do the same for $|x| \geq 6$.

231. Translate "x is 12 units from 20" into an equation. What are the values of x being described?

231. Translate the sentence "x and y are twelve units apart" into algebraic code. Find a pair (x, y) that fits this description. How many pairs are there?

232. The equation $|x - 7| = 2$ is a translation of "the distance from x to 7 is 2."
(a) Translate $|x - 7| \leq 2$ into English, and graph its solutions on a number line.
(b) Convert "the distance from -5 to x is at most 3" into symbolic form, and solve it.

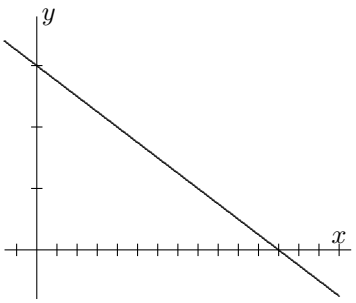
233. Verify that $(0, 4)$ is on the line $3x + 2y = 8$. Find another point on this line. Use these points to calculate the slope of the line. Is there another way to find the slope of the line?

234. Graph a horizontal line through the point $(3, 5)$. Choose another point on this line. What is the slope of this line? What is the y-intercept of this line? What is an equation for this line? Describe a context that could be *modeled* by this line.

235. Graph a vertical line through the point $(3, 5)$. Does this line have a slope or y-intercept? What is an equation for this line? Describe a context that could be modeled by this line.

236. After successfully solving an absolute-value problem, Ariel spilled Heath Bar CrunchRCall over the problem. All that can be read now is, "The distance between x and (mess of ice cream) is (another mess of ice cream)." Given that Ariel's answers are x = −3 and x = 7, reconstruct the missing parts of the problem.

237. The figure shows the graph of $20x+40y = 1200$. Find the x- and y-intercepts, the slope of the line, and the distances between tick marks on the axes. Duplicate this figure on a graphing tool. What window settings did you use?



238. Is the point (8.4, 23) below, on, or above the line $3x - y = 2$? Justify your answer.

239. An access ramp starts at ground level and rises 27 inches over a distance of 30 feet. What is the slope of this ramp?

240. Jay thinks that the inequality $k < 3$ implies the inequality $k^2 < 9$, but Val thinks otherwise. Who is right, and why?

241. The specifications for machining a piece of metal state that it must be 12 cm long, within a 0.01-cm tolerance. What is the longest the piece is allowed to be? What is the shortest? Using l to represent the length of the finished piece of metal, write an absolute-value inequality that states these conditions.

242. *O'Neil Cinema* charges \$10.50 for each adult and \$8.50 for each child. If the total amount in ticket revenue one evening was \$3136 and if there were 56 more children than adults, then how many children attended?

243. If $|x + 1| = 5$, then $x + 1$ can have two possible values, 5 and -5. This leads to two equations, $x + 1 = 5$ and $x + 1 = -5$. If $|2x - 7| = 5$, what possible values could the expression $2x - 7$ have? Write two equations using the expression $2x - 7$ and solve them.

244. Write two equations without absolute value symbols that, in combination, are equivalent to $|3x + 5| = 12$. Solve each of these two equations.

245. Given that $0.0001 \leq n \leq 0.01$ and $0.001 \leq d \leq 0.1$, what are the largest and smallest values that $\frac{n}{d}$ can possibly have? Write your answer $smallest \leq \frac{n}{d} \leq largest$.

246. A *lattice point* is defined as a point whose coordinates are integers. If $(-3, 5)$ and $(2, 1)$ are two points on a line, find three other lattice points on the same line.

247. The equation $13x+8y = 128$ expresses a linear relationship between x and y. The point $(5, 8)$ is on, or above, or below the linear graph. Which is it? How do you know?

248. Show that the equation $y = \frac{7}{3}x - \frac{11}{8}$ can be rewritten in the standard form $ax+by = c$, in which a, b, and c are all integers.

249. Fill in the blanks:
(a) The inequality $|x - 1.96| < 1.04$ is equivalent to "x is between _____ and_____."
(b) The inequality $|x - 2.45| \geq 4.50$ is equivalent to "x is not between _____ and _____."

250. Find the value for h for which the slope of the line through $(-5, 6)$ and $(h, 12)$ is $3/4$.

251. Solve the equation 0.05x + 0.25(30 – x) = 4.90. Invent a context for the equation.

252. When two quantities are proportional to one another, they are also said to *vary directly*. The data in each table fits a *direct variation*. Complete each table, write an equation to model its data, and sketch a graph.

(a)

x	2	4	6	
y	3	6		18

(b)

x	2	3		8
y	-8	-12	-20	

253. For each of the following equations, find the x-intercept and y-intercept. Then use them to calculate the slope of the line.

(a) $3x + y = 6$ **(b)** $x - 2y = 10$ **(c)** $4x - 5y = 20$ **(d)** $ax + by = c$

254. Blair's average on the first five in-class tests is 67. If this is not pulled up to at least a 70, Blair will not be allowed to watch any more *Netflix*. To avoid losing those privileges, what is the lowest score Blair can afford to make on the last in-class test? Assume that all tests carry equal weight.

255. Sketch the graphs of $y = 2x$, $y = 2x + 1$, and $y = 2x - 2$ all on the same coordinate-axis system. Find the slope of each line. How are the lines related to one another?

256. I have 120 cm of framing material to make a picture frame, which will be most pleasing to the eye if its height is 2/3 of its width. What dimensions should I use?

257. Describe the relationship between the following pairs of numbers:

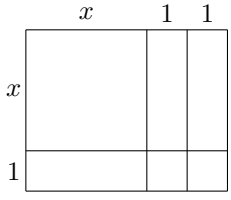
(a) $24 - 11$ and $11 - 24$ **(b)** $x - 7$ and $7 - x$ **(c)** $|x - 7|$ and $|7 - x|$

258. In each case, decide whether the three points given are collinear:

(a) $(-4, 8)$, $(0, 2)$, and $(2, -1)$ **(b)** $(350, 125)$, $(500, 300)$, and $(650, 550)$

259. A horse thief riding at 8 mph has a 32-mile head start. The posse in pursuit is riding at 10 mph. In how many hours will the thief be overtaken? [From *The New Arithmetic*, Sey-mour Eaton, 1885]

260. Write $(x + 1)(x + 2)$ without parentheses. Explain how the diagram at right illustrates this product.



261. Solve the equation $C = \frac{5}{9}(F - 32)$ for F.

262. The manager at Jen and Berry's Ice Cream Company estimates that the cost C (in dollars) of producing n quarts of ice cream in a given week is given by the equation $C = 560 + 1.20n$.
(a) During one week, the total cost of making ice cream was \$1070. How many quarts were made that week?
(b) Explain the meanings of the "560" and the "1.20" in the cost equation.

263. As mountain climbers know, the higher you go, the cooler the temperature gets. At noon on July 4th last summer, the temperature at the top of Mt. Washington — elevation 6288 feet — was $56°F$. The temperature at base camp in Pinkham Notch — elevation 2041 feet — was $87°F$. It was a clear, still day. At that moment, a group of hikers reached Tuckerman Junction — elevation 5376 feet. To the nearest degree, calculate the temperature the hikers were experiencing at that time and place. When you decided how to model this situation, what assumptions did you make?

264. Draw a line through the origin with a slope of 0.4. Draw a line through the point $(1, 2)$ with a slope of 0.4. How are these two lines related? What is the vertical distance between the two lines? Find an equation for each line.

265. Solve $\frac{3m}{4} + \frac{3}{8} = \frac{m}{3} - \frac{5}{6}$ for m, expressing your answer as a fraction in lowest terms.

266. Find two different ways of determining the slope of the line $11x + 8y = 176$.

267. When weights are placed on the end of a spring, the spring stretches. If a three-pound weight stretches the spring to a length of 4.25 inches, a five-pound weight stretches the spring to a length of 5.75 inches, and a nine-pound weight stretches the spring to a length of 8.75 inches, what was the initial length of the spring?

268. Given that y is proportional to x and that $y = 60$ when $x = 20$, find y when $x = 12$.

269. Solve for x: $\frac{1}{2}(x-2)+\frac{1}{3}(x-3)+\frac{1}{4}(x-4)=10$

270. A chemist would like to dilute a 90-cc solution that is 5% acid to one that is 3% acid. How much water must be added to accomplish this task?

271. A cube measures x cm on each edge.
(a) Find a formula in terms of x for the volume of this cube in cubic centimeters (cc).**(b)** Evaluate this formula when $x = 1.5$ cm; when $x = 10$ cm.
(c) Write an expression for the area of one of the faces of the cube. Write a formula for the total surface area of all six faces.
(d) Evaluate this formula when $x = 1.5$ cm; when $x = 10$ cm.
(e) Although area is measured in square units and volume in cubic units, is there any cube for which the number of square units in the total surface area equals the number of cubic units in the volume?

272. Apply the distributive property to write without parentheses and collect like terms:

(a) $x(x-3)+2(x-3)$ **(b)** $2x(x-4)-3(x-4)$ **(c)** $x(x-2)+2(x-2)$

273. Asked to solve the inequality $3 < |x-5|$ at the board, Corey wrote "$8 < x < 2$," Sasha wrote "$x < 2$ or $8 < x$," and Avery wrote "$x < 2$ and $8 < x$." What do you think of these answers? Do any of them agree with your answer?

274. Apply the distributive property to write without parentheses and collect like terms:

(a) $(x+2)(x-3)$ **(b)** $(2x-3)(x-4)$ **(c)** $(x+2)(x-2)$

275. If the width and length of a rectangle are both increased by 10%, by what percent does the area of the rectangle increase? By what percent does the perimeter of the rectangle increase?

276. A train is leaving in 11 minutes and you are one mile from the station. Assuming you can walk at 4 mph and run at 8 mph, how much time can you afford to walk before you must begin to run in order to catch the train?

277. Sandy was told by a friend that "absolute value makes everything positive." So Sandy rewrote the equation $|x - 6| = 5$ as $x + 6 = 5$. Do you agree with the statement, or with what Sandy did to the equation? Explain your answer.

278. For each of the following points, find the distance to the y-axis:

(a) $(11, 7)$ **(b)** $(-5, 9)$ **(c)** $(4, y)$ **(d)** $(x, -8)$

279. To mail a first-class letter in 2016, the rate was 47 cents for the first ounce or fraction thereof, and 21 cents for each additional ounce or fraction thereof. Let p be the number of cents needed to mail a first-class letter that weighed w ounces. Make a table that includes some non-integer values for w. Then graph p versus w, with w on the horizontal axis.

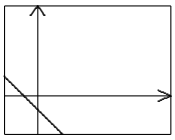
280. Given the line $y = \frac{1}{2}x + 6$, write an equation for the line through the origin that has the same slope. Write an equation for the line through $(2, -4)$ that has the same slope.

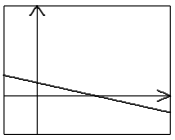
281. In a suitably large container, there are 100 liters of vinegar that is 12% acetic acid. How many liters of pure water need to be added to the container in order to dilute this solution to a 5% acetic acid solution?

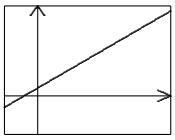
282. Twelve math students were each given an individual bag of M&Ms. Each bag has a stated net weight of 47.9 grams. The students emptied each bag and weighed the contents on an electronic scale. The weights are recorded in the chart below. Why do the weights differ from 47.9 g? Are you surprised? If you buy a bag in the store yourself tonight, would you expect it to weigh 47.9 g? Is there another weight that you think is more likely?

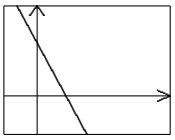
Bag Weights (grams)					
47.35	48.63	52.36	50.13	46.71	48.69
47.94	48.96	47.30	46.96	47.83	51.72

283. Which of the following screens could represent the graph of $9x + 5y = 40$?

(a) 

(b) 

(c) 

(d) 

284. For each of these absolute-value equations, write two equations without absolute-value symbols that are equivalent to the original. Solve each of the equations.

(a) $2|x + 7| = 12$ **(b)** $3 + |2x + 5| = 17$ **(c)** $6 - |x + 2| = 3$ **(d)** $-2|4 - 3x| = -14$

285. Hearing Yuri say "This line has no slope," Tyler responds "Well, 'no slope' actually means slope 0." What are they talking about? Do you agree with either of them?

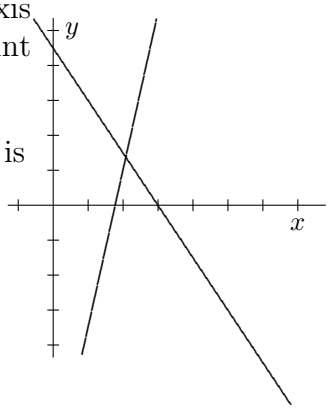
286. The edges of a solid cube are 3p cm long. At one corner of the cube, a small cube is cut away. All its edges are p cm long. In terms of p, what is the total surface area of the remaining solid? What is the volume of the remaining solid? Make a sketch.

287. Lee's pocket change consists of x quarters and y dimes. Put a dot on every lattice point (x, y) that signifies that Lee has exactly one dollar of pocket change. What equation describes the line that passes through these points? Notice that it does not make sense to connect the dots in this context, because x and y are discrete variables, whose values are limited to integers.

288. (Continuation) Put a dot on every lattice point (x, y) that signifies that Lee has at most one dollar in pocket change. How many such dots are there? What is the relationship between Lee's change situation and the inequality $0.25x + 0.10y \leq 1.00$?

289. (Continuation) Write two inequalities that stipulate that Lee cannot have fewer than zero quarters or fewer than zero dimes.

290. The figure shows the graphs of two lines. Use the graphs (the axis markings are one unit apart) to estimate the coordinates of the point that belongs to both lines.



291. (Continuation) The *system of equations* that has been graphed is

$$\begin{cases} 9x - 2y = 16 \\ 3x + 2y = 9 \end{cases}$$

Jess took one look at these equations and knew right away what to do. "Just add the equations and you will find out quickly what x is." Follow this advice, and explain why it works.

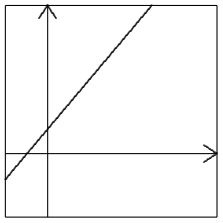
292. (Continuation) Find the missing y-value by inserting the x-value you found into either of the two original equations. Do the coordinates of the *point of intersection* agree with your estimate? These coordinates are called a *simultaneous solution* of the original system of equations. Explain the terminology.

293. Consider the line with equation $y = 2(x + 3) - 1$. Write an equation for the line which has the same slope and contains the point $(3, -1)$.

294. Most linear equations can be rewritten in slope-intercept form y = mx + b. Give an example that shows that not all linear equations can be so rewritten.

295. Given that *2x - 3y = 17* and *4x + 3y = 7*, and without using paper, pencil, or calculator, find the value of *x*.

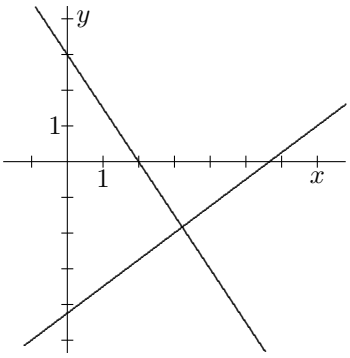
296. Which of the following could be the equation whose graph is shown at right? To support your answer, explain what portion of the x-axis and y-axis are shown?



(a) $3y - 7x = 28$ **(b)** $x + 2y = 5$
(c) $12x = y + 13$ **(d)** $y - 0.01x = 2000$

297. Find values for x and y that fit both of the equations 2x − 3y = 8 and 4x + 3y = −2.

298 The figure at right shows the graphs of two lines. First use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is



$$3x + 2y = 6$$
$$3x - 4y = 17$$

Randy took one look at these equations and knew right away what to do. "Just subtract the equations and you will find out quickly what y is." Follow this advice.

299. (Continuation) Find the missing x-value by inserting the y-value you found into one of the two original equations. Does it matter which one? Compare the intersection coordinates with your estimate.

300. (Continuation) If you *add* the two given equations, you obtain the equation of yet another line. Add its graph to the figure. You should notice something. Was it expected?

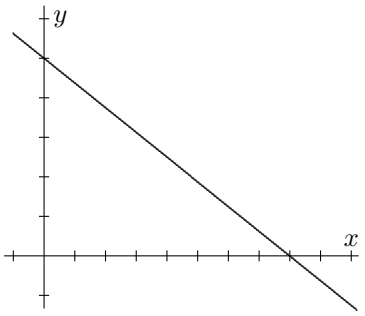
301. Record the number of letters in the first name of everyone in your math class. Create a *frequency table* as shown, where the frequency is the number of students with a name of a particular length. Make your graph similar to the one in #29. Is there a name length that you might consider typical for your math class?

Length of First Name (# of letters)	Frequency (# of students)

302. Find the value of x that fits the equation: $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 26$.

303. Fitness Universe has a membership fee of $50, after which individual visits to the gym are $5.50. Non-members pay $8.00 per visit. Stuart is going to exercise at the gym regularly, and is wondering whether it makes sense to become a member. How regularly would Stuart need to visit this gym, in order for a membership to be worth it?

304. What is the slope of the line graphed at the right, if
(a) the distance between the x-tick marks is 2 units and the distance between the y-tick marks is 1 unit?
(b) the distance between the x-tick marks is 100 units and the distance between the y-tick marks is 5 units?



305. My sleeping bag is advertised to be suitable for temperatures T between 20 degrees below zero and 20 degrees above zero (Celsius). Write an absolute-value inequality that describes these temperatures T.

306. Pat has x quarters and y dimes, and, in addition, has no more than two dollars. Write several inequalities that represent this situation and then graph all points in the coordinate plane that satisfy this condition.

307. Graph the equation $2x + 3y = 6$. Now graph the inequality $2x + 3y \leq 6$ by shading all points (x, y) that fit it. Notice that this means shading all the points on one side of the line you drew. Which side? Use a test point like $(0, 0)$ to decide.

308. Some questions about the line that passes through the points $(-3, -2)$ and $(5, 6)$:**(a)** Find the slope of the line.
(b) Is the point $(10, 12)$ on the line? Justify your answer.
(c) Find y so that the point $(7, y)$ is on the line.

309. Find values for x and y that fit both of the equations $5x + 3y = 8$ and $4x + 3y = -2$.

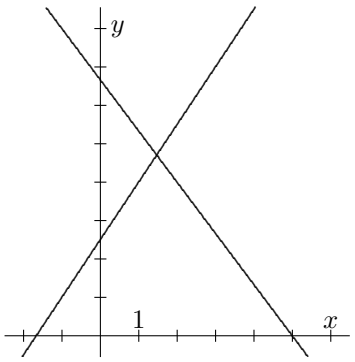
310. A 100-liter barrel of vinegar is 8% acetic acid. Before it can be bottled and used in cooking, the acidity must be reduced to 5% by diluting it with pure water. In order to produce 64 liters of usable vinegar, how many liters of vinegar from the barrel and how many liters of pure water should be combined?

311. Express each as a single fraction:**(a)** $\frac{1}{a} + \frac{2}{b} + \frac{3}{c}$ **(b)** $\frac{1}{a} + \frac{1}{b+c}$ **(c)** $1 + \frac{2}{a+b}$

312. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20 \\ 3x - 2y = -5 \end{cases}$$

Lee took one look at these equations and announced a plan: "Just multiply the first equation by 2 and the second equation by 3." What does changing the equations in this way do to their graphs?



313. (Continuation) Lee's plan has now created a familiar situation. Do you recognize it? Complete the solution to the system of equations. Do the coordinates of the point of intersection agree with your initial estimate?

314. Sandy's first four test scores this term are 73, 87, 81 and 76. To have at least a B test grade, Sandy needs to average at least 80 on the five term tests (which count equally). Let t represent Sandy's score on the fifth test, and write an inequality that describes the range of t-values that will meet Sandy's goal.

315. Graph solutions on a number line: **(a)** $|x+8| < 20$ **(b)** $|2x-5| \leq 7$ **(c)** $3|4-x| \geq 12$

316. Shade the points in the plane whose x-coordinates are greater than their y-coordinates. Write an inequality that describes these points.

317. Algebraically solve the system of equations $2x + y = 5$ and $5x - 2y = 8$. Check your answer graphically.

318. A large telephone company sent out an offer for pre-paid phone cards. The table below accompanied the ad and summarized their offer. Does this data form a linear relationship? Explain your answer. Which offer has the best rate per minute?

75-minute card	150-minute card	300-minute card	500-minute card	1000-minute card	1500-minute card
\$4.95	\$9.90	\$19.80	\$30.00	\$56.00	\$75.00

319. Find an equation for each of the following lines. When possible, express your answer in both point-slope form and slope-intercept form.
(a) The line passes through $(3, 5)$, and has -1.5 as its slope.
(b) The line is parallel to the line through $(-8, 7)$ and $(-3, 1)$, and has 6 as its x-intercept.
(c) The line is parallel to the line $x = -4$, and it passes through $(4, 7)$.

320. Which of the following has *variability*?
(a) The answer to the problem: $2 + 3 = ?$
(b) The number of students at assembly this past Friday.
(c) Surface area of maple leafs.
(d) The number of students who attend assembly from week to week during the school year.
(e) The volume of soda found in 12 oz. bottles of Squamscott.
What could you add to this list that exhibits variability? Do some types of data have more variability than others? Explain.

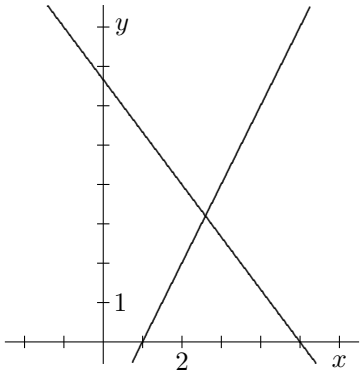
321. Jess and Wes used to race each other when they were younger. Jess could cover 8 meters per second, but Wes could cover only 5 meters per second, so Jess would sportingly let Wes start 60 meters ahead. They would both start at the same time and continue running until Jess caught up with Wes. How far did Jess run in those races?

322. Use a different color for the regions described in parts (a) and (b):
(a) Shade all points whose x- and y-coordinates sum to less than 10.
(b) Shade all points whose x- and y-coordinates are both greater than zero.
(c) Write a system of three inequalities that describe where the two regions overlap.

323. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20 \\ y = 2x - 2 \end{cases}$$

Min took one look at these equations and offered a plan: "The second equation says you can *substitute* $2x - 2$ for y in the first equation. Then you have only one equation to solve." Explain the logic behind Min's substitution strategy. Carry out the plan, and compare the exact coordinates of the intersection point with your estimates.



324. Farmer MacGregor wants to know how many cows and ducks are in the meadow. After counting 56 legs and 17 heads, the farmer knows. How many cows and ducks are there?

325. If the dimensions of a rectangle are $2x + 1$ by $x + 1$, find its area, in terms of x.

326. How much money do you have, if you have d dimes and n nickels? Express your answer in **(a)** cents; **(b)** dollars.

327. Find the point (x, y) that fits both of the equations $y = 1.5x + 2$ and $9x + 4y = 41$.

328. Sam boards a ski lift, and rides up the mountain at 6 miles per hour. Once at the top, Sam immediately begins skiing down the mountain, averaging 54 miles per hour, and does not stop until reaching the entrance to the lift. The whole trip, up and down, takes 40 minutes. Assuming the trips up and down cover the same distance, how many miles long is the trip down the mountain?

329. If the price of a stock goes from \$4.25 per share to \$6.50 per share, by what percent has the value of the stock increased?

330. Your company makes spindles for the space shuttle. NASA specifies that the length of a spindle must be 12.45 ± 0.01 cm. What does this mean? What are the smallest and largest acceptable lengths for these spindles? Write this range of values as an inequality, letting L stand for the length of the spindle. Write another inequality using absolute values that models these constraints.

331. *Factor* each of the following quadratic expressions:

(a) $x^2 + 4x$ **(b)** $2x^2 - 6x$ **(c)** $3x^2 - 15x$ **(d)** $-2x^2 - 7x$

332. (Continuation) The *zero-product property* says that $a \cdot b = 0$ is true if $a = 0$ or $b = 0$ is true, and *only* if $a = 0$ or $b = 0$ is true. Explain this property in your own words (looking up the word *or* in the Reference section if necessary). Apply it to solve these *quadratic equations*:

(a) $x^2 + 4x = 0$ **(b)** $2x^2 - 6x = 0$ **(c)** $3x^2 - 15x = 0$ **(d)** $-2x^2 - 7x = 0$

333. Use the distributive property to multiply $(x+p)(x+q)$. The result of this multiplication can be expressed in the form $x^2 + \nabla x + \Delta$; what do ∇ and Δ stand for?

334. (Continuation) When attempting to factor $x^2 + 5x + 4$ into a product of two *binomials* of the form $(x + p)(x + q)$, Dylan set up the *identity* $x^2 + 5x + 4 = (x + \,)(x + \,)$. Using a *trial-and-error* process, try to figure out what numbers go in the blank spaces. What is the connection between the numbers in the blank spaces and the coefficients 5 and 4 in the quadratic expression being factored?

335. Expand the following products:
(a) $(x - 4)(x + 4)$ **(b)** $(x + 7)(x - 7)$ **(c)** $(3x - 2)(3x + 2)$

Use the pattern to predict the factors of $x^2 - 64$ and $4x^2 - 25$. Explain why this pattern is called *the difference of two squares*.

336. When taking an algebra quiz, Dale was asked to factor the trinomial $x^2 + x + 6$. Dale responded that this particular trinomial was not factorable. Decide whether Dale is correct, and justify your response.

337. What is unusual about the graphs of the equations $9x - 12y = 27$ and $-3x + 4y = -9$?

338. What percent decrease occurs when a stock goes from \$6.50 per share to \$4.25 per share?

339. With parental assistance, Corey buys some snowboarding equipment for \$500, promising to pay \$12 a week from part-time earnings until the 500-dollar debt is retired. How many weeks will it take until the outstanding debt is under \$100? Write an inequality that models this situation and then solve it algebraically.

340. Factor each expression:

(a) $3x^2 + 12x$ **(b)** $x^2 + 8x + 15$ **(c)** $4xy + 2y^2$

341. On 3 January 2004, after a journey of 300 million miles, the rover Spirit landed on Mars and began sending back information to Earth. It landed only six miles from its target. This accuracy is comparable to shooting an arrow at a target fifty feet away and missing the exact center by what distance?

342. Find an equation for the line that passes through the point $(-3, 6)$, parallel to the line through the points $(0, -7)$ and $(4, -15)$. Write your answer in point-slope form.

343. Sid has a job at Morgan Motors. The salary is \$1200 a month, plus 3% of the sales price of every car or truck Sid sells (this is called a *commission*).
(a) The total of the sales prices of all the vehicles Sid sold during the first month on the job was \$72000. What was Sid's income (salary plus commission)?
(b) In order to make \$6000 in a single month, how much selling must Sid do?
(c) Write a linear equation that expresses Sid's monthly income y in terms of the value x of the vehicles Sid sold.
(d) Graph this equation. What are the meanings of its y-intercept and slope?

344. I recently paid \$85.28 for 12.2 pounds of coffee beans. What was the price per pound of the coffee? How many pounds did I buy per dollar?

345. Find the value of x that fits the equation $1.24x - (3 - 0.06x) = 4(0.7x + 6)$.

346. At the East Candy Shop, Jess bought 5.5 pounds of candy — a mixture of candy priced at \$4 per pound and candy priced at \$3.50 per pound. Given that the bill came to\$20.75, figure out how many pounds of each type of candy Jess bought.

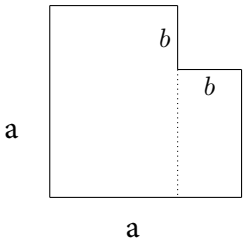
347. Explain how to evaluate 4^3 by hand. The superscript 3 is called an *exponent*, and 4^3 is *a power of 4*. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ as a power of 4. Write the product $4^3 \cdot 4^5$ as a power of 4.

348. Does every system of equations $px + qy = r$ and $mx + ny = k$ have a simultaneous solution (x, y)? Explain.

349. Sketch the region that is common to the graphs of $x \geq 2$, $y \geq 0$, and $x + y \leq 6$, and find its area.

350. Use the trial-and-error method to factor the following *trinomials* into the product of two binomials:
(a) $x^2 + 2x - 8$ **(b)** $x^2 - x - 6$ **(c)** $2x^2 + 7x + 6$

351. By rearranging the two parts of the diagram shown at right, show that $a^2 - b^2$ is equivalent to $(a + b)(a - b)$.

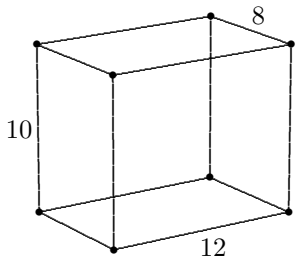


352. Wes walks from home to a friend's house to borrow a bicycle, and then rides the bicycle home along the same route. By walking at 4 mph and riding at 8 mph, Wes takes 45 minutes for the whole trip. Find the distance that Wes walked.

353. Riley also orders a burrito every week from Las Olas and records the weight in ounces in the table below. Calculate the mean and median of Riley's burrito weights.

Riley's Burritos (oz)										
12.2	10.5	10.4	10.0	12.5	12.7	11.1	12.0	12.1	13.7	13.6
13.0	15.0	15.0	9.90	9.20	19.2	10.7	11.5	13.1	12.5	12.9
8.90	12.7	12.4	9.80	11.8	13.0	13.1	11.4	11.0	10.1	11.3

354. The figure at the right shows a rectangular box whose dimensions are 8 cm by 10 cm by 12 cm.
(a) Find the volume of the solid.
(b) What is the combined area of the six faces?
(c) If you were to outline the twelve edges of this box with decorative cord, how much would you need?



355. The area of a rectangle can be found by multiplying its dimensions. For example, if the dimensions are 2 by 7, the area is 14. Going in the other direction, one can say the factors of the area give possible dimensions for the rectangle. So, finding factors of 14 give possible dimensions of a rectangle with area 14. Similarly, to find dimensions of a rectangle with area $x^2 + 2x - 3$, one can ask for factors of $x^2 + 2x - 3$. In this case, $x^2 + 2x - 3 = (x - 1)(x + 3)$, so possible dimensions are $x - 1$ by $x + 3$. Find possible dimensions of rectangles with area:

(a) $x^2 + 5x + 6$ **(b)** $2x^2 + 5x + 2$

356. The population of Pittsburgh is about 301 thousand people. The population of the United States is about 300 million people. What percent of the US population lives in "The Burgh"?

357. A rectangle is four times as long as it is wide. If its length were diminished by 6 meters and its width were increased by 6 meters, it would be a square. What are the rectangle's dimensions?

358. Factor:

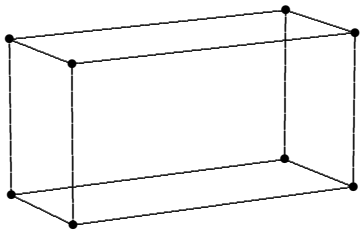
(a) $x^2 + 9x + 20$ **(b)** $3x^2 + x - 4$ **(c)** $x^2 - 81$

359. A rectangle width x and length 2x − 1. Its area is 21. Find its dimensions.

360. After a weekend of rock-climbing in the White Mountains, Dylan is climbing down a 400-foot cliff. It takes 20 minutes to descend the first 60 feet. Assuming that Dylan makes progress at a steady rate, write an equation that expresses Dylan's height h above level ground in terms of t, the number of minutes of descending from the top. Use your equation to find how much time it will take Dylan to reach level ground.

361. In 2016 the length of The Appalachian Trail stretched 2190 miles from Georgia to Maine. A hiker completed this hike in 46 days. The hiker averaged 3.4 miles per hour. How many hours a day did the hiker average?

362. The diagram at the right shows the wire framework for a rectangular box. The length of this box is 8 cm greater than the width and the height is half the length. A total of 108 cm of wire was used to make this framework.

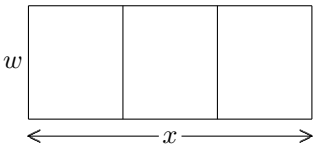


(a) What are the dimensions of the box?
(b) The faces of the box will be panes of glass. What is the total area of the glass needed for the six panes?
(c) What is the volume of the box?

363. Write a formula that expresses the distance between p and 17. Describe all the possible values for p if this distance is to be greater than 29.

364. Use the zero-product property to solve:
(a) $x^2 + 2x - 8 = 0$ **(b)** $x^2 - x - 20 = 0$ **(c)** $81 - x^2 = 0$

365. A farmer has 90 meters of fencing material with which to construct three rectangular pens side-by-side as shown at right. If w were 10 meters, what would the length x be? Find a general formula that expresses x in terms of w.



366. Find how many pairs (x, y) satisfy the equation $x + y = 25$, assuming that
(a) there is no restriction on the values of x and y;
(b) both x and y must be positive integers;
(c) the values of x and y must be equal.

367. The table at the right shows the value of a car as it depreciates over time. Does this data satisfy a linear relationship? Explain.

year	*value*
1992	24 000
1993	20 400
1994	16 800
1995	13 200

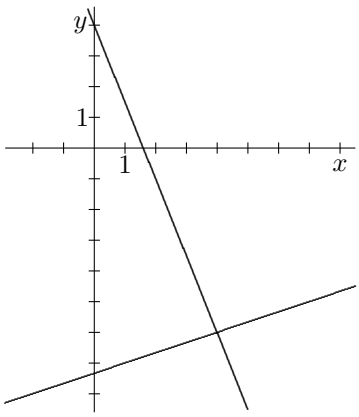
368. Write an inequality that describes all the points that are more than 3 units from 5.

369. If x varies directly with y, and if x = 5 when y = 27, find x when y = 30.

370. When asked to solve the system of equations

$$\begin{cases} 5x + 2y = 8 \\ x - 3y = 22 \end{cases}$$

Kelly said "Oh that's easy — you just set them equal to each other." Looking puzzled, Wes replied "Well, I know the method of *linear combinations*, and I know the method of *substitution*, but I do not know what method you are talking about." First, explain each of the methods to which Wes is referring, and show how they can be used to solve the system. Second, explain why Wes did not find sense in Kelly's comment. Third, check that your answer agrees with the diagram.



371. Factor:

(a) $2x^2 + x - 21$ **(b)** $4x^2 - 15x - 4$ **(c)** $4x^2 - 81$ **(d)** $0.04x^2 - 81$

372. Giant Eagle carries two types of apple juice. One is 100% fruit juice, while the other is only 40% juice. Yesterday there was only one 48-ounce bottle of the 100% juice left. I bought it, along with a 32-ounce bottle of the 40% juice. I am about to mix the contents of the two bottles together. What percent of the mixture will be actual fruit juice?

373. (Continuation) On second thought, I want the mixture to be at least 80% real fruit juice. How much of the 32-ounce bottle can I add to the mixture and be satisfied?

374. Solve each of the systems of equations below

(a) $\begin{cases} 3x + 4y = 1 \\ 4x + 8y = 12 \end{cases}$ **(b)** $\begin{cases} 2x + 3y = -1 \\ 6x - 5y = -7 \end{cases}$

375. The difference between the length and width of a rectangle is 7 cm. The perimeter is 50 cm. Find the length and width.

376. Randy has 25% more money than Sandy, and 20% more money than Mandy, who has \$1800. How much money does Sandy have?

377. A restaurant has 23 tables. Some of the tables seat 4 people and the rest seat 2 people. In all, 76 people can be seated at once. How many tables of each kind are there?

378. Solve each of the following systems of equations:

(a) $\begin{cases} 3r + 5s = 6 \\ 9r = 13s + 4 \end{cases}$ **(b)** $\begin{cases} 3a = 1 + \frac{1}{3}b \\ 5a + b = 11 \end{cases}$

379. Use the distributive property to write each of the following in *factored form*:

(a) $ab^2 + ac^2$ **(b)** $3x^2 - 6x$ **(c)** $wx + wy + wz + w$

380. Most of Conservative Casey's money is invested in a savings account that pays 1% interest a year, but some is invested in a risky stock fund that pays 7% a year. Casey's total initial investment in the two accounts was \$10000. At the end of the first year, Casey received a total of \$250 in interest from the two accounts. Find the amount initially invested in each.

381. Faced with the problem of multiplying 5^6 times 5^3, Brook is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18}, or 25^9. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are able to formulate the *common-base principle for multiplication* of expressions $b^m \cdot b^n$.

382. A large family went to a restaurant for a buffet dinner. The price of the dinner was \$12 for adults and \$8 for children. If the total bill for a group of 13 people came to \$136, how many children were in the group?

383. Write each of the following in factored form:

(a) $2x^2 + 3x^3 + 4x^4$ **(b)** $5xp + 5x$ **(c)** $2\pi r^2 + 2\pi rh$

384. Find values for a and b that make ax + by = 14 parallel to 12 − 3y = 4x. Is there more than one answer? If so, how are the different values for a and b related?

385. Exponents are routinely encountered in scientific work, where they help investigators deal with large numbers:
(a) The human population of Earth is roughly 7000000000, which is usually expressed in scientific notation as 7 × 109. The average number of hairs on a human head is 5 × 105. Use scientific notation to estimate the total number of human head hairs on Earth.
(b) Light moves very fast — approximately 3 × 108 meters every second. At that rate, how many meters does light travel in one year, which is about 3×107 seconds long? This so-called light-year is used in astronomy as a yardstick for measuring even greater distances.

386. Solve the equation 1.2x + 0.8(20 − x) = 17.9 for x. Make up a word problem that could use this equation in its solution. In other words, the equation needs a context.

387. Write the following sentence using mathematical symbols:

"The absolute value of the sum of two numbers a and b is equal to the sum of the absolute values of each of the numbers a and b." Is this a true statement? Explain.

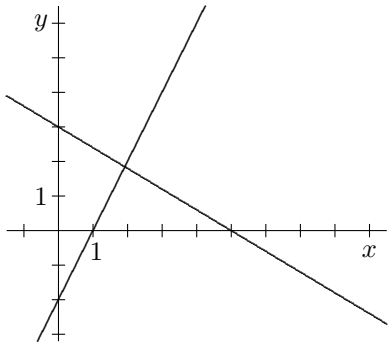
388. The perimeter of a square is p inches. Write expressions, in terms of p, for the length of the side of the square and the area of the square.

389. Fill in all of the blanks with the same number in order to make a true statement: $x^2 + 8x + 16 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

390. Solve:

(a) $x^2 + 8x + 16 = 0$ **(b)** $x^2 + 8x + 15 = 0$ **(c)** $x^2 + 8x + 7 = 0$

391. The figure shows the graphs of two lines, whose axis intercepts are integers. Use the graphs to estimate the coordinates of the point that belongs to both lines, then calculate the exact value. You will of course have to find equations for the lines.



392. Find three lattice points on the line $x + 3y = 10$. How many others are there?

393. In a coordinate plane, shade the region that consists of all points that have positive x- and y-coordinates whose sum is less than 5. Write a system of three inequalities that describes this region.

394. A laser beam is shot from the point $(0, 2.35)$ along the line whose slope is 3.1. Will it hit a very thin pin stuck in this coordinate plane at the point $(10\,040, 31126)$? Explain your reasoning and any assumptions that you may have made.

395. The Pgh. Tree Company charges a certain amount per *cord* for firewood and a fixed amount for each delivery, no matter how many cords are delivered. My bill from PTC last winter was \$155 for one cord of wood, and my neighbor's was \$215 for one and one-half cords. What is the charge for each cord of wood and what is the delivery charge?

396. A *monomial* is a constant or a product of a constant and variables. If some variable factors occur more than once, it is customary to use positive integer exponents to consolidate them. Thus 12, $3ax^2$, and x^5 are monomials, but $3xy^4 + 3x^4y$ is not. Rewrite each of these monomials:

(a) $x \cdot x^2 \cdot x^3 \cdot x^4$ **(b)** $(2x)^7$ **(c)** $(2w)^3 \cdot 5w^3$ **(d)** $3a^4 \cdot (\frac{1}{2}b)^3 \cdot ab^6$

397. The point $(2, 3)$ lies on the line $2x + ky = 19$. Find the value of k.

398. Do the three lines $5x - y = 7$, $x + 3y = 11$, and $2x + 3y = 13$ have a common point of intersection? If so, find it. If not, explain why not.

399. Using an absolute-value inequality, describe the set of numbers whose distance from 4 is greater than 5 units. Draw a graph of this set on a number line. Finally, describe this set of numbers using inequalities without absolute value signs.

400. A polynomial is obtained by adding (or subtracting) monomials. Use the distributive property to rewrite each of the following polynomials in factored form. In each example, you will be finding a common monomial factor.

(a) $x^2 - 2x$ (b) $6x^2 + 21x$ (c) $80t - 16t^2$ (d) $9x^4 - 3x^3 + 12x^2 - x$

A binomial is the sum of two unlike monomials, and a trinomial is the sum of three unlike monomials. The monomials that make up a polynomial are often called its terms.

401. The simultaneous conditions $x - y < 6$, $x + y < 6$, and $x > 0$ define a region R. How many lattice points are contained in R?

402. In $7^4 \cdot 7^4 \cdot 7^4 = (7^4)\Delta$ and $b^9 \cdot b^9 \cdot b^9 \cdot b^9 = (b^9)\nabla$, replace the triangles by correct exponents. The expression $(p^5)^6$ means to write p^5 as a factor how many times? To rewrite this expression without exponents as $p \cdot p \cdot p \cdots$, how many factors would you need?

403. Graph the system of equations shown at right. What special relation-ship exists between the two lines? Confirm this by solving the equations algebraically.

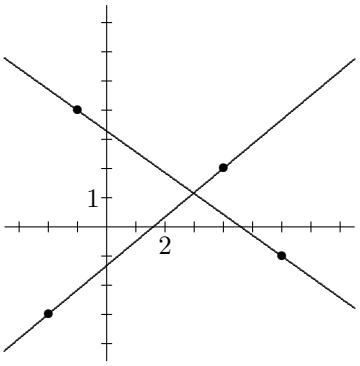
$$\begin{cases} 3x - y = 10 \\ 6x = 20 + 2y \end{cases}$$

404. Faced with the problem of calculating $(5^4)^3$, Brook is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12}, or 5^7. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the principle that tells how to write $(b^m)^n$ as a power of b.

405. The diameter of an atom is so small that it would take about 10^8 of them, arranged in a line, to span one centimeter. It is thus a plausible estimate that a cubic centimeter contains about $10^8 \times 10^8 \times 10^8 = (10^8)^3$ atoms. Write this huge number as a power of 10.

406. During a phone call about the system of equations $\{5x + 2y = 8\,,\ 8x + 4y = 8\}$, Dylan told Max, "It's easy, just set them equal to each other." But Max replied, "That doesn't help — I get $-2y = 3x$. What good is that?" Help these two students solve the problem.

407. The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines, then calculate the exact value. You will of course have to find equations for the lines, which both go through designated lattice points.



408. A math teacher is designing a test, and wants $(3, -4)$ to be the solution to the system of equations $\{3x - 5y = a\,,\ 7x + y = b\}$. What values should the teacher use for a and b?

409. In each part, use the same number in each blank to make a true statement. Compare the number you put in the blanks with the original expression. What do you notice?

(a) $x^2 + 10x + 25 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

(b) $x^2 + 12x + 36 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

(c) $x^2 + 14x + 49 = (x + \_\_)(x + \_\_) = (x + \_\_)^2$

410. In 2015 the world was consuming approximately 97 million barrels of oil per day. The United States was consuming approximately 19 million barrels of oil per day.
(a) It is estimated that oil shale in the Green River basin of the Rocky Mountains holds approximately 800 billion barrels of recoverable oil. At the 2015 rate of consumption, how long would this supply the world with oil?
(b) Production of each barrel of oil from oil shale requires between 2 and 3 barrels of water. How many barrels of water would be required annually to supply the United States from oil shale?

411. A catering company offers three monthly meal contracts:
Contract A costs a flat fee of \$480 per month for 90 meals;
Contract B costs \$200 per month plus \$4 per meal;
Contract C costs a straight \$8 per meal.
If you expect to eat only 56 of the available meals in a month, which contract would be best for you? When might someone prefer contract A? contract B? contract C?

412. You are buying some cans of juice and some cans of soda for the dorm. The juice is \$0.60 per can while the soda is \$0.75. You have \$24 of dorm funds, all to be spent.
(a) Write an equation that represents all the different combinations of juice and soda you can buy for \$24.
(b) Is it possible to buy exactly 24 cans of juice and spend the remainder on soda? Explain.

(c) How many different combinations of drinks *are* possible?

413. Jan had the same summer job for the years 1993 through 1996, earning \$250 in 1993, \$325 in 1994, \$400 in 1995, and \$475 in 1996.
(a) Plot the four data points, using the horizontal axis for "year". You should be able to draw a line through the four points.
(b) What is the slope of this line? What does it represent?
(c) Which points on this line are meaningful in this context?
(d) Guess what Jan's earnings were for 1992 and 1998, assuming the same summer job.
(e) Write an inequality that states that Jan's earnings in 1998 were within 10% of the amount you guessed.

414. Replace the triangles in $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^{\Delta}$ and $\frac{6^9}{6^4} = 6^{\nabla}$ by correct exponents.

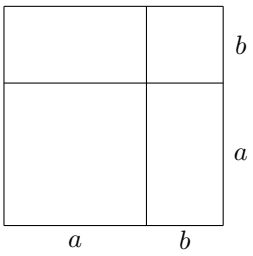
415. Rewrite each of the following polynomials as a product of two factors. One of the factors should be the greatest common monomial factor.

(a) $24x^2 + 48x + 72$ **(b)** $\pi r^2 + \pi re$ **(c)** $7m - 14m^2 + 21m^3$

416. If possible, find values for x and y for which

(a) $|x + y| < |x| + |y|$ **(b)** $|x + y| = |x| + |y|$ **(c)** $|x| + |y| < |x + y|$

417. Refer to the diagram at right, which shows a large square that has been subdivided into two squares and two rectangles. Write formulas for the areas of these four pieces, using the dimensions a and b marked on the diagram. Then write an equation that states that the area of the large square is equal to the combined area of its four pieces. Do you recognize this equation?



418. Find the equations of at least three lines that intersect each other at the point $(6, -2)$.

419. Factor the following:

(a) $2x^2 - 4x$ **(b)** $x^2 + 3x$ **(c)** $2x^3 - 32x$ **(d)** $x^2 + 24x + 144$

420. Faced with the problem of dividing 5^{24} by 5^8, Brook is having trouble deciding which of these four answers is correct: 5^{16}, 5^3, 1^{16}, or 1^3. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the *common-base principle for division* that tells how to divide b^m by b^n and get another power of b. Then apply this principle to the following situations:

(a) Earth's human population is roughly 6×10^9, and its total land area, excluding the polar caps, is roughly 5×10^7 square miles. If the human population were distributed uniformly over all available land, approximately how many people would be found per square mile?

(b) At the speed of light, which is 3×10^8 meters per second, how many seconds does it take for the Sun's light to travel the 1.5×10^{11} meters to Earth?

421. Cameron bought some 39-cent, 24-cent, and 13-cent stamps at the Post Office. The 100 stamps cost \$33.40, and there were twice as many 24-cent stamps in the sale as there were 13-cent stamps. How many stamps of each denomination did Cameron buy?

422. Given the equation $3x + y = 6$, write a second equation that, together with the first, will create a system of equations that
(a) has one solution;
(b) has an infinite number of solutions;
(c) has no solution;
(d) has the ordered pair $(4, -6)$ as its only solution.

423. Factor the following *perfect-square trinomials*:
(a) $x^2 - 12x + 36$ **(b)** $x^2 + 14x + 49$ **(c)** $x^2 - 20x + 100$
As suggested, these should all look like either $(x - r)^2$ or $(x + r)^2$. State the important connection between the *coefficients* of the given trinomials and the values you found for r.

424. (Continuation) Complete the following to make a true statement:

$$x^2 + 2nx + \_\_\_\_ = (x + \_\_)(x + \_\_) = (x + \_\_)^2$$

425. (Continuation) In the following, choose k to create a perfect-square trinomial:
(a) $x^2 - 16x + k$ **(b)** $x^2 + 10x + k$ **(c)** $x^2 - 5x + k$

426. In each of the following, find the correct value for ∇:
(a) $y^4 y^7 = y^{\nabla}$ **(b)** $y^{12} y^{\nabla} = y^{36}$ **(c)** $y^4 y^4 y^4 y^4 = y^{\nabla}$ **(d)** $\left(y^{\nabla}\right)^3 = y^{27}$

427. Find three consecutive odd numbers whose sum is 627.

428. The distance from here to the beach at Little Boar's Head is 10 miles. If you walked there at 4 mph and returned jogging at 8 mph, how much time would the round trip take?What would your overall average speed be?

429. Ten cc of a solution of acid and water is 30% acid. I wish to dilute the acid in the mixture by adding water to make a mixture that is only 6% acid. How much pure water must I add to accomplish this?

430. Corey is out on the roads doing a long run, and also doing some mental calculations at the same time. Corey's pace is 3 strides per second, and each stride covers 5 feet.
(a) How much time does it take Corey to cover a mile?
(b) If Corey's stride increased to 5.5 feet per step, how much time would be needed to cover a mile?
(c) At five feet per step, how many steps would Corey need to run the marathon distance, which is 26 miles and 385 yards?

431. What are the dimensions of a square that encloses the same area as a rectangle that is two miles long and one mile wide? Answer to the nearest inch, please.

432. When I ask my calculator for a decimal value of $\sqrt{2}$, it displays 1.41421356237. What is the meaning of this number? To check whether this square root is correct, what needs to be done? Can the square root of 2 be expressed as a ratio of whole numbers — for example as $\frac{17}{12}$? Before you say "impossible", consider the ratio $\frac{665857}{470832}$.

433. What happens if you try to find an intersection point for the linear graphs $3x - 2y = 10$ and $3x - 2y = -6$? What does this mean?

434. A jeweler has 10 ounces of an alloy that is 50% gold. How much more pure gold does the jeweler need to add to this alloy, to increase the percentage of gold to 60%?

435. Pat and Kim are having an algebra argument. Kim is sure that −x2 is equivalent to (−x)2, but Pat thinks otherwise. How would you resolve this disagreement? What evidence does your calculator offer?

436. What is the value of $\frac{5^7}{5^7}$? Of $\frac{8^3}{8^3}$? Of $\frac{c^{12}}{c^{12}}$? What is the value of any number divided by itself? If you apply the common-base rule dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what power? and $\frac{c^{12}}{c^{12}}$ should equal c raised to what power? It therefore makes sense to define c^0 to be what?

437. One morning, Ryan remembered lending a friend a bicycle. After breakfast, Ryan walked over to the friend's house at 3 miles per hour, and rode the bike back home at 7 miles per hour, using the same route both ways. The round trip took 1.75 hours. What distance did Ryan walk?

438. Write the following monomials without using parentheses:
(a) $(ab)^2\,(ab^2)$ **(b)** $(-2xy^4)\,(4x^2y^3)$ **(c)** $(-w^3x^2)\,(-3w)$ **(d)** $(7p^2q^3r)\,(7pqr^4)^2$

439. Taylor starts a trip to the mall with $160 cash. After 20% of it is spent, seven-eighths of the remainder is lost to a pickpocket. This leaves Taylor with how much money?

440. A box with a square base and rectangular sides is to be 2 feet and 6 inches high, and to contain 25.6 cubic feet. What is the length of one edge of the square base?

441. Equations such as $A = 40x - x^2$ and $h = 300 - 16t^2$ define *quadratic functions*. The word *function* means that assigning a value to one of the variables (x or t) determines a *unique* value for the other (A or h). It is customary to say that "A is a function of x." In this example, however, it would be incorrect to say that "x is a function of A." Explain.

442. Water pressure varies linearly with the depth of submersion. The pressure at the surface is 14.7 pounds per square inch. Given that a diver experiences approximately 58.8 pounds per square inch of pressure at a depth of 100 feet, what pressure will a submarine encounter when it is one mile below the surface of the Atlantic Ocean?

443. When two rational numbers are multiplied together, their product is also a rational number. Explain. Is it necessarily true that the product of two irrational numbers is irrational? Explore this question by evaluating the following products.
(a) $\sqrt{3} \cdot \sqrt{27}$ **(b)** $\sqrt{2} \cdot \sqrt{6} \cdot \sqrt{3}$ **(c)** $\sqrt{6} \cdot \sqrt{12}$ **(d)** $\left(\sqrt{6}\right)^3$ **(e)** $\sqrt{3}\left(\sqrt{3}\right)^2$

444. Evaluate each of the following expressions by substituting s = 30 and t = −4.

(a) $t^2 + 5t + s$ **(b)** $2t^2 s$ **(c)** $3t^2 - 6t - 2s$ **(d)** $s - 0.5t^2$

445. Evaluate $\sqrt{x^2 + y^2}$ using $x = 24$ and $y = 10$. Is $\sqrt{x^2 + y^2}$ equivalent to $x + y$, in this case? Does the square-root operation "distribute" over addition?

446. Evaluate $\sqrt{(x + y)^2}$ using $x = 24$ and $y = 10$. Is $\sqrt{(x + y)^2}$ equivalent to $x + y$, in this case? Explain.

447. Evaluate $\sqrt{(x + y)^2}$ using $x = -24$ and $y = 10$. Is $\sqrt{(x + y)^2}$ equivalent to $x + y$, in this case? Explain.

448. A car traveling at 60 miles per hour is covering how many feet in one second? A football field is 100 yards long. At 60 mph, how many seconds does it take to cover this distance?State your answer to the nearest tenth of a second.

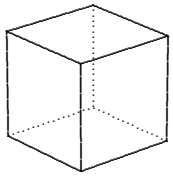
449. Perform the indicated operations, and record your observations :
(a) $\sqrt{2} \cdot \sqrt{18}$ **(b)** $\sqrt{8} \quad \overline{8}$ **(c)** $2\sqrt{5}\left(\sqrt{20}\right)$

450. (Continuation) Suggest a rule for multiplying numbers in the form $\sqrt{a} \cdot \sqrt{b}$. Extend your rule to problems in the form of $p\sqrt{a}\left(q\sqrt{b}\right)$. Use your rule to simplify the following:
(a) $\sqrt{3} \cdot \sqrt{15}$ **(b)** $\sqrt{12} \cdot \sqrt{7}$ **(c)** $4\sqrt{8}\left(5\sqrt{7}\right)$

451. (Continuation) Use what you have just seen to explain why $\sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$. Rewrite the following square roots in the same way — as the product of a whole number and a square root of an integer *that has no perfect square factors*. The resulting expression is said to be in *simplest radical form*.
(a) $\sqrt{50}$ **(b)** $\sqrt{108}$ **(c)** $\sqrt{125}$ **(d)** $\sqrt{128}$

452. At Sam's Warehouse, a member pays $25 a year for membership, and buys at the regular store prices. A non-member does not pay the membership fee, but does pay an additional 5% above the store prices. Under what conditions would it make sense to buy a membership?

453. The total area of six faces of a cube is 1000 sq cm. What is the length of one edge of the cube? Round your answer to three decimal places.



454. Solve each of the following equations. Answers should either be exact, or else accurate to three dec-imal places.

(a) $x^2 = 11$ **(b)** $5s^2 - 101 = 144$ **(c)** $x^2 = 0$ **(d)** $30 = 0.4m^2 + 12$

455. When asked to solve the equation $(x - 3)^2 = 11$, Jess said, "That's easy — just take the square root of both sides." Perhaps Jess also remembered that there are two numbers whose squares are 11, one positive and the other negative. What are the two values for x, in exact form? (In this situation, "exact" means no decimals.)

456. (Continuation) When asked to solve the equation $x^2 - 6x = 2$, Deniz said, "Hmm . . . not so easy, but I think that adding something to both sides of the equation is the thing to do." This is indeed a good idea, but what number should Deniz add to both sides? How is this equation related to the previous one?

457. Suppose that m and n stand for positive numbers, with $n < m$. Which of the following expressions has the largest value? Which one has the smallest value?

(a) $\frac{m+1}{n+1}$ **(b)** $\frac{m+1}{n}$ **(c)** $\frac{m}{n}$ **(d)** $\frac{m}{n+2}$ **(e)** $\frac{m}{n+1}$

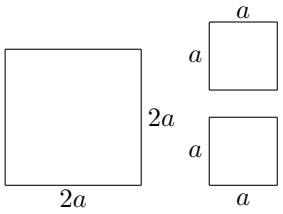
458. Use the distributive property to factor each of the following:

(a) $x^2 + x^3 + x^4$ **(b)** $\pi r^2 + 2\pi rh$ **(c)** $25x - 75x^2$ **(d)** $px + qx^2$

459. Solving a quadratic equation by rewriting the left side as a perfect-square trinomial is called solving by *completing the square*. Use this method to solve each of the following equations. Leave your answers in exact form.

(a) $x^2 - 8x = 3$ **(b)** $x^2 + 10x = 11$ **(c)** $x^2 - 5x - 2 = 0$ **(d)** $x^2 + 1.2x = 0.28$

460. Write $(2a)^2$ without parentheses. Is $(2a)^2$ larger than, smaller than, or the same as $2a^2$? Make reference to the diagram at right in writing your answer. Draw a similar diagram to illustrate the non-equivalence of $(3a)^2$ and $3a^2$.

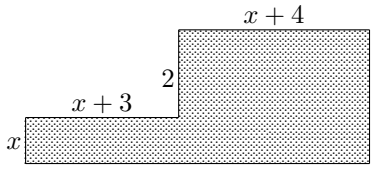


461. Solve each of the following quadratic equations by hand:

(a) $(x+4)^2 = 23$ **(b)** $7x^2 - 22x = 0$ **(c)** $x^2 + 4x = 21$ **(d)** $1415 - 16x^2 = 0$

462. The cost of a ham-and-bean supper at a local church was \$6 for adults and \$4 for children. At the end of the evening, the organizers of the supper found they had taken in a total of \$452 and that 86 people had attended. How many of these people were adults?

463. In the diagram, the dimensions of a piece of carpeting have been marked in terms of x. All lines meet at right angles. Express the area and the perimeter of the carpeting in terms of x.

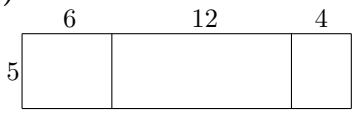


464. The work at right shows the step-by-step process used by a student to solve $x^2+6x-5=0$ by the method of completing the square. Explain why the steps in this process are reversible. Apply this understanding to find a quadratic equation $ax^2 + bx + c = 0$ whose solutions are $x = 7 + \sqrt{6}$ and $x = 7 - \sqrt{6}$.

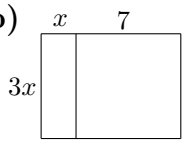
$$x^2 + 6x - 5 = 0$$
$$x^2 + 6x + 9 = 5 + 9$$
$$(x+3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

465. Express the areas of the following large rectangles in two ways. First, find the area of each small rectangle and add the expressions. Second, multiply the total length by the total width.

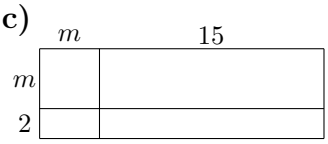
(a)



(b)



(c)



466. Apply the zero-product property to solve the following equations:

(a) $(x-2)(x+3) = 0$ **(b)** $x(2x+5) = 0$ **(c)** $5(x-1)(x+4)(2x-3) = 0$

467. Solve the following equations for x by hand.

(a) $x^2 - 5x = 0$ **(b)** $3x^2 + 6x = 0$ **(c)** $ax^2 + bx = 0$

468. Multiply: **(a)** $(3x)(7x)$ **(b)** $(3x)(7+x)$ **(c)** $(3+x)(7+x)$

469. Factor each expression completely.

(a) $4x^4 - 16x^2$ **(b)** $4x^2 - 18x + 8$ **(c)** $4x^2y - 19xy + 12y$

470. Given $P = (1, 4)$, $Q = (4, 5)$, and $R = (10, 7)$, decide whether or not PQR is a straight line, and give your reasons.

471. Given the equation $h = pea + pa$, solve the formula for: **(a)** e **(b)** a

472. *Completing the square*. Confirm that the equation $ax^2 + bx + c = 0$ can be converted into the form $x^2 + (b/a)\,x = -\frac{c}{a}$. Describe the steps. To achieve the goal suggested by the title, what should now be added to both sides of this equation?

473. (Continuation) Working with the equation $x^2 + \frac{b}{a}\,x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$,

(a) show how the left side of the equation can be factored as a perfect square trinomial, and
(b) show how the right side of the equation can be combined over a common denominator.
(c) Finish the solution of the general quadratic equation by taking the square root of both sides of your most recent equation. The answer is called *the quadratic formula*.
(d) Apply your formula to solve $x^2 + 2x - 3 = 0$ by letting $a = 1$, $b = 2$, and $c = -3$.

474. Solve the following quadratic equations:

(a) $x^2 + 6x + 5 = 0$ **(b)** $x^2 - 7x + 12 = 0$ **(c)** $3x^2 + 14x + 8 = 0$ **(d)** $2x^2 + 5x - 3 = 0$

475. Perform the following operations and combine like terms where possible:

(a) (x + 6) (x - 7) (b) $(x - 5)^2$ (c) (x + 9) (x - 9)

476. Solve $x^2 - 4x - 96 = 0$ by using:
(a) the zero-product property (b) completing the square (c) the quadratic formula

477. When taking an algebra quiz, Casey was asked to factor the trinomial $x^2 + 3x + 4$. Casey responded that this particular trinomial was not factorable. Decide whether Casey was correct, and justify your response.

478. Find the value for c that forces the graph of $3x + 4y = c$ to go through $(2, -3)$.

479. Write in as compact form as possible:

(a) $x^4 \cdot \frac{1}{x^3}$ **(b)** $\left(\frac{2}{x^3}\right)^4$ **(c)** $(2x + x + 2x)^3$ **(d)** $\frac{x^6}{x^2}$

480. By using square roots, express the solutions to $(x - 5)^2 - 7 = 0$ exactly (no decimals).

481. The *degree* of a monomial counts how many variable factors would appear if it were written without using exponents. For example, the degree of $6ab$ is 2, and the degree of $25x^3$ is 3, since $25x^3 = 25xxx$. The degree of a polynomial is the largest degree found among its monomial terms. Find the degree of the following polynomials:
(a) $x^2 - 6x$ **(b)** $5x^3 - 6x^2$ **(c)** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ **(d)** $4\pi r^2 h$

482. Find at least three integers (positive or negative) that, when put in the blank space, make the expression $x^2 + x -$ 36 a factorable trinomial. Are there other examples? How many?

483. (Continuation) Find at least three integers that, when put in the blank space, make the expression $x^2 + 4x -$ a factorable trinomial. Are there other examples? How many?What do all these integers have in common?

484. Combine into one fraction:
(a) $\frac{1}{3} + \frac{1}{7}$ **(b)** $\frac{1}{15} + \frac{1}{19}$ **(c)** $\frac{1}{x-2} + \frac{1}{x+2}$
Evaluate your answer to part (c) with $x = 5$ and $x = 17$. How do these answers compare to your answers in parts (a) and (b)?

485. Jessie walks a certain distance to a friend's house at 4 miles per hour and returns by bicycle on a route that is 8 miles longer. Jessie bikes at an average rate of 12 miles per hour. If the total time for the round trip is 90 minutes, what is the exact distance Jessie walked?

486. Plot a point near the upper right corner of a sheet of graph paper. Move your pencil 15 graph-paper units (squares) to the left and 20 units down, then plot another point. Use your ruler to measure the distance between the points. Because the squares on your graph paper are probably larger or smaller than the squares on your classmates' graph paper, it would not be meaningful to compare ruler measurements with anyone else in class. You should therefore finish by converting your measurement to graph-paper units.

487. (Continuation) Square your answer (in graph-paper units), and compare the result with the calculation $15^2 + 20^2$.

488. (Continuation) Repeat the entire process, starting with a point near the upper left corner, and use the instructions "20 squares to the right and 21 squares down." You should find that the numbers in this problem again fit the equation $a^2 + b^2 = c^2$. These are instances of the *Pythagorean Theorem*, which is a statement about right-angled triangles. Write a clear statement of this useful result. You will need to refer to the longest side of a right triangle, which is called the *hypotenuse*.

489. The product of two polynomials is also a polynomial. Explain. When a polynomial of degree 3 is multiplied by a polynomial of degree 2, what is the degree of the result?

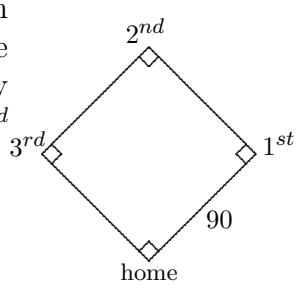
490. Starting at school, you and a friend ride your bikes in different directions — you ride 4 blocks north and your friend rides 3 blocks west. At the end of this adventure, how far apart are you and your friend?

491. From the library, you ride your bike east at a rate of 10 mph for half an hour while your friend rides south at a rate of 15 mph for 20 minutes. How far apart are you? How is this problem similar to the preceding problem? How do the problems differ?

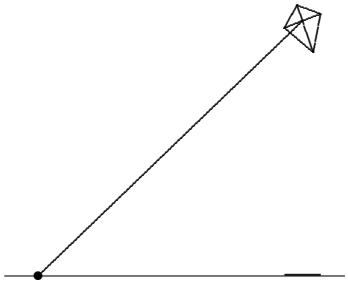
492. Imagine a circle of rope, which has twelve evenly spaced knots tied in it. Suppose that this rope has been pulled into a taut, triangular shape, with stakes anchoring the rope at knots numbered 1, 4, and 8. Make a conjecture about the angles of the triangle.

493. Combine over a common denominator: **(a)** $\frac{1}{x-3}+\frac{2}{x}$ **(b)** $\frac{1}{x-3}+\frac{2}{x+3}$

494. In baseball, the infield is a square that is 90 feet on a side, with bases located at three of the corners, and home plate at the fourth. If the catcher at home plate can throw a baseball at 70 mph, how many seconds does it take for the thrown ball to travel from home plate to 2^{nd} base?



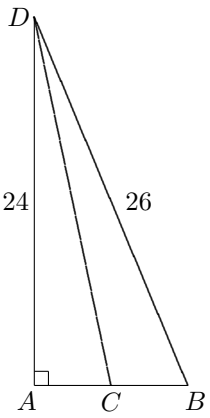
495. While flying a kite at the beach, you notice that you are 100 yards from the kite's shadow, which is directly beneath the kite. You also know that you have let out 150 yards of string. How high is the kite?



496. Starting from home, Jamie haphazardly walks 2 blocks north, 3 blocks east, 1 block north, 3 blocks east, 1 block north, 5 blocks east, and 1 block north. How far is Jamie from home if each block is 150 meters long?

497. The sides of Fran's square are 5 cm longer than the sides of Tate's square. Fran's square has 225 sq cm more area. What is the area of Tate's square?

498. In the figure at right, BAD is a right angle, and C is the midpoint of segment AB. Given the dimensions marked in the figure, find the length of CD.



499. Solve each of the following by the method of completing the square:
(a) $3x^2 - 6x = 1$ **(b)** $2x^2 + 8x - 17 = 0$

500 Use a calculator to evaluate the following: **(a)** $\frac{\sqrt{50}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{28}}{\sqrt{7}}$ **(c)** $\frac{\sqrt{294}}{\sqrt{6}}$
Explain why your results make it reasonable to write $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$. Check that this rule also works for: **(d)** $\frac{\sqrt{48}}{\sqrt{6}}$ **(e)** $\frac{\sqrt{84}}{\sqrt{12}}$ **(f)** $\frac{\sqrt{180}}{\sqrt{15}}$

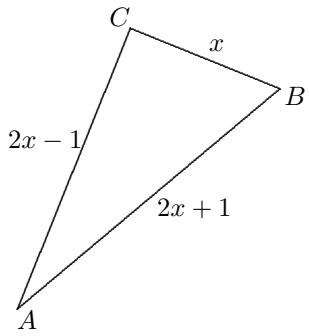
501. *Rationalizing denominators*. How are the decimal approximations for $\frac{6}{\sqrt{}}$ $\overline{6}$ and $\sqrt{6}$ related? Was this predictable? Verify that the decimal approximations for $\frac{1}{\sqrt{8}}$ and $\frac{\sqrt{2}}{4}$ are equal. Was this predictable? What is the effect of multiplying $\frac{1}{\sqrt{8}}$ and $\frac{\sqrt{2}}{\sqrt{2}}$? To show equivalence of expressions, you might have to transform one *radical expression* to make it look like another.

502. By hand, decide whether the first expression is equivalent to the second:
(a) $\sqrt{75}$ and $5\sqrt{3}$ **(b)** $\frac{\sqrt{800}}{2}$ and $10\sqrt{2}$ **(c)** $\frac{2}{\sqrt{8}}$ and $\frac{2}{2}$ **(d)** $\sqrt{\frac{1000}{6}}$ and $\frac{10\sqrt{15}}{3}$

503. The *period* of a pendulum is the time T it takes for it to swing back and forth once. This time (measured in seconds) can be expressed as a function of the pendulum length L, measured in feet, by the physics formula $T = \frac{1}{4}\pi\sqrt{2L}$.
(a) To the nearest tenth of a second, what is the period for a 2-foot pendulum?
(b) To the nearest inch, how long is a pendulum whose period is 2.26 seconds?

504. In each of the following, supply the missing factor:
(a) $2x^2 + 5x - 12 = (2x - 3)(\quad)$ **(b)** $3x^2 - 2x - 1 = (3x + 1)(\quad)$
(c) $4y^2 - 8y + 3 = (2y - 1)(\quad)$ **(d)** $6t^2 - 7t - 3 = (3t + 1)(\quad)$

505. Refer to the diagram at right and find the value of x for which triangle ABC has a right angle at C.



506. The expression $4x + 3x$ can be combined into one term, but $4x + 3y$ cannot. Explain. Can $4\sqrt{5} + 3\sqrt{5}$ be combined into one term? Can $\sqrt{2} + \sqrt{2}$ be combined into one term? Can $\sqrt{2} + \sqrt{3}$ be combined into one term? At first glance, it may seem that $\sqrt{2} + \sqrt{8}$ cannot be combined into one term. Take a close look at $\sqrt{8}$ and show that $\sqrt{2} + \sqrt{8}$ can in fact be combined into one term.

507. Jordan is thinking of a right triangle, whose sides can be represented by $x - 5$, $2x$, and $2x + 1$. Find the lengths of the three sides.

508. Because $\sqrt{8}$ can be rewritten as $2\sqrt{2}$, the expression $\sqrt{8} + 5\sqrt{2}$ can be combined into a single term $7\sqrt{2}$. By hand, combine each of the following into one term.
(a) $\sqrt{12} + \sqrt{27}$ **(b)** $\sqrt{63} - \sqrt{28}$ **(c)** $\sqrt{6} + \sqrt{54} + \sqrt{150}$ **(d)** $2\sqrt{20} - 3\sqrt{45}$

509. Solve each of the following for x. Leave your answers in exact form.
(a) $x\sqrt{2} = \sqrt{18}$ **(b)** $x\sqrt{6} = -\sqrt{30}$ **(c)** $\sqrt{2x} = 5$ **(d)** $2\sqrt{5x} = \sqrt{30}$

510. Show by finding examples that it is hardly ever true that $\sqrt{a + b}$ is the same as $\sqrt{a} + \sqrt{b}$.

511. Expand each of the following expressions and collect like terms:
(a) $(x + 2)^3$ **(b)** $(x + 3)(x^2 - 3x + 9)$ **(c)** $1 - (x + 1)^2$ **(d)** $(2x + 1)^2 - 2(x + 1)^2$

512. Given that $\sqrt{72} + \sqrt{50} - \sqrt{18} = \sqrt{h}$, find h by hand.

513. Expand and simplify the following products of two factors:
(a) $(x - 1)(x + 1)$ **(b)** $(x - 1)(x^2 + x + 1)$ **(c)** $(x - 1)(x^3 + x^2 + x + 1)$

514. Find $\sqrt{4 + \frac{1}{16}}$ on your calculator. Is the result equivalent to $\sqrt{4} + \sqrt{\frac{1}{16}}$? Explain.

515. Factor each of the following as completely as you can:

(a) $p^4 - 4p^2$ **(b)** $w^3 - 2w^2 - 15w$ **(c)** $16y - 9yz^2$ **(d)** $2x^2 + 20x + 50$

516. By hand, combine each of the following into one term.

(a) $\sqrt{\frac{1}{2}} - 2\sqrt{2} + \frac{3}{\sqrt{8}}$ **(b)** $-\sqrt{\frac{1}{12}} + \sqrt{27} - \sqrt{\frac{1}{3}}$

517. Find the solution to each equation:

(a) $\frac{x}{3} + \frac{x}{5} = 12$ **(b)** $\frac{x-2}{-2} = \frac{4x-3}{4}$ **(c)** $\frac{x+1}{3} + \frac{x-1}{x} = 2$

518. Sam is a guest on the TV show *Math Jeopardy*, and has just chosen the \$300 question in the category "Quadratic Equations." The answer is "The solutions are $x = 3$ and $x = -2$." What question could Sam ask that would win the \$300? Is there more than one possible correct question?

519. What is the distance from the point $(4, 2)$ to the point $(-3, -2)$? Be prepared to explain your method.

520. Calculate the following distances, and briefly explain your method:

(a) from $(2, 1)$ to $(10, 10)$ **(b)** from $(-2, 3)$ to $(7, -5)$

(c) from $(0, 0)$ to $(9, 8)$ **(d)** from $(4, -3)$ to $(-4, 6)$

521. Pat and Kim are having another algebra argument. Pat is quite sure that $\sqrt{x^2}$ is equivalent to x, but Kim thinks otherwise. How would you resolve this disagreement?

522. A mathematics teacher wants to make up a quadratic equation $ax^2 + bx + c = 0$, so that a, b, and c are integers, and the correct solutions are $x = \frac{1}{2}$ and $x = -3$. Find values for a, b, and c that will do the job. Is there more than one equation that will work?

523. Given four numbers a, b, c, and d, one can ask for the distance from (a, b) to (c, d). Write a procedure for computing this distance, using the four numbers.

524. Write an expression for the distance

(a) from $P = (3, 1)$ to $Q = (x, 1)$; **(b)** from $P = (3, 1)$ to $Q = (x, y)$.

525. Both legs of a right triangle are 8 cm long. In simplest radical form, how long is the hypotenuse? How long would the hypotenuse be if both legs were k cm long?

526. The hypotenuse of a right triangle is twice as long as the shortest side, whose length is m. In terms of m, what is the length of the intermediate side?

527. Can you find integer lengths for the legs of a right triangle whose hypotenuse has length $\sqrt{5}$? What about $\sqrt{7}$? Explain your reasoning.

528. Let a and b be numbers such that $0 < a < 1$ and $1 < b < 2$. Explain why $0 < a^2 < a$. Using similar inequalities, write possible intervals for each of the following numbers. Try to keep the intervals as small as possible. **(a)** $\sqrt{a}$ **(b)** $\sqrt{b}$ **(c)** b^2 **(d)** $\sqrt{\frac{b}{a}}$

529. A rectangle has an area of 36 square meters. Its length is $2\sqrt{3}$ meters. In exact form, what is the perimeter of the rectangle?

530. At noon one day, AJ decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 5 miles east and 12 miles north of its point of departure.
(a) What was AJ's position at two o'clock?
(b) How far had AJ traveled between noon and two o'clock?
(c) What was AJ's speed?

531. (Continuation) Assume that the gas tank initially held 12 gallons of fuel, and that the boat got 4 miles to the gallon.
(a) How far did AJ get before running out of fuel? When did this happen?
(b) How did AJ describe the boat's position to the Coast Guard when radioing for help?

532. What is the y-intercept of the line ax + by = c? What is the x-intercept?

533. Wes and Kelly decide to test their new walkie-talkies, which have a range of six miles. Leaving from the spot where Kelly is standing, Wes rides three miles east, then four miles north. Can Wes and Kelly communicate with each other? What if Wes rides another mile north? How far can Wes ride on this northerly course before communication breaks down?

absolute value: The absolute value of x is denoted $|x|$ and is the distance between x and zero on a number line. The absolute value of a quantity is never negative.

additive inverse: See *opposite*.

average speed: The average speed during a time interval is $\frac{\text{total distance}}{\text{total time}}$.

average a list of numbers: Add them and divide by how many numbers in the list.

axis of symmetry: A line that separates a figure into two parts that are equivalent by reflection across the line. Every *parabola* has an axis of symmetry.

balance diagram: A diagram displaying a scale that is in equilibrium.

bar chart: See *bar graph*.

bar graph: A graph used to depict categorical data where each vertical or horizontal bar displays the total, frequency or relative frequency of the variable for each category. Also know as a *bar chart*.

binomial: The sum of two unlike monomials, *e.g.* $x + 2$ or $3x^3y - 7z^5$.

box and whisker plot: See *box plot*.

box plot: A graphical rendition of statistical data based on the minimum, first quartile, median, third quartile, and maximum.

British Thermal Unit: A BTU is a unit of energy, approximately the amount needed to raise the temperature of a gallon of water by 1 degree Celsius.

cc: Abbreviation for cubic centimeter. See *conversions*.

Celsius: A scale for recording temperatures. It is defined by the stipulation that water freezes at 0 degrees and boils at 100 degrees.

center: The *center of data* or *center of distribution*. When viewing a histogram, the center of the distribution under consideration is the approximate middle. One way to visualize the center is to think about the balance point. If you were to draw the histogram on a piece of paper and cut it out, approximately where would you put your finger in order to balance the cutout? That balance point is the approximate center. If a complete data set is available, then the center can be defined as the median or the mean, and it can be calculated.

class: Data is sorted into groups, or classes, as a way to consolidate the data in order to display it in a frequency table. Typically 5 to 15 classes are used. Classes are also referred to as categories, bins or class intervals.

coefficient: See *monomial*.

collinear: Three (or more) points that all lie on a single line are collinear.

combine over a common denominator: To create a single fraction that is equal to a given sum of fractions.

commission: This is a supplementary payment to a salesperson for making a sale.

common denominator: Given a set of fractions, a common denominator is divisible by every one of the given denominators.

common monomial factor: A *monomial* that divides every term of a *polynomial*.

completing the square: Adding a quantity to a trinomial so that the new trinomial can be factored as a perfect square.

conjecture: An unproven statement that seems likely to be true.

consecutive integers: Two integers are consecutive if their difference is 1.

continuous: A variable whose values fill an *interval*. Continuous variables represent quantities that are divisible, such as time and distance. See also *discrete*.

conversions: 1 mile = 5280 feet; 1 foot = 12 inches; 1 inch = 2.54 centimeters; one liter is 1000 milliliters; a milliliter is the same as a cubic centimeter.

coordinate: A number that locates a point on a number line or describes the position of a point in the plane with respect to two number lines (axes).

cord: A unit of measure typically used for split firewood. When the wood is stacked tightly it 4 ft by 4 ft by 8 ft and is approximately 128 cubic feet in volume.

dependent variable: When the value of one variable determines a unique value of another variable, the second variable is sometimes said to *depend* on the first variable. See also *function*

degree: For a monomial, this counts how many variable factors would appear if the monomial were written without using exponents. The degree of a polynomial is the largest degree found among its monomial terms.

direct variation: Two quantities *vary directly* if one quantity is a constant multiple of the other. Equivalently, the ratio of the two quantities is constant. The graph of two quantities that vary directly is a straight line passing through the origin.

discrete: A variable that is restricted to a finite number of values. See also *continuous*.

distributive property: Short form of "multiplication distributes over addition," a special property of arithmetic. In algebraic code: $a(b+c)$ and $ab+ac$ are equivalent, as are $(b+c)a$ and $ba + ca$, for any three numbers a, b, and c. Multiplication also distributes over subtraction, of course.

endpoint convention: If an interval includes an endpoint (as in $6 \leq x$ or $y \leq -4$), this point is denoted graphically by filling in a circle. If an interval excludes an endpoint (as in $6 < x$ or $y < -4$), this point is denoted by drawing an empty circle.

equation: A statement that two expressions are equivalent. For example, $3x + 5 = 2x - 4$, $\frac{3}{4} = \frac{15}{20}$, and $(x + 3)^2 = x^2 + 6x + 9$ are all equations. The last one is an *identity*.

evaluate: Find the numerical value of an expression by *substituting* numerical values for the *variables*. For example, to evaluate $2t + 3r$ when $t = 7$ and $r = -4$, substitute the values 7 and -4 for t and r, respectively.

exponent: An integer that indicates the number of equal factors in a product. For example, the exponent is 3 in the expression w^3, which means $w \cdot w \cdot w$.

exponents, rules of: These apply when there is a *common base*: $a^m \cdot a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$; when there is a *common exponent*: $a^m \cdot b^m = (a \cdot b)^m$ and $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$; or when an exponential expression is raised to a power: $(a^m)^n = a^{mn}$. Notice the special case of the common-base rules: $a^0 = 1$.

extrapolate: To enlarge a table of values by going outside the given range of data.

factor: *Noun*: a number or expression that divides another number or expression without remainder. For example, 4 is a factor of 12, $2x$ is a factor of $4x^2 + 6xy$. *Verb*: to rewrite a number or an expression as a product of its factors. For example, 12 can be factored as $2 \cdot 2 \cdot 3$, and $4x^2 + 6xy$ can be factored as $2x(2x + 3y)$.

factored form: Written as a product of factors. For example, $(x - 3)(2x + 5) = 0$ is written in factored form. If an equation is in factored form it is particularly easy to find the solutions, which are $x = 3$ and $x = -\frac{5}{2}$ in this example.

factored form of a quadratic function: For variables x and y, and real numbers a, p and q, with $a \neq 0$, the equation $y = a(x - p)(x - q)$ is commonly called factored form or *intercept form of a quadratic function.*

Fahrenheit: A scale for recording temperatures. It is defined by the stipulation that water freezes at 32 degrees and boils at 212 degrees.

feasible region: A region of the plane defined by a set of inequalities. The coordinates of any point in the feasible region satisfy all the defining inequalities.

frequency table: A table that displays a total for each group, category, or class.

function: A function is a rule that describes how the value of one quantity (the dependent variable) is determined uniquely by the value of another quantity (the independent variable). A function can be defined by a formula, a graph, a table, or a text.

greatest common (integer) factor: Given a set of integers, this is the largest integer that divides all of the given integers. Also called the *greatest common divisor*.

greatest common (monomial) factor: Given a set of *monomials*, this is the largest monomial that divides all of the given monomials.

guess-and-check: A method for creating equations to solve word problems. In this approach, the *equation* emerges as the way to check a *variable* guess. Initial practice is with constant guesses, so that the checking can be done with ordinary arithmetic.

histogram: A specific type of *bar chart* where the widths of the bars are the *class* intervals or bins and the height is the frequency or relative frequency of the data values within each bin. Often the data is continuous, thus the bars are drawn adjacent to one another.

hypotenuse: In a right triangle, the side opposite the right angle. This is the longest side of a right triangle.

identity: An *equation*, containing at least one *variable*, that is true for all possible values of the variables that appear in it. For example, $x(x + y) = x^2 + xy$ is true no matter what values are assigned to x and y.

income: See *revenue*.

inequality: A statement that relates the positions of two quantities on a number line. For example, $5 < x$ or $t \leq 7$.

integer: The whole number and their opposites, that is, the set $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

intercept form of a quadratic function: see *factored form of a quadratic function*

interpolate: To enlarge a table of values by staying within the given range of data.

intersection point: See *point of intersection*.

interval: A connected piece of a number line. It might extend infinitely far in the positive direction (as in $-1 < x$), extend infinitely far in the negative direction (as in $t \leq 7$), or be confined between two endpoints (as in $2 < m \leq 7$).

irrational number: A number that cannot be expressed exactly as the ratio of two integers. Two familiar examples are π and $\sqrt{2}$. See *rational number*.

lattice point: A point both of whose coordinates are integers. The terminology derives from the rulings on a piece of graph paper, which form a lattice.

light-year: Approximately 5.88 trillion miles, this is a unit of length used in astronomical calculations. As the name implies, it is the distance traveled by light during one year.

like terms: These are *monomials* that have the same variables, each with the same exponents, but possibly different numerical coefficients. Like terms can be combined into a single monomial; unlike terms cannot.

linear: A polynomial, equation, or function of the first degree. For example, $y = 2x - 3$ defines a linear function, and $2x + a = 3(x - c)$ is a linear equation.

linear combinations: A method for solving systems of linear equations.

loss: This is a negative *profit*.

lowest terms: A fraction is in lowest terms if the greatest common factor of the numerator and denominator is 1. For example, $\frac{14}{}$ is not in lowest terms because 14 and 21 have 7 as a common factor. When numerator and denominator are each divided by 7 the resulting fraction $\frac{2}{3}$ is equal to $\frac{14}{21}$, and is in lowest terms.

mean: The average of a set of numbers or data set.

median: The midpoint of a data set or the strip of land between the lanes of opposing traffic on a divided highway. Fifty percent of the data lies above the median and 50% lies below. The median may or may not be a data value.

model: An equation (or equations) that describe a context quantitatively.

monomial: A constant (real number) or a product of a constant and variables. In the case when the monomial is not simply a constant, the constant part is called the *coefficient*. Any exponents of variables are restricted to be non-negative integers. For example: 3, x^3, $\frac{4}{5}y^3x^2$, and $3x^5$ are monomials. See also *binomial*, *polynomial*, and *trinomial*. [486]

multiplicative inverse: See *reciprocal*.

number line: A line on which two points have been designated to represent 0 and 1. This sets up a one-to-one correspondence between numbers and points on the line.

opposite: When the sum of two quantities is zero, they are called opposites (or *additive inverses*); each is the opposite of the other. On a number line, zero is exactly midway between any number and its opposite.

or: Unless you are instructed to do otherwise, interpret this word *inclusively* in mathematical situations. Thus a phrase "...(something is true) or (something else is true) ..." allows for the possibility that *both* (something is true) and (something else is true).

parabola: The shape of a graph of the form $y = ax^2 + bx + c$. All parabolas have a *vertex* and an *axis of symmetry*.

PEA: An abbreviation for Phillips Exeter Academy.

percentile of a data set: The percentage of data that fall at or below that data value. For example, if 10% of scores are at or below 68, then the value of 68 is the 10th percentile of the data set or of the distribution of scores.

perimeter: The total length of the sides of a figure. The perimeter of a rectangle is twice the length plus twice the width. In algebraic code, $p = 2l + 2w = 2(l + w)$.

period of a pendulum: The time needed for a pendulum to swing back and forth once.

point of intersection: A point where one line or curve meets another. The coordinates of a point of intersection must satisfy the equations of the intersecting curves.

point-slope form: The line with slope m that passes through the point (h, k) can be described in point-slope form by either $y - k = m(x - h)$ or $y = m(x - h) + k$.

polynomial: A sum of *monomials*. See also *binomial* and *trinomial*.

population: The entire set of people, animals or things that share something in common so that information or data may be collected from them to answer a question. See *sample*.

prep: A term used at Phillips Exeter Academy to refer to a ninth grader. Historically the term *junior* has also been used.

profit: The result of deducting total costs from total *revenues*. See also *loss*.

proportion: An *equation* stating that two *ratios* are equal. For example, $\frac{4}{6} = \frac{6}{9}$ is a proportion.

proportional: Two quantities are proportional if one can be expressed as a fixed constant times the other. This constant is called a *proportionality constant*. For example if $y = kx$, y and k are proportional and k is a proportionality constant.

proportionality constant: See *proportional*.

Pythagorean Theorem: The square of the length of the *hypotenuse* of a right triangle equals the sum of the squares of the lengths of the other two sides.

quadratic equation: A polynomial equation of degree 2.

quadratic formula: The solution to the quadratic equation $ax^2 + bx + c = 0$, which can be written as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

quadratic function: A function defined by an equation of the form $y = ax^2 + bx + c$, where y is the dependent variable. The word quadratic comes from a Latin word that means "to make square", and it refers to the presence of a squared variable in the equation.

quadrillion: To an English speaker, this is $1000000000000000 = 1.0 \times 10^{15}$. [511]

quartiles: The values that divide a set of ranked data into quarters. The first quartile is the median of the lower half of the data set. The second quartile is the median. The third quartile is the median of the upper half of the data set. If there are an odd number, n, of data values, the lower half is the lowest $\dfrac{(n-1)}{2}$ data values, while the upper half is the highest $\dfrac{(n-1)}{2}$ data values. If there are an even number, n, of data values, the lowest half is the lowest $\dfrac{n}{2}$ data values and the upper half is the highest $\dfrac{n}{2}$ data values.

radical expression: An expression containing roots, like $\sqrt{2}$ or $\sqrt{x-3}$.

rate (of change): Rate often denotes speed, *i.e.* units of distance per unit of time. For example, 60 miles per hour, 50 feet per second, 67 furlongs per fortnight. A general rate of change is similar: number of units of A *per* one unit of B. For example, 5 liters per student, 24 angels per pinhead, 1.3 thousand people per year, 70 passengers per lifeboat.

ratio: The ratio of *a to b* is the expression $\dfrac{a}{b}$; also written $a{:}b$ or a/b or $a \div b$.

rational number: A number that can be written as the ratio of two integers. For example, 5, $\dfrac{7}{13}$, and 0.631 are rational numbers. See also *irrational number*.

reciprocal: When the product of two quantities is 1, they are called reciprocals (or *multiplicative inverses*); each is the reciprocal of the other. For example, 0.2 is the reciprocal of 5, and $\dfrac{a}{b}$ is the reciprocal of $\dfrac{b}{a}$. Any nonzero number has a reciprocal.

relatively prime integers have no common divisor that is larger than 1.

revenue: This is money received as a result of sales; also known as *income*.

sample A subset of a *population*.

Scandinavian flags are all based on the *Dannebrog*.

scatter plot: The graph of a discrete set of data points.

scientific notation: The practice of expressing numbers in the form $a \times 10^n$, in which n is an integer, and a is a number whose magnitude usually satisfies $1 \leq |a| < 10$.

simplest radical form: An expression $a\sqrt{b}$ is in simplest radical form if b is a positive integer that has no factors that are perfect squares. For example, $18\sqrt{5}$ is in simplest radical form, but $5\sqrt{18}$ is not.

simultaneous solution: A solution to a system of equations must satisfy *every* equation in the system. [329]

skewed left: See *skewed right*. [630]

skewed right: The shape of a distribution of a data set is described as being skewed right if most of the data points are smaller in value with a few data points being much larger. These large values skew, or distort, the right tail of the distribution. Similarly a distribution is *skewed left* if most of the data points are larger in value with a few data points being much smaller. [630]

slope: The slope of a line is a measure of its steepness. It is computed by the ratio $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$. A line with positive slope rises as the value of x increases. If the slope is negative, the line drops as the value of x increases. [140]

slope-intercept form: The line whose slope is m and whose y-intercept is b can be described in slope-intercept form by $y = mx + b$. [180]

solve: To find the numerical values of the variables that make a given equation or inequality a true statement. Those values are called *solutions*. [39]

square: To multiply a number by itself; *i.e.* b^2 is the square of b.

square root: A square root of a nonnegative number k is a number whose square is k. If k is positive, there are two such roots. The positive root is denoted $\sqrt{k}$, and sometimes called "the square root of k." The negative root is denoted $-\sqrt{k}$.

standard form: A linear equation in the form $ax + by = c$. Notice that this refers to a linear equation, which should not be confused with *standard form of a quadratic function*.

standard form of a quadratic function: For variables x and y, and real numbers a, b and c, with $a \neq 0$, the equation $y = ax^2 + bx + c$ is commonly called standard form of a quadratic function.

substitution: Replacing one algebraic expression by another of equal value.

system of equations: A set of two or more equations. The solution to a system of linear equations is the coordinates of the point where the lines meet. The solution is the values of the variables that satisfy all the equations of the system at the same time.

texting: Texting while driving a vehicle is illegal in nearly all US states. As of 2016, according to the National Council of State Legislatures, the exceptions were Montana, Arizona, Missouri (for people under 21) and Texas (for bus drivers when a passenger 17 and younger is present, for intermediate license holders for first 12 months, or for all drivers in school crossing zones). More importantly, texting while driving is a terrible idea regardless of your state.

trial-and-error factoring: Factoring a trinomial $ax^2 + bx + c$ into the product of two binomials $(px + q)(rx + s)$ by using trial-and-error to find numbers p, q, r, and s such that $pr = a$, $qs = c$, and $ps + qr = b$.

triangular number: Any integer obtained by summing $1 + 2 + \cdots + n$, for some positive integer n.

trinomial: The sum of three unlike monomials, e.g. $x^2 - x + 2$ or $3x^3y - 7x^5 + 8qrs$.

variable: A letter (such as x, y, or n) used to represent a number. A few letters (such as m and n) tend to be associated with integers, but this is not a rule.

variability: A measure of how spread out a data set is about its center.

vary directly: See *direct variation*.

versus: This was once the name of a television sports network. It is also a word that frequently appears when describing graphs, as in "the graph of volume versus time." This book follows the convention of associating the first-named variable with the vertical axis, and the second-named variable with the horizontal axis. The first-named variable is *dependent* on the second-named variable.

vertex: A "corner" point on an absolute-value graph. The vertex of the graph $y = a|x-h|+k$ is (h, k). [259] The vertex of the graph of a quadratic function is the point whose y-coordinate is extreme (highest or lowest). It is the point on the parabola that is also on the axis of symmetry. The vertex of the graph $y = a(x - h)^2 + k$ is (h, k).

vertex form of a quadratic function: For variables x and y, and real numbers a, h, and k with $a \neq 0$, the equation $y = a(x - h)^2 + k$ is commonly called vertex form of a quadratic function. The ordered pair (h, k) denotes the coordinates of the vertex.

voluntary response: A survey is considered to be a voluntary response survey if it is sent to, or made available to, a population of interest and those individuals decide whether or not they want to participate in the survey, that is, they volunteer.

water lily: In 1849, Henry Wadsworth Longfellow wrote his novel *Kavanagh*, which contained several mathematical puzzles. One was about water lilies.

whole numbers: The numbers $\{0, 1, 2, 3, \ldots\}$.

x-intercept: The x-coordinate of a point where a line or curve meets the x-axis. The terminology is sometimes applied to the point itself.

y-intercept: The y-coordinate of a point where a line or curve meets the y-axis. The terminology is sometimes applied to the point itself.

zero-product property: If the product of a set of factors is zero, then at least one of the factors must be zero. In symbols, if $ab = 0$ then either $a = 0$ or $b = 0$.

Math 1 Extra Practice Name_____ ID: 1

1. Dimension Analysis

Date_____

1) If you are 13 years old, how many seconds have you been alive?

2) Convert 65 miles per hour to feet per second.

3) The price of a Big Mac meal in the US is $5.99. This price of a Big Mac meal in France is 9 euros. How many dollars does the Big Mac meal cost in France?

4) The price of a apples in the US is $5.50 per 3 pounds. The price of apples in Germany is 1.2 euros/kilogram. Where are apples cheaper? (One kilogram is equivalent to 2.20462 pounds.)

5) Many major-league baseball pitchers can throw the ball at 95 miles per hour. At that speed, how long does it take a pitch to travel from the pitcher's mound to home plate, a distance of 60 feet, 6 inches? Give your answer to the nearest hundredth of a second. There are 5280 feet in a mile and 12 inches in a foot.

6) The number of miles represented on a map is directly proportional to the number of centimeters you measure on that map. Suppose that the scale is 2 cm = 15 miles. What is the distance on the map from Pittsburgh to Buffalo 220 miles away?

7) (Continuation) What is the distance on the map between Pittsburgh and Nashville, which are 580 miles apart?

8) Convert 75 miles per hour to kilometers per hour. (One mile is approximately 1.61 kilometers.)

9) Convert 3/4 cup to liters. (There are 8 ounces in a cup and 1 ounce to 0.0295735 liters.)

10) A gallon of water weighs 8.34 pounds at room temperature. Convert this to kilograms.

Math 1 Extra Practice Name_____ ID: 1

2. Solving for a Variable

Date_____

Solve each equation for the indicated variable.

1) $g = a - c$, for a

2) $c + x = d - r$, for x

3) $c - x = xg$, for x

4) $a + m = az$, for a

Solve each equation.

5) $p + 20 = 33$

6) $-16 = -18 + a$

7) $-8p = -32$

8) $10 = 6x - x$

9) $5m - 4m = 2$

10) $-7x - 6(4x - 6) = 253$

11) $182 = 7(1 + 5n)$

12) $7(x + 5) = 30 + 8x$

13) $7(1 + x) = 16 + 4x$

14) $6(5x - 2) + 6(6x - 1) = -18$

15) $3(1 - n) = -(n - 3) - 6$

16) $7(5 - 3x) = -5(5x + 1)$

Math 1 Extra Practice Name_____ ID: 1

3. Distributive Property

Date_____

Simplify each expression.

1) $4r - 8 - 5r - 2$

2) $-4x - 2x$

3) $2(1 + 10p)$

4) $9(1 - 4m)$

5) $2(5b + 5) - 3$

6) $m - 10(1 + 10m)$

7) $7(k - 3) - 5(2k + 10)$

8) $10(9 - 6b) - 7(b + 5)$

9) $\frac{5}{3}p + \frac{6}{5}p$

10) $\frac{1}{2}\left(\frac{7}{2}r - \frac{1}{4}\right)$

Math 1 Extra Practice Name_____ ID: 1

4. Percent

Date_____

Find each percent change. State if it is an increase or a decrease.

1) From 67 to 48

2) From 95 to 24

3) From 66 to 91.1

4) From 19 to 57

5) From 67 to 45

6) From 61 to 84

Solve each problem.

7) What percent of 24 is 67?

8) 33 is what percent of 78?

9) 95% of 157 is what?

10) What is 82% of 77?

11) 45 is 81% of what?

12) 30% of what is 46?

13) Mary bought a coat that cost $72 and was labeled 40% off. What is the original price of the coat?

14) A stock went from $10,235 per share to $11,120. What was the percent increase?

5. Scientific Notation

Date_____

Write each number in scientific notation.

1) 1920

2) 0.0000392

3) 0.578

4) 7300

Write each number in standard notation.

5) 5.25×10^{1}

6) 4.9×10^{2}

7) 5×10^{-4}

8) 8.4×10^{-3}

Simplify. Write each answer in scientific notation.

9) $\dfrac{3 \times 10^{4}}{1.5 \times 10^{3}}$

10) $(5 \times 10^{6})(9 \times 10^{4})$

11) $(4.55 \times 10^{-6})(8.1 \times 10^{4})$

12) $\dfrac{7.91 \times 10^{4}}{7 \times 10^{2}}$

Mathematics 1 Extra Practice Name_____ ID: 1

6. Rational Expressions and Equations

Date_____

Simplify each expression.

1) $\frac{x+4y}{20x}+\frac{5x}{20x}$

2) $\frac{x+2y}{8xy}-\frac{2x-4y}{8xy}$

3) $\frac{4u-2v}{15v^2}-\frac{u+5v}{15v^2}$

4) $\frac{6}{3xy}-\frac{6}{5xy}$

5) $2n+\frac{5m}{12n}$

6) $\frac{4x}{6y}-\frac{5y}{2x^3}$

7) $\frac{3x}{5y}-\frac{3x}{6}$

8) $\frac{5}{2}+\frac{5x}{4y}$

9) $\frac{4}{5y}+\frac{3}{4xy}$

10) $\frac{5x}{6}-\frac{6x-4y}{2xy}$

Mathematics 1 Extra Practice Name_____ ID: 1

7. Ratio and Proportion

Date_____

Solve each proportion.

1) $\frac{6}{3} = \frac{5}{x}$

2) $\frac{6}{10} = \frac{8}{n}$

3) $\frac{10}{2} = \frac{p}{10}$

4) $\frac{9}{4} = \frac{6}{5x}$

5) $\frac{2}{k+8} = \frac{6}{9}$

6) $\frac{6}{8} = \frac{4}{n-2}$

7) $\frac{x}{2} = \frac{x+5}{7}$

8) $\frac{10}{3} = \frac{n}{n-7}$

9) $\frac{k+2}{k-1} = \frac{2}{8}$

10) $\frac{x+6}{6} = \frac{x-1}{8}$

11) $\frac{k-8}{2} = \frac{k-6}{5}$

12) $\frac{x+5}{7x} = \frac{3}{10}$

13) Write and solve a proportion to answer this question. Given that y varies directly with x and $y = 80$ when $x = 15$, find y when $x = 40$.

14) Write and solve a proportion to answer this question. Given that a varies directly with b and $a = 220$ when $b = 35$, find a when $b = 133$.

15) You want to make 1/3 of a recipe of brownies that call for $1\frac{3}{4}$ cups of flour. How much flour should you add?

Mathematics 1 Extra Practice Name_____ ID: 1

8. Factoring

Date_____

Factor the common factor out of each expression.

1) $-5n^3 + 15$

2) $-70a + 90$

3) $-63n^3 + 14n$

4) $2n^2 - 2n$

Factor each completely.

5) $2x^3 - 7x^2 + 6x - 21$

6) $4v^3 - 6v^2 + 14v - 21$

7) $49n^3 - 35n^2 - 56n + 40$

8) $4v^3 + 3v^2 - 8v - 6$

9) $n^2 + 5n$

10) $v^2 + 8v - 9$

11) $n^2 + 2n - 8$

12) $x^2 + 6x - 40$

9. Absolute Value Equations and Inequalities

Date_____

Solve each equation.

1) $|b| = 3$

2) $|x| = 6$

3) $|5 + r| = 1$

4) $|x - 5| = 2$

5) $|-1 + p| = 4$

6) $|-4m| = 12$

7) $\left|\frac{n}{7}\right| = 3$

8) $|k - 9| = 4$

9) $|-8 + x| = 9$

10) $|n + 1| = 9$

11) $|x + 4| = 4$

12) $|b - 3| = 11$

13) $|7 + a| = 17$

14) $|-5r| = 45$

10. Solving Quadratic Equations

Date_____

Solve each equation by taking square roots.

1) $m^2 = 81$

2) $25x^2 = 25$

3) $-1 + 100r^2 = 15$

4) $25v^2 + 9 = 45$

Solve each equation by completing the square.

5) $n^2 + 6n - 26 = 0$

6) $x^2 - 18x + 45 = 0$

7) $n^2 + 10n + 26 = 10$

8) $a^2 + 18a - 38 = 2$

Mathematics 1 Extra Practice Name_____ ID: 1

11. Laws of Exponents

Date_____

Simplify. Your answer must contain only positive exponents.

1) $4 \cdot 4^4$

2) $4^2 \cdot 4^4$

3) $(-4)^3 \cdot (-4)^2$

4) $4^2 \cdot 4^3$

5) $-3x \cdot -3x^2$

6) $a^3 \cdot -3a^3$

7) $4x \cdot 2x^3$

8) $-2k \cdot 2k^2$

9) $n^3 \cdot -2n^4 \cdot -2n^2$

10) $-3x^3 \cdot 2x^3$

11) $2m^2n^2 \cdot 4mn^2$

12) $-x^3 \cdot -4x^4y^4$

13) $-yx^4 \cdot x^3y^4$

14) $-3yx^3 \cdot -4x^3y^3$

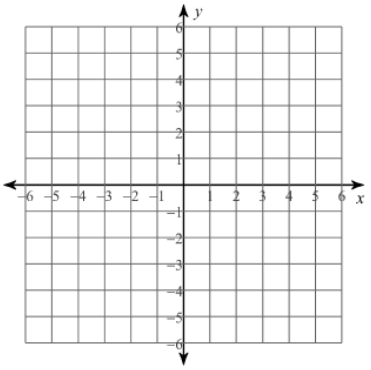
Mathematics 1 Extra Practice Name_____ ID: 1

12. Lines

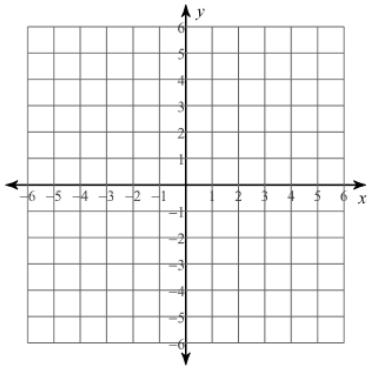
Date_____

Sketch the graph of each line.

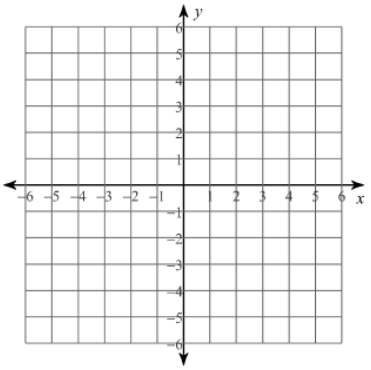
1) $x = 4$



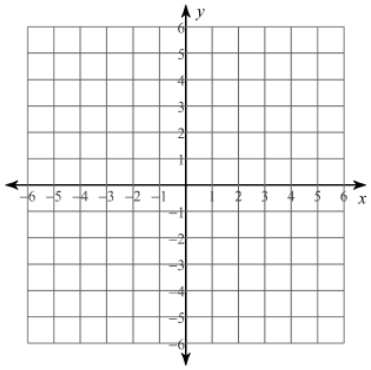
2) $y = -4x + 5$



3) $y = \frac{3}{5}x + 4$



4) $y = -\frac{3}{2}x + 5$



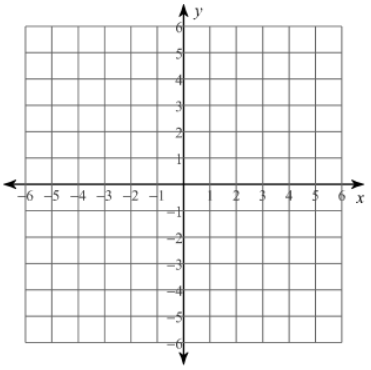
Mathematics 1 Extra Practice Name_____ ID: 1

13. Linear Inequalities

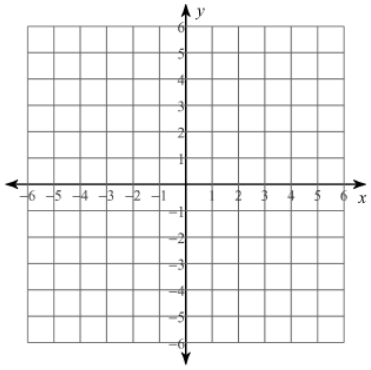
Date_____

Sketch the graph of each linear inequality.

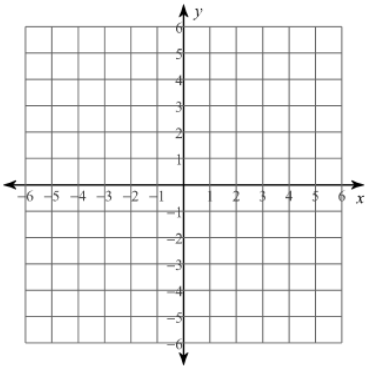
1) $y > -\frac{1}{2}x - 2$



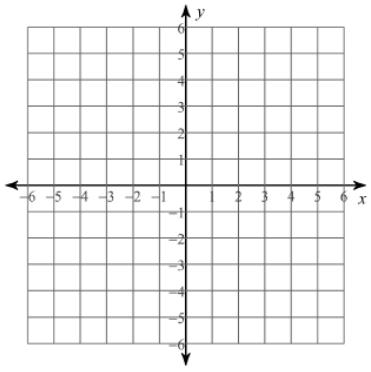
2) $y \geq \frac{3}{2}x - 3$



3) $y > \frac{1}{2}x - 3$



4) $y \leq -3$



Mathematics 1 Extra Practice Name_____ ID: 1

14. Systems of Linear Equations

Date_____

Solve each system by graphing.

1) $y = \frac{1}{2}x + 4$
$y = -2x - 1$

2) $y = \frac{5}{2}x + 3$
$y = -x - 4$

3) $y = -8x - 4$
$y = -x + 3$

4) $y = -\frac{5}{4}x - 1$
$y = -\frac{1}{4}x + 3$

Solve each system by elimination (linear combination).

5) $3x + 4y = -28$
$-3x + 2y = 4$

6) $2x - y = 3$
$6x + y = -27$

7) $x - 7y = -17$
$x + 8y = 13$

8) $-2x + 6y = 0$
$-2x + 2y = 12$

9) $-x + 10y = -5$
$-7x + 10y = 25$

10) $-10x + 8y = 0$
$2x + 4y = -28$

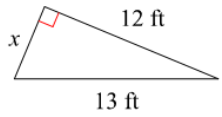
Mathematics 1 Extra Practice Name_____ ID: 1

15. The Pythagorean Theorem and the Distance Formula

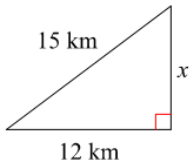
Date_____

Find the missing side of each triangle.

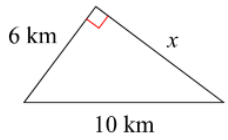
1)



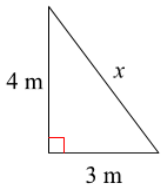
2)



3)

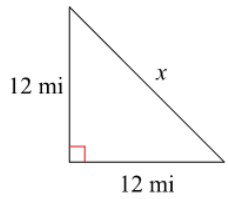


4)

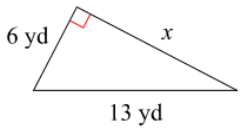


Find the missing side of each triangle. Leave your answers in simplest radical form.

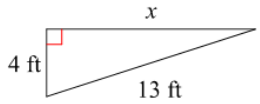
5)



6)



7)



Mathematics 1 Extra Practice Name_____ ID: 1

16. Radicals

Date_____

Simplify and leave your answer in simplest radical form.

1) $\sqrt{441}$

2) $\sqrt{112}$

3) $\sqrt{8}$

4) $\sqrt{392}$

5) $\sqrt{500}$

6) $\sqrt{900}$

7) $\sqrt{108}$

8) $\sqrt{200}$

9) $\sqrt{24}$

10) $\sqrt{250}$

11) $\sqrt{810}$

12) $\sqrt{28}$

Mathematics 1 Extra Practice Name_____ ID: 1

17. Mean, Mode and Median

Date_____

Find the mode, median, and mean for each data set.

1) # Words in Book Titles

2 6 1 2 4 3 4 4
2 3

2) Games per World Series

6 4 7 7 5 4 5 5
6

3) Games per World Series

6 7 6 7 4 7 4 7
6 4 5

4) Academy Awards

Movie	# Awards
The Best Years of Our Lives	7
Mrs. Miniver	6
It Happened One Night	5
Gone with the Wind	8

Movie	# Awards
Titanic	11
Patton	7
The Artist	5
Marty	4

Movie	# Awards
Shakespeare in Love	7
West Side Story	10
In the Heat of the Night	5

5) Nobel Laureates

Name	Age
Barack Hussein Obama Jr.	48
Koichi Tanaka	43
Hartmut Michel	40
James McGill Buchanan Jr.	67
James Watson Cronin	49
Roger Bruce Myerson	56
Kenneth Joseph Arrow	51
Konstantin Sergeevich Novoselov	36
Eric Francis Wieschaus	48

6) Sales Tax

State	Percent
Massachusetts	6.25
Wisconsin	5
Washington	6.5

State	Percent
Alabama	4
Maryland	6

State	Percent
Utah	4.7
Alaska	0

State	Percent
Hawaii	4
Ohio	5.75

State	Percent
North Dakota	5
New Jersey	7

Mathematics 1 Extra Practice Name_____ ID: 1

18. Triangle Inequality

Date_____

State if the three numbers can be the measures of the sides of a triangle.

1) 8, 11, 3

2) 13, 11, 11

3) 7, 4, 8

4) 6, 15, 9

5) 7, 4, 7

6) 12, 12, 8

7) 8, 11, 18

8) 10, 9, 20

9) 6, 7, 16

10) 11, 6, 6

Two sides of a triangle have the following measures. Find the range of possible measures for the third side.

11) 7, 10

12) 6, 12

13) 7, 8

14) 8, 9

15) 6, 8

16) 10, 8

17) 11, 6

18) 12, 9

19) 11, 8

20) 10, 11