MATH III

Mathematics Department
Shady Side Academy
Pittsburgh, PA
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[Problems originated with the Mathematics Department at Phillips Exeter Academy, NH.]

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SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

MATH III

MISSION/HISTORY: As part of an on-going curriculum review the Mathematics Department of Shady Side Academy sent two members of the senior school math faculty to visit Phillips Exeter Academy [PEA] in 2008 to observe their classes. After this observation and much reflection, the department decided to adopt this problem-based curriculum. The materials used in the Mathematics I and II courses are taken directly from PEA. We thank the teachers at PEA for the use of their materials.

RATIONALE: The Shady Side Academy Mathematics Department Goals are as follows:

Students will develop the habit of using mathematical reasoning based on logical thinking.
Students will develop adequate skills necessary to solve problems mathematically.
Students will recognize that the structure and order of mathematics can be discovered in the world around us.
Students will recognize the connections of mathematics to other disciplines.
Students will express themselves clearly in mathematical discourse.
Students will be familiar with and proficient in appropriate technology.
Students will achieve their highest mathematical goals.
Students will gain an appreciation for the study of mathematics.

In addition, the teachers in the Department of Mathematics want you to be an articulate student of mathematics. We want you to be able to speak and write mathematics well. We want you to be a fearless problem solver so that you approach problems with curiosity and not trepidation. The Mathematics II classroom is student-centered. The curriculum is problem-based with an integrated design. You will continually learn new material while reviewing prior topics.

EXPECTATIONS: In order for you to be successful in this course, the Mathematics Department has the following suggestions and expectations. First, we expect you to attempt every problem. More than merely writing the problem number, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. Next, seek help wherever you can find it. We expect you to cooperate with your peers and teachers. The Mathematics Department is a team of teachers striving to help all students reach their potential. You are encouraged to ask any teacher for help if your own is not available. Finally, as stated in the Student Handbook on page 13: "Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets." We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

To the Student

Contents: Members of the PEA Mathematics Department have written the material in this book. As you work through it, you will discover that algebra and geometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a section on tangents to circles. The curriculum is problem-centered, rather than topic-centered. Techniques and theorems will become apparent as you work through the problems, and you will need to keep appropriate notes for your records — there are no boxes containing important theorems. You will begin the course with this binder of problems, graph paper, and a protractor. All of your solutions are to be kept in this binder. It will be periodically collected and will factor into your term grade. There is no index in your binder but the reference section at the end should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

Comments on problem-solving: You should approach each problem as an exploration. Reading each question carefully is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. It is important to make accurate diagrams whenever appropriate. Useful strategies to keep in mind are: create an easier problem, guess and check, work backwards, and recall a similar problem. It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day.

Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher. If you have taken the time to think about a problem, you should bring to class a written record of your efforts, not just a blank space in your notebook. The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. Proper spelling is essential for clear written communication.

About technology: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind: write before you calculate, so that you will have a clear record of what you have done; store intermediate answers in your calculator for later use in your solution; pay attention to the degree of accuracy requested; refer to your calculator's manual when needed; and be prepared to explain your method to your classmates. Also, if you are asked to "graph $y = (2x - 3)/(x + 1)$", for instance, the expectation is that, although you might use your calculator to generate a picture of the curve, you should sketch that picture in your notebook or on the board, with correctly scaled axes.

Shady Side Academy
Introductory Math Guide for Students

Homework

First, we expect you to attempt every problem. More than merely writing the problem number in your notebook, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. As stated in the Student Handbook: “Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets.” We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

Going to the Board

It is very important to go to the board to put up homework problems. Usually, every homework problem is put on the board at the beginning of class, presented, and then discussed in class. By doing this, you will develop your written and oral presentation skills.

Plagiarism

You can get help from almost anywhere, but make sure that you cite your help, and that all work shown or turned in is your own, even if someone else showed you how to do it. Never copy work from others. Teachers do occasionally give problems/quizzes/tests to be completed at home. You may not receive help on these assessments, unless instructed to by your teacher; it is imperative that all the work is yours. More information about plagiarism can be found on page vii in your binder.

Math Extra-Help

Getting help is an integral part of staying on top of the math program here at Shady Side Academy. It can be rather frustrating to be lost and stuck on a problem. Teachers, peer tutors, study groups, the internet, your resource book and classmates are all helpful sources.

Teachers and Meetings

The very first place to turn for help should be your teacher. Teachers at SSA are always eager to help you succeed. The Math Department office is located on the 3rd floor of Rowe Hall. Individual meetings can be arranged with teachers during study halls, free periods, or after school. You can always ask or email any teacher in the department for help. Getting help from your teacher is the first and most reliable source to turn to for extra help.

SSA Student Quotes

"This program really helped me learn and understand the concepts of Algebra II. It helped us as a group because we covered materials together. We all said our own ideas and accepted when they were wrong. It gave each individual confidence in their understanding of the material. Some days we did not check over every problem like I would have liked to, but this allowed me to be a frequent visitor in the math office. It was a different approach that ran very smoothly in this class."
--Betsy Vuchinich, '12

"I loved Math II this year because the curriculum was completely different from anything I've previously encountered in math. We didn't use a book for the majority of the year, instead we focused on more complicated word problems and worked together in small groups to solve these difficult problems. This forced us to think through the problems and think about "Why?" more so than "How?" and this was a much different look for a math class. Working with your peers in a setting that promoted group work was refreshing, and I enjoyed it very much. I hope the Math Department continues to use this curriculum."
--Jonathan Laufe, '12

"I came into the year unsure of what to think about this approach to mathematics. I had criticism and positive words about the packet, and I didn't know what to expect. Though sometimes I was confused, in the end, everything worked out."
--Erin Gorse, '12

"The curriculum definitely took some getting used to but once you figure it out, it has a balance of being challenging and easy at the same time."
--Elijah Williams, '13

"I thought the word problems were unnecessary at first, then I found my mind starting to expand."
--Guy Philips, '13

"The curriculum for Math II under Ms. Whitney was not easy, but the use of a packet full of word problems that challenged our minds to apply concepts previously learned really expanded our knowledge much easier than traditional out of the book teaching. The packet introduced to me a new way of learning that I was not familiar with, but even if students are struggling to understand concepts of problems then teachers make themselves available to work with you very often. You will not get by easily in this course by daydreaming, but this hands on experience in the classroom of interacting with your classmates and teacher will show how much easier learning is because you stress previously learned material and open windows to other, more complex problems."
--Christopher Bush, '13

“Math I is a great way to learn and if I had to describe it in one word it would be ‘Utopian’: The classroom environment motivates me to do better and it teaches you to either accept your method or to abandon it for a better one. The fact that the teachers act as moderators in the classroom makes it a better way to learn because it really gets you to think. I loved Math One and look forward to doing more Exeter problems in Math II.”
--Adam D'Angelo, ‘14

“I think Math II will teach you a lot about not only math. Even coming out of a year of "the binder", Math I, I found that I actually learned math a lot differently this year than last; this required me to be malleable with how I approached things and studied. "Doing math differently", for lack of a better word, was something that confronted me this year, and it challenged things I already did in a healthy way: communicating via email, asking questions, organizing things differently, learning how to take notes in new ways, and meeting new people. A beneficial thing I recommend is visiting the math office even once a week after school to meet with your teacher(or any teacher) and go over homework, do practice problems, and chat. There are a lot of cool people in that office, and the more you communicate with them, the more you will enjoy and feel confident with math. Overall, you will probably grow a lot as a person throughout taking Math II, and learn many valuable life skills; be excited for that.”
--Felicia Reuter, ‘17

- “ I have found with the Math II binder that it is much easier to approach the problems with other people. Doing homework with friends mimics an actual class and confusion is more easily avoided. It is also helpful to bounce ideas off each other and you might even find a new way to solve a problem!
- If you are struggling with one type of problem (i.e. proofs) go back to the basics and get worksheets or practice problems from a teacher. You will find that once you have mastered the foundation of the problem you will more easily be able to attempt the harder stuff.
- Always have the full answer to the problems written down by the end of class. Or using a smartphone, take a picture of the board. This will ensure a solid reference for studying.
- Take advantage of your resources! Go to your teacher outside of class, or any other teacher in the math department, they are more than willing to help. It will pay off to put in the extra effort.
- Lean into discomfort. It is okay to not have a full solution to a problem, explain what you know and trust that your class and teacher are there to support you and help you find an answer without judgement.
- Stay organized! Often in class teachers will reference problems from previous pages to make connections or to draw conclusions so it is most helpful to organize your binder in a way that makes the most sense to you. Also, tests and quizzes are pulled from a number of different pages so find a system of organization that can help you succeed in your studies.”

--Caroline Benec, ‘17

SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

Policy on Plagiarism and Cheating

At the beginning of each course, each teacher in the Mathematics Department will explain to the class what is expected with regard to the daily completion of homework, the taking of in-class tests, make-up tests, and take-home tests. Students will be told whether or not they may use books and/or other people when completing in-class or out-of class assignments/tests. The consequences listed below will take effect if a teacher suspects that a student is in violation of the instructions given for a particular assignment or test.

PLAGIARISM

Plagiarism is the act of representing something as one's own without crediting the source. This may be manifest in the mathematics classroom in the form of copying assignments, fabricating data, asking for or giving answers on a test, and using a "cheat sheet" on an exam.

CHEATING

If, during an in-class test, the teacher in that room considers that a student has violated the teacher's instructions for the test, the teacher will instruct the student that there is a suspicion of cheating and the teacher will initiate the consequences below. If a student is taking a make-up test out of class and any teacher considers that the student is, or has been, cheating the teacher will bring the issue to the notice of the Department Chair, and initiate the consequences below. Sharing the content of a particular test with an individual who has not taken the test is considered by the department to be cheating by both parties.

CONSEQUENCES

When a teacher suspects plagiarism or academic dishonesty, the teacher and Department Chair will speak with the student. The Department Chair, in conjunction with the Dean of Student Life, will determine whether plagiarism or academic dishonesty has occurred. If plagiarism or academic dishonesty is determined, the Dean of Student Life and the Department Chair make the decision about the appropriate response to the situation, which will likely include referral to the Discipline Committee. The Department Chair will contact the family to discuss the infraction and consequence. If a Discipline Committee referral is made, the Dean of Student Life will follow up with the family as well.

In any case of plagiarism or cheating, the student concerned will likely receive a failing grade for that piece of work, as well as any other appropriate steps deemed necessary by the Department Chair and the Dean of Academic Life.

Math III Guided Notes

The following pages are a place for you to organize the concepts, topics, formulas and ideas you learn this year. You can use them in any way you wish. It is suggested that when you come upon an important finding or result in class or on your own, that you write it in these notes so that it is easily accessible when it comes time to study for an exam or review material. These notes are not a substitute for taking notes in other ways, and the Mathematics Department encourages you to use a notebook to have a record of your work, corrections and any notes you get in class. We hope this is useful to you, and we welcome any feedback.

--SSA Senior School Math Department

Special Right Triangles

Type	Ratio/Notes/Drawing
45-45-90	
30-60-90	

Circles

A circle is a collection of _____________ that are all ____________________ from a given point.

The equation of a circle with radius r centered at (h,k) is _____________________________

Probability, Permutations, and Combinations

Define the following terms in your own words:

- Sample space
- Probability
- Permutation
- Combination

Vectors:

Topics	Notes
Finding a vector from one point to another	
Finding the slope/direction of a vector $\langle a, b\rangle$	
Finding a vector going in the opposite direction of $\langle a, b\rangle$	
Finding vector going perpendicularly to $\langle a, b\rangle$	
Finding the length of a vector $\langle a, b\rangle$	
Finding a unit vector going in the same direction as $\langle a, b\rangle$	
Changing the length of the vector $\langle a, b\rangle$ OR making it a specific length	
Finding the dot product of two vectors $\langle a, b\rangle$ and $\langle m, n\rangle$	
What does it mean if the dot product of two vectors is zero?	
Adding and subtracting vectors $\boldsymbol{u} = \langle a, b\rangle$ and $\boldsymbol{v} = \langle c, d\rangle$	
What is the geometric interpretation of $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$?	

Parametric Equations: Let's think about the generic parametric $\begin{cases} x = at + b \\ y = ct + d \end{cases}$

Topics	Notes
Finding the starting point of a parametric equation	
Finding the slope/direction of a parametric equation	
Finding the speed of a parametric equation	
Changing the speed or direction (which way it's going) of a parametric equation	
Converting a parametric equation to a nonparametric equation	

Functions

- A function is a relation that has a ___________________ output for each of its inputs.
- The graph of a function passes the _________________ ___________________ test.

	Function	**Not a Function**
Example		
Graph		

Domain and Range

The domain of a graph is the …

The range of a graph is the …

Interval Notation

Let a and b be two real numbers.

Number Line Shading	**Inequality Notation**	**Interval Notation**
	$x \leq b$	
	$x > a$	
	$a \leq x < b$	
	$x < a$ or $x \geq b$	

Inverse Functions

To find the inverse of a function $y = f(x)$, perform the following set of steps:

A function is invertible if its inverse is also a ________________. The graph of an invertible function passes the HLT (__________________ _____________ test).

For two functions that are inverses of each other, what can be said about

- Their graphs:
- The ordered pairs (a,b) on their graphs:
- Their domains and ranges:

Trigonometric Ratio Functions

	$\sin\theta$	$\cos\theta$	$\tan\theta$
Ratio definition			
Diagram/Example			

Inverse Trig Functions

	$\sin^{-1}\theta$	$\cos^{-1}\theta$	$\tan^{-1}\theta$
Definition			
Diagram/Example			

Absolute Value

Absolute value has to do with ____________________________

Notes on solving absolute value equations:	Notes on solving absolute value inequalities:

Graphing: In the general absolute value equation $y = a|x - h| + k$:

- a indicates:
- h indicates:
- k indicates:

Sketch:

Quadratics

Form of quadratic (Name)	Form of quadratic (Algebraic form)	What each variable indicates and what elements of the graph are quickest to get from each form. (Hint: Vertex, axis of symmetry, intercepts)

The three ways we have of solving quadratics when they are set equal to zero are:

- ____
- ____
- ____

How to graph a quadratic:

Transformations

What kind of transformation will move a graph up/down? Left/right?

Distance, Work, Rate, Time

Sample chart: What kinds of things do you put in each box and how do they relate to each other?

Exponents

Exponent rule	Explanation/Proof/Reason	Exponent rule	Explanation/Proof/Reason

The graph of an exponential function looks like:

What type of values can you get out of an exponential function? What can you not?

Logarithms

Log rule	Explanation/Proof/Reason	Log rule	Explanation/Proof/Reason

The graph of a logarithmic function looks like:

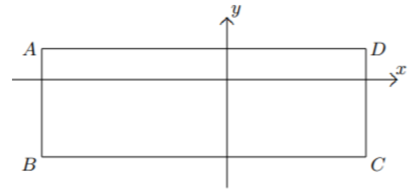
What type of values can you put into logarithmic functions? What can you not?

Math III

UNLESS OTHERWISE STATED, EXPRESS ALL ROUNDED DECIMAL VALUES TO THE NEAREST THOUSANDTH (3 DECIMAL PLACES).

1. (I.62.8) Give two examples of linear functions. Why are they called linear?

2. (I.137) The rectangle ABCD shown at right has sides that are parallel to the coordinate axes. Side AD is three times the length of side AB and the perimeter of ABCD is 56 units.
   a. Find the dimensions of ABCD.
   b. Given the information D = (9, 2), find the coordinates for points A, B, and C.



Problem 2

3. (I.138) A ladder is leaning against the side of a building. Each time I step from one rung to the next, my foot moves 6 inches closer to the building and 8 inches further from the ground. The base of the ladder is 9 ft from the wall. How far up the wall does the ladder reach?

4. (I.139) Each step of the stairs leading from room 9 to room 107 in the Academy Building has a vertical *rise* of 7 inches and a horizontal *run* of 12 inches. Each step of the marble staircase leading to the Assembly Hall has a vertical rise of 5.5 inches and a horizontal run of 13 inches.
   a. Which flight of stairs do you think is steeper? Why?
   b. Calculate the ratio *rise/run* for each flight of stairs, and verify that the greater ratio belongs to the flight you thought to be steeper.

5. (I.252) Is the point (8.4, 23) below, on, or above the line $3x - y = 2$? Justify your answer.

6. (I.140) The *slope* of a line is a measure of how steep the line is. It is calculated by dividing the change in *y*-coordinates by the corresponding change in *x*-coordinates between two points on the line: $\text{slope} = \frac{\text{change in y}}{\text{change in x}}$. Calculate the slope of the line that goes through the two points (1, 3) and (7, 6). Calculate the slope of the line that goes through the two points (0, 0) and (9, 6). Which line is steeper?

7. *Probability* is a number between 0 and 1, often expressed as a percent, that expresses the likelihood that a given event will occur. For example, the probability that two coins will both fall showing heads is 25%. A probability of 1 will absolutely happen. A probability of 0 will absolutely not happen. Give a real life example of each probability.
   a. probability = 1 b. probability = 0 c. probability = 0.5

8. (III.66.5) There are twenty-three red marbles and two blue marbles in a box. A marble is randomly chosen from the box, its color noted, then put back in the box. This process is repeated. What is the probability that **(a)** the first marble is blue? **(b)** the first four marbles are red? **(c)** the first four marbles are red and the fifth is blue?

9. (I.238) As you know, temperatures can be measured by either Celsius or Fahrenheit units; $30°C$ is equivalent to $86°F$, $5°C$ is equivalent to $41°F$, and $-10°C$ is equivalent to $14°F$.
   a. Plot this data with C on the horizontal axis and F on the vertical axis.
   b. Verify that these three data points are *collinear*.
   c. Find a linear equation that relates C and F.
   d. Graph F versus C. In other words, graph the linear equation you just found.
   e. Graph C versus F. You will need to re-plot the data, with C on the vertical axis.
   f. On New Year's Day, I heard a weather report that said the temperature was a balmy $24°C$. Could this have happened? What is the corresponding Fahrenheit temperature?
   g. Water boils at $212°F$ and freezes at $32°F$ at sea level. Find the corresponding Celsius temperatures.
   h. Is it ever the case that the temperature in degrees Fahrenheit is the same as the temperature in degrees Celsius?

10. (I.148) Draw the segment from (3,1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

11. (I.247) Graph a horizontal line through the point (3, 5). Choose another point on this line. What is the slope of this line? What is the y-intercept of this line? What is an equation for this line? Describe a context that could be *modeled* by this line.

12. (I.248) Graph a vertical line through the point (3, 5). Does this line have a slope or y-intercept? What is an equation for this line? Describe a context that could be modeled by this line.

13. (I.168) A car and a small truck started out from Exeter at 8:00 am. Their distances, in miles, from Exeter, recorded at hourly intervals, are recorded in the tables at right. Plot this information on the same set of axes and draw two lines connecting the points in each set of data. What is the slope of each line? What is the meaning of these slopes in the context of this problem?

time	*car*	*truck*
8 : 00	0	0
9 : 00	52	46
10 : 00	104	92
11 : 00	156	138
12 : 00	208	184

Problem 13

14. (SAT problem) A jar contains a red marble, a blue marble, and six green marbles. Alex draws one marble from the jar, and then Chris draws a marble from those remaining. What is the probability that Alex draws the red marble and Chris draws the blue marble?

15. (I.221) Let $P = (x, y)$ and $Q = (1,5)$. Write an equation that states that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y - 5 = 3(x - 1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

16. (I.222) (Continuation) What do the lines $y = 3(x - 1) + 5, y = 2(x - 1) + 5$, and $y = -12(x - 1) + 5$ all have in common? How do they differ from each other?

17. (I.179) Drivers in distress near Exeter have two towing services to choose from: Brook's Body Shop charges \$3 per mile for the towing, and a fixed \$25 charge regardless of the length of the tow. Morgan Motors charges a flat \$5 per mile. On the same system of axes, represent each of these choices by a *linear* graph that plots the cost of the tow versus the length of the tow. If you needed to be towed, which service would you call, and why?

18. (I.180) Compare the graph of y = 2x + 5 with the graph of y = 3x + 5.
    a. Describe a context from which the equations might emerge.
    b. Linear equations that look like y = mx+b are said to be in slope-intercept form. Explain. The terminology refers to which of the two intercepts?

19. (I.175) By hand, find coordinates for the points where the line $3x + 2y = 12$ intersects the *x*-axis and the *y*-axis. These points are called the *x-intercept* and *y-intercept*, respectively. Use these points to make a quick sketch of the line.

20. (I.397) Find an equation for the line that passes through the point (−3, 6), parallel to the line through the points (0, −7) and (4, −15). Write your answer in point-slope form.

21. (I.466) Find values for *a* and *b* that make $ax + by = 14$ parallel to $12 - 3y = 4x$. Is there more than one answer? If so, how are the different values for *a* and *b* related?

22. Given $A = (-3,10)$ and $B = (9,-2)$. Find the equation, in point-slope form, of the perpendicular bisector of segment *AB*.

23. (I.271) For each of the following equations, find the x-intercept and y-intercept. Then use them to calculate the slope of the line.
    a. $3x + y = 6$ b. $x - 2y = 10$ c. $4x - 5y = 20$ d. $ax + by = c$

24. (IV.1.2) In many states, automobile license plates display six characters—three letters followed by a three-digit number, as in SSA-127. Would this system work adequately in Pennsylvania?

25. *Permutations* and *combinations* are the various ways that you can select objects from a set. When order matters, your set of possible outcomes is a permutation, such as when the first person is selected and they are awarded a gold metal while the second and third would receive silver and bronze metals, respectively. When order does not matter, the set of possible outcomes is smaller and is a combination. Give an example in real life of a permutation and another example that is a combination.

26. The *sample space* of a random experiment is the collection of all possible outcomes. The probability of a specific outcome, or event, is the number of successful outcomes in the sample space divided by the total number of possible outcomes. If a single roll of a six-sided die is the random experiment, what is the sample space of possible outcomes? If Morgan is going to roll a six-sided die once, determine the following outcomes.
    a. a two (also known as a deuce) is rolled
    b. an odd number is rolled
    c. a number other than 6 is rolled (or could be stated as "not rolling a 6", or just "*not* 6")
    d. a deuce *or* an odd number

27. If a set contains only the letter A, then when one letter is selected, the only result possible is A. The permutation where there is one object and you select one from the set gives us 1. This is sometimes is denoted ${}_1P_1 = 1$. If a set contains two letters, A and B, then when one letter is selected, the two options are A or B (${}_2P_1 = 2$). If the same set of two letters are given, A and B, and you want to select two, then the two unique outcomes, or permutations are AB and BA (${}_2P_2 = 2$). Using this logic, complete the chart of permutations and conclude with a general formula.

	Number of objects in the *permutation*:					
Number of objects to select from:	**1**	**2**	**3**	**4**	**...**	**k**
1: A	${}_1P_1 = 1$ A	X	X	X		X
2: A, B	${}_2P_1 = 2$ A, B	${}_2P_2 = 2$ AB, BA	X	X		X
3: A, B, C	${}_3P_1 = 3$ A, B, C	${}_3P_2 = 6$ AB, BA, BC, CB, AC, CA	${}_3P_3 = 6$ ABC, ACB, BAC, BCA, CAB, CBA	X		X
4: A, B, C, D	${}_4P_1 =$	${}_4P_2 =$	${}_4P_3 =$	${}_4P_4 =$		X
...						
n						${}_nP_k =$

Problem 27

28. (Continuation) While AB and BA are different permutations, they would not be considered different or unique solutions if they were combinations. The order of the objects in a combination does not matter. Complete the chart of combinations and conclude with a general formula.

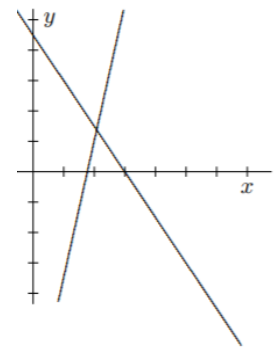
	Number of objects in the *combination*:					
Number of objects to select from:	**1**	**2**	**3**	**4**	**…**	**k**
1: A	${}_1C_1 = 1$ A	X	X	X		X
2: A, B	${}_2C_1 = 2$ A, B	${}_2C_2 = 1$ AB/BA (same)	X	X		X
3: A, B, C	${}_3C_1 =$	${}_3C_2 =$	${}_3C_3 =$	X		X
4: A, B, C, D	${}_4C_1 =$	${}_4C_2 =$	${}_4C_3 =$	${}_4C_4 =$		X
…						
n						${}_nC_k =$

Problem 28

29. (II.14.13) Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of k that makes the line AB
    a. parallel to PQ; b. perpendicular to PQ.

30. (II.57.11) Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is 13.

31. (I.327) The figure to the right shows the graphs of two lines. Use the graphs (the axis markings are one unit apart) to estimate the coordinates of the point that belongs to both lines.



Problems 31, 32, 33

32. (I.328) (Continuation) The system of equations that has been graphed is

$$\begin{cases} 9x - 2y = 16 \\ 3x + 2y = 9 \end{cases}$$

Jess took one look at these equations and knew right away what to do. "Just add the equations and you will find out quickly what x is." Follow this advice, and explain why it works.

33. (I.329) (Continuation) Find the missing *y*-value by inserting the *x*-value you found into either of the two original equations. Do the coordinates of the *point of intersection* agree with your estimate? These coordinates are called a *simultaneous solution* of the original system of equations. Explain the terminology.

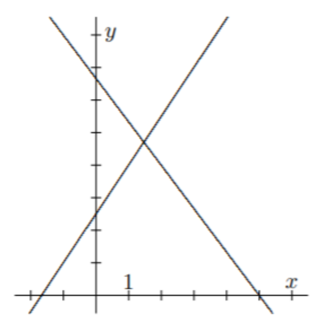
34. Five members of a club are seeking leadership positions. If the first place winner is awarded President, and the second place winner is awarded Vice-President, is this an example of a permutation or combination? Explain your logic. How many unique outcomes are possible when two of five club members will be selected as President and Vice-President of the club?

35. (Continuation) Ten members of a club are interested in participating in a volunteer opportunity. The car that will be transporting the volunteers will only hold five students. How many unique outcomes are possible when five of ten members will be selected to participate in the volunteer work? Is this an example of a permutation or combination? Explain your logic.

36. (I.355) The figure at right shows the graphs of two lines. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20 \\ 3x - 2y = -5 \end{cases}$$

Lee took one look at these equations and announced a plan: "Just multiply the first equation by 2 and the second equation by 3." What does changing the equations in this way do to their graphs?



Problem 36

37. (I.356) (Continuation) Lee's plan has now created a familiar situation. Do you recognize it? Complete the solution to the system of equations. Do the coordinates of the point of intersection agree with your initial estimate?

38. (I.365) Find an equation for each of the following lines. When possible, express your answer in both point-slope form and slope-intercept form.
    a. The line passes through (3, 5) and has −1.5 as its slope.
    b. The line is parallel to the line through (−8, 7) and (−3, 1) and has 6 as its x-intercept.
    c. The line is parallel to the line $x = -4$, and it passes through (4, 7).
    d. The line is perpendicular to the line $y = 3x + 5$ and passes through (1, 2).

39. (II.58.3) Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1, x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

40. (I.362) Algebraically solve the system of equations $2x + y = 5$ and $5x - 2y = 8$. Check your answer graphically.

41. (I.49.4) Find the value of p that makes the linear graph $y = p - 3x$ pass through the point where the lines $4x - y = 6$ and $2x - 5y = 12$ intersect.

42. If Morgan is going to roll two six-sided dice, what is the sample space of possible outcomes? Determine the probability of the following.
    a. both dice will show a 1, or "snake-eyes"
    b. a deuce on one die and a three on the other
    c. the sum of the two rolled numbers will be 7
    d. the sum of the two rolled numbers will be greater than 7
    e. the sum will not be 7 or 11

43. (II.7.9) Draw the following segments on the same graph. What do these segments have in common?
    a. from $(3, -1)$ to $(10, 3)$
    b. from $(1.3, 0.8)$ to $(8.3, 4.8)$
    c. from $\left(\pi, \sqrt{2}\right)$ to $\left(7 + \pi,\ 4 + \sqrt{2}\right)$

44. (II.7.10) (Continuation) The *directed segments* have the *same* length and the *same* direction. Each represents the *vector* $\langle 7,4 \rangle$. The *components* of the vector are the numbers 7 and 4.
    a. Find another example of a directed segment that represents this vector. The initial point of your segment is called the *tail* of the vector, and the final point is called the *head*.
    b. Which of the following directed segments represents $\langle 7,4 \rangle$? from (– 2, – 3) to (5, – 1); from (– 3, – 2) to (11, 6); from (10, 5) to (3, 1); from (– 7, – 4) to (0, 0).

45. (II.11.1) Instead of saying that Cary moves *3 units left and 2 units up*, you can say that Cary's position is *displaced* by the vector $\langle -3,2 \rangle$. Identify the following displacement vectors for each scenario.
    a. Stacey starts at (2, 3) at 1 PM, and has moved to (5, 9) by 6 am.
    b. At noon, Eugene is at (3, 4); two hours earlier, Eugene was at (6, 2).
    c. During a single hour, a small airplane flew 40 miles north and 100 miles west.

46. (II.57.11) Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is r.

47. (I.51.3) If it costs d dollars to buy p gizmos, how much will it cost to buy k gizmos?

48. (II.71.12) Find an equation for the circle of radius 5 whose center is at $(3, -1)$.

49. (SAT problem) A Monopoly player rolls two standard 6-sided dice and is astonished to discover that both individual dice show prime numbers, and their sum is also a prime number. What is the probability of this outcome?

50. A six-sided die is to be rolled once. What is the probability of obtaining…
    a. a "2" (called a deuce)?
    b. a non-deuce? Next, the same die is to be rolled twice.
    c. two deuces?
    d. two non-deuces?
    e. exactly one deuce?

51. A single die is to be rolled three times. What is the probability of obtaining…
    a. three deuces?
    b. three non-deuces?
    c. exactly one deuce?
    d. at least one deuce?

52. The distance between the tail and the head of a vector is known as the *magnitude*, or length, of the vector. Find the magnitude of each vector.
    a. $\langle -5, -12 \rangle$ b. $\langle 2, -3 \rangle$ c. $\langle 1, 1 \rangle$ d. $\langle 4, -6 \rangle$

53. (Continuation) Compare and contrast the direction and magnitude of vectors $\langle 2, -3 \rangle$ and $\langle 4, -6 \rangle$. How would the vector $\langle 10, -15 \rangle$ compare to the previous two vectors? What about $\langle 2k, -3k \rangle$?

54. *M* is the *midpoint* of segment *AB*. If $A = (1, 2)$, $M = (4, -2)$, and $B = (7, -6)$, graph vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$. Find the component form of each vector. Find the magnitude of each vector. How the magnitudes of these vectors compare? Compare and contrast the direction of these vectors.

55. When a vector is given in component form without specific points for the tail and head, we draw the vector starting at the origin. This is called *standard position*. Draw the vector $\langle -5, 1 \rangle$ in standard position. Draw a vector in standard position going in the opposite direction. What is the component form of the vector drawn in the opposite direction?

56. (Continuation) Draw a vector in the same direction as $\langle -5, 1 \rangle$, but twice as long. What is the component form of this new vector? What would the component form be for a vector in the same direction as and three times as long as the vector $\langle -5, 1 \rangle$? What about "k" times as long as vector $\langle -5, 1 \rangle$?

57. (Continuation) Draw a vector in standard position that is perpendicular to the vector $\langle -5, 1\rangle$. What is the component form of your new vector? Is there more than one vector perpendicular to $\langle -5, 1\rangle$? Explain.

58. (II.24.7) Find a vector that is perpendicular to the line $3x - 4y = 6$.

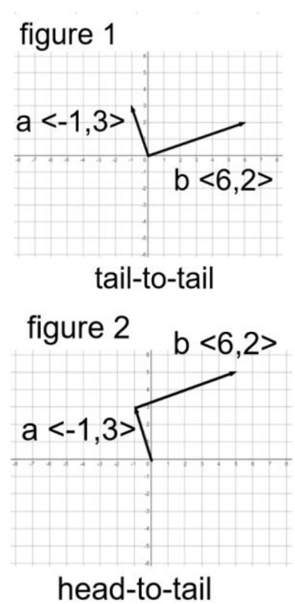
59. (III.9.7) The *dot product* of vectors $\boldsymbol{u} = \langle a, b\rangle$ and $\boldsymbol{v} = \langle m, n\rangle$ is the number $\boldsymbol{u} \cdot \boldsymbol{v} = am + bn$. In general, the dot product of two vectors is the sum of all the products of corresponding components. For example, $\langle 2, -5\rangle \cdot \langle 1, 9\rangle = 2(1) + (-5)(9) = -43$. Let $\boldsymbol{u} = \langle 1,5\rangle, \boldsymbol{v} = \langle 0,4\rangle$, and $\boldsymbol{w} = \langle -5,1\rangle$. Calculate the following.
    a. $\boldsymbol{u} \cdot \boldsymbol{v}$ b. $\boldsymbol{v} \cdot \boldsymbol{w}$ c. $\boldsymbol{w} \cdot \boldsymbol{v}$ d. $\boldsymbol{u} \cdot \boldsymbol{w}$

60. (Continuation) A dot product value of zero is a significant result. What is significant about two vectors with a dot product of zero? Find a new vector that would have a dot product of zero with the given vector $\langle 2, -3\rangle$.

61. (II.29.10) The *sum* of two vectors $\langle a, b\rangle$ and $\langle p, q\rangle$ is defined as $\langle a + p, b + q\rangle$. Find the components of the vector $\langle 2,3\rangle + \langle -7,5\rangle$. Graph $\langle 2,3\rangle$, $\langle -7,5\rangle$, and their sum on the same set of axes.

62. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are shown in the given diagram. Figure 1 shows the vectors drawn *tail-to-tail* while figure 2 shows the vectors drawn *head-to-tail*. Find the component form for the addition of the two vectors, $\boldsymbol{a} + \boldsymbol{b}$. This new vector is called the *resultant vector*. Add the resultant vector, drawn in standard position, to figures 1 and 2. What do you notice? The resultant vector is the diagonal of what type of quadrilateral, with $\boldsymbol{a}$ and $\boldsymbol{b}$ being two of the sides?

63. (Continuation) What would the resultant vector for $\boldsymbol{a} - \boldsymbol{b}$ look like in the diagrams?



Problems 62 and 63

64. (II.12.3) *Some terminology*: When the components of the vector $\langle 5, -7\rangle$ are multiplied by a given number *t*, the result may be written either as $\langle 5t, -7t\rangle$ or as $t\langle 5, -7\rangle$. This is called the *scalar multiple* of vector $\langle 5, -7\rangle$ by the *scalar t*. Find components for the following scalar multiples:
    a. $\langle 12, -3\rangle$ by scalar 5 b. $\left\langle \sqrt{5}, \sqrt{10}\right\rangle$ by scalar $\sqrt{5}$
    c. $\left\langle -\frac{3}{4}, \frac{2}{3}\right\rangle$ by scalar $-\frac{1}{2} + \frac{2}{6}$ d. $\langle p, q\rangle$ by scalar $\frac{p}{q}$

65. (II.12.4) Find the magnitude (length) for each of the following vectors.
    a. $\langle 3,4\rangle$ b. $2018\langle 3,4\rangle$ c. $\frac{2018}{5}\langle 3,4\rangle$ d. $t\langle 3,4\rangle$ e. $t\langle a, b\rangle$

Math III

66. Graph the vector $\boldsymbol{u} = \langle 3,4 \rangle$ in standard position. Verify htat the magnitude is 5 units. Find the component form of a new vector in the same direction as the vector $\langle 3,4 \rangle$ but with a magnitude equal to 1 unit. The new vector you just found is called the *unit vector* in the direction of u.

67. Find a unit vector in the same direction as the given vector.
    a. $\langle -5, -12 \rangle$ b. $\langle 2, -3 \rangle$ c. $\langle 1,1, \rangle$

68. (II.23.8) Find *k* so that the vectors $\langle 4, -3 \rangle$ and $\langle k, -6 \rangle$ **(a)** point in the same direction; **(b)** are perpendicular.

69. Are the vectors $\langle 2,4 \rangle$ and $\langle 6, -3 \rangle$ perpendicular? Why or why not? Try to give two forms of proof.

70. Given the vector $\langle 2, -3 \rangle$, find the following vectors.
    a. same direction, twice as long
    b. opposite direction, same length
    c. perpendicular, triple the length
    d. same direction, length 1 unit
    e. same direction, length 5 units

71. (II.12.2) Given the vector $\langle -5, 12 \rangle$, find the following vectors.
    a. same direction, twice as long
    b. same direction, length 1
    c. opposite direction, length 10
    d. opposite direction, length *c*

72. (III.80.2) If you were to ask for the birthday of a random SSAer, what is the probability that the response will be the 18th of August? (Assume that there are 365 birthdates in a year.) What is the probability that it will *not* be 18 August? If you ask two random SSAers to state their birthdays, what is the probability that *neither* will say 18 August? If you ask all 485 SSAers this birthday question, what is the probability that *no one* will say 18 August? What is the probability that *someone* (that means at least one person) will say 18 August? Final question: What is the probability that tomorrow's birthday list would be empty?

73. (III.88.6) What is the probability that the thirty-two residents of Lamont have thirty-two different birthdays? What is the probability that there is at least one birthday coincidence in Lamont?

74. (III.88.8) If five standard (six-sided) dice are tossed onto the table, what is the probability that
    a. all of them will show an odd number on top?
    b. no aces or deuces (that means ones or twos) will show on top?
    c. the five dice will show five different values on top?

Math III

75. (III.85.1) Una is going to roll ten standard (six-sided) dice, one after another. What is the probability that…
    a. *none* of the dice land showing an ace (a single spot) on top?
    b. at least one of the dice land showing an ace on top?
    c. the first die shows an ace, but none of the others do?
    d. the last die shows an ace, but none of the others do?
    e. *exactly one* of the ten dice shows an ace on top?

76. (II.11.3) A bug is initially at (– 3, 7). Where is the bug after being displaced by vector $\langle -7, 8\rangle$?

77. *Special right triangles:* Recall that all isosceles right triangles (also called 45-45-90 triangles) have congruent legs and a hypotenuse that is $\sqrt{2}$ times the leg length. Verify this result by finding the length of the hypotenuse of a 45-45-90 triangle whose leg length is k. Your answer will also be in terms of k.

78. (Continuation): For every right triangle with a shorter leg that is half the length of the hypotenuse, the longer leg is $\sqrt{3}$ times the short leg length. Such right triangles are also known as 30-60-90 triangles for the measures of the angles in the triangles. If the short leg of a 30-60-90 triangle has a length of n, find the length of the longer leg and the hypotenuse in terms of n.

79. (I.83.12) I have been observing the motion of a really tiny red bug on my graph paper. When I started watching, the bug was at the point (3, 4). Ten seconds later it was at (5, 5). Another ten seconds later it was at (7, 6). After another ten seconds it was at (9, 7).
    a. Draw a picture that illustrates what is happening.
    b. Write a description of any pattern that you notice. What assumptions are you making?
    c. Where was the bug 25 seconds after I started watching it?
    d. Where was the bug 26 seconds after I started watching it?

80. (I.84.11) From its initial position at (1, 6), an object moves linearly with constant speed. It reaches (7, 10) after two seconds and (13, 14) after four seconds.
    a. Predict the position of the object after six seconds; after nine seconds; after t seconds.
    b. Will there be a time when the object is the same distance from the x-axis as it is from the y-axis? If so, when, and where is the object at that time?

81. (I.86.6) The x- and y-coordinates of a point are given by the equations shown to the right. The position of the point depends on the value assigned to t. Use your graph paper to plot points corresponding to the values $t = -3, -2, -1, 0, 1, 2,$ and 3. These points should appear to be collinear. Convince yourself that this is the case, and calculate the slope of this line. The displayed equations are called *parametric*, and t is called a *parameter*. How is the slope of a line determined from its parametric equations?

$$\begin{cases} x = 3 + t \\ y = 5 - 2t \end{cases}$$

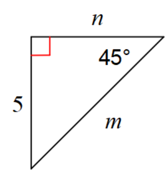
Problem 81

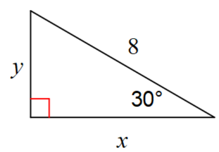
Math III

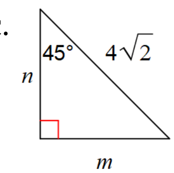
82. (I.86.7) (Continuation) Plot the points (1, 2), (2, 5), and (3, 8) on the coordinate plane. Write parametric equations, similar to those in the preceding exercise, that produce these points when *t* values are assigned. There is more than one correct answer.

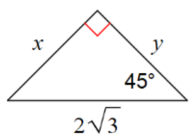
83. (I.62.8) Give two examples of linear functions. Why are they called linear? How are linear equations and parametric equations alike? How are they different?

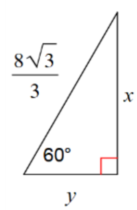
84. For each special right triangle, find the indicated side lengths.

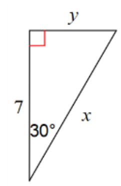
a.


b.


c.


d.


e.


f.


85. (II.8.2) Points (x, y) described by the equations $x = 1 + 2t$ and $y = 3 + t$ form a line. Is the point (7, 6) on this line? How about (−3, 1)? How about (6, 5.5)? How about (11, 7)?

86. (II.4.10) A bug moves linearly with constant speed across my graph paper. I first notice the bug when it is at (3, 4). It reaches (9, 8) after two seconds and (15, 12) after four seconds.
   a. Predict the position of the bug after six seconds; after nine seconds; after *t* seconds.
   b. Is there a time(s) when the bug is equidistant from the *x*- and *y*-axes? If so, at what time(s), and where is the bug?

87. (II.3.4) Una recently purchased two boxes of ten-inch candles—one box from a discount store, and the other from an expensive boutique. One evening, Una noticed that the inexpensive candles last only three hours each, while the expensive candles last five hours each. The next evening, Una hosted a dinner party and lit two candles—one from each box—at 7:30 PM. During dessert, a guest noticed that one candle was twice as long as the other. At what time was this observation made? Hint: This is similar to the tiny red bug problem.

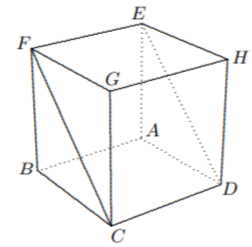
88. (II.6.8) In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair's position at 4 AM is (2, 5) and at 6 AM it is (6, 3). What is Blair's position at 8:15 AM when the alarm goes off?

89. (II.8.5) Find parametric equations to describe the line that goes through the points *A* = (5, −3) and *B* = (7, 1). There is more than one correct answer to this question.

Math III

90. (II.9.1) In another dream, Blair is moving along the line $y = 3x + 2$. At midnight, Blair's position is (1, 5), the x-coordinate increasing by 4 units every hour. Write parametric equations that describe Blair's position t hours after midnight. What was Blair's position at 10:15 PM when the nightmare started? Find Blair's speed, in units per hour.

91. (2.629) A rhombus has four 6-inch sides and two 120-degree angles. From one of the vertices of the obtuse angles, the two altitudes are drawn, dividing the rhombus into three pieces. Find the areas of these pieces.



Problem 92

92. (2.712) The figure at right shows a cube ABCDEFGH. Square ABCD and rectangle EFCD form an angle that is called *dihedral* because it is formed by two intersecting planes. The line of intersection here is CD. Calculate the size of this angle.

93. (II.9.2) The parametric equations $x = -2 - 3t$ and $y = 6 + 4t$ describe the position of a particle, in meters and seconds. How does the particle's position change each second? each minute? What is the speed of the particle, in meters per second? Write parametric equations that describe this motion, using meters and *minutes* as units.

94. (II.10.2) A bug is moving along the line $3x + 4y = 12$ with constant speed 5 units per second. The bug crosses the x-axis when $t = 0$ seconds. It crosses the y-axis later. When? Where is the bug when $t = 2$? when $t = -1$? when $t = 1.5$? What does a negative t-value mean?

95. (II.10.4) Find parametric equations to describe the line $3x + 4y = 12$. Use your equations to find coordinates for the point that is three-fifths of the way from (4, 0) to (0, 3). By calculating some distances, verify that you have the correct point.

96. Draw a rectangle with a width that is 1 unit more than twice the length. Write an equation where the width is in terms of the length. Draw a circle with a diameter of 2 inches. Write an equation where the circumference of the circle is in terms of the diameter.

97. (Continuation) In the previous problem, the width depends on the length. The width is two times the length plus 1 unit. $w = 2l + 1$. The circumference depends on the diameter of the circle. $C = \pi d$. The word *function* means that assigning a value to one of the variables (l or d) determines a unique value for the other (w or C). It is customary to say that "C is a function of d." In this example, however, it would be incorrect to say that "d is a function of C." Explain.

98. (II.11.2) Kirby moves with constant speed 5 units per hour along the line $y = \frac{3}{4}x + 6$ crossing the y-axis at midnight and the x-axis later. When is the x-axis crossing made? What does it mean to say that *Kirby's position is a function of time*? What is Kirby's position 1.5 hours after midnight? What is Kirby's position t hours after midnight?

Math III

99. Determine if each situation is a function or not. If it is a function, identify the independent and dependent variables, or which variable is a function of the other. If it is not an example of a function, explain why.
    a. The gross money you earn is determined by the hours that you work. You earn $15 per hour.
    b. On the equator, the time of day can be determined by the length of your shadow.
    c. In your class, the number of minutes spent studying for the last exam is recorded as well as the percent grade for each student.

100. (II.12.1) A bug moves at 13 cm/sec, and its position is described by the parametric equations to the right. How does the bug's position change with respect to time? Change the equations to obtain the description of a bug moving along the same line with speed 26 cm/second.

$$\begin{cases} x = 2 - 12t \\ y = 1 + 5t \end{cases}$$

Problem 100

101. (II.9.5) Find parametric equations that describe the following lines.
    a. through (3, 1) and (7, 3)
    b. through (7, −1) and (7, 3)

102. (2.490) Mark A = (0, 0) and B = (10, 0) on your graph paper, and use your protractor to draw the line of positive slope through A that makes a 25-degree angle with AB. By making suitable measurements, calculate (approximately) the slope of this line.

103. (2.491) (Continuation) With your calculator in degree mode, find a decimal approximation for the expression $\tan(25°)$ (read as *tangent of 25 degrees*). How does this value compare with your answer from the previous problem?

104. (2.492) (Continuation) Repeat the preceding construction and tangent verification for $72°$ and $45°$. Summarize your findings, including a definition in your own words of the *tangent ratio* of an angle.

105. (II.24.1) An object moves with constant *velocity* (which means constant speed and direction) from (−3, 1) to (5, 7), taking five seconds for the trip.
    a. What is the speed of the object?
    b. Where does the object cross the *y*-axis?
    c. Where is the object three seconds after it leaves (−3, 1)?

106. (II.12.7) The initial position of an object is $P(0) = (7, -2)$. Its position after being displaced by the vector $t\langle -8,7\rangle$ is $P(t) = (7,-2) + t\langle -8,7\rangle$. Notice that the position is a function of *t*. Calculate $P(3)$, $P(2)$, and $P(-2)$. Describe the configuration of all possible positions $P(t)$.

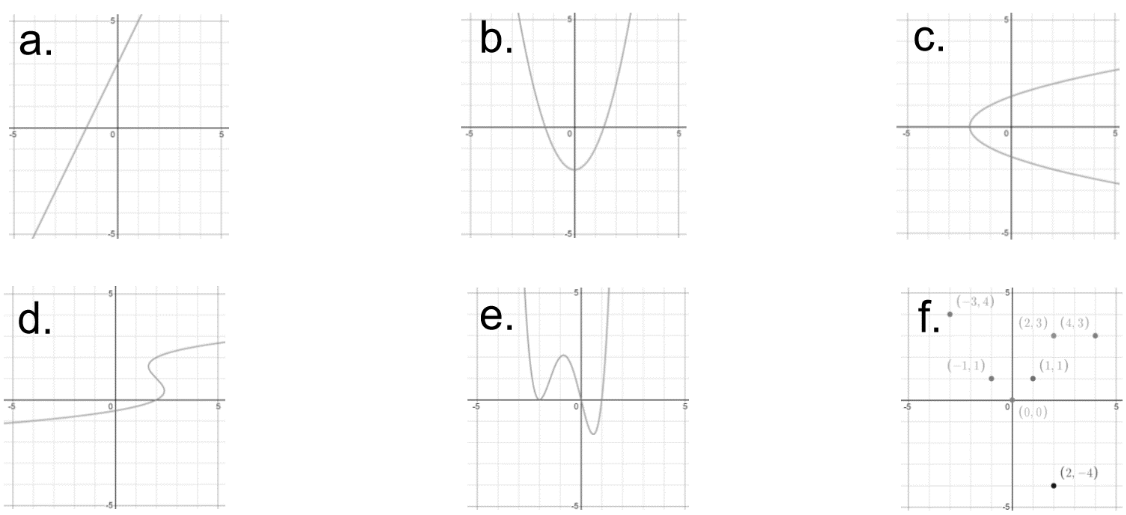
107. (II.14.11) The lines defined by $P(t) = (4 + 5t, -1 + 2t)$ and $Q(u) = (4 - 2u, -1 + 5u)$ intersect perpendicularly. Justify this statement. What are the coordinates of the point of intersection?

108. (II.15.5) On the same coordinate-axis system, graph the line defined by $P(t) = (3t - 4, 2t - 1)$ and the line defined by $4x + 3y = 18$. The graphs should intersect in the first quadrant.
   a. Calculate $P(2)$, and show that it is not the point of intersection.
   b. Find the value of t for which $P(t)$ is on the line $4x + 3y = 18$.

109. (II.16.10) Motions of three particles are described by the following three pairs of equations
   a. $\begin{cases} x = 2 - 2t \\ y = 5 + 7t \end{cases}$ b. $\begin{cases} x = 4 - 2t \\ y = -2 + 7t \end{cases}$ c. $\begin{cases} x = 2 - 2(t + 1) \\ y = 5 + 7(t + 1) \end{cases}$
   How do the positions of these particles compare at any given moment?

110. If moving at a constant rate of 5 ft/sec, the distance an object travels is a function of time. The equation is $d = 5t$, where d is the distance in feet and t is the time in seconds. Function notation can be used to represent this problem by writing $f(t) = 5t$. The *f(t)* is another way of representing the distance, which depends on the time. When t = 1 second, the value of $f(1) = 5 \cdot 1 = 5$ feet. Using $f(t) = 5t$, determine the value of $f(0)$, $f(2)$, and $f(10)$. Explain the meaning of $f(0)$ in the context of this problem.

111. When graphing two variables (a relation), we say that the relation is a function if each variable for *x* graphs a unique value for *y*. In other words, *x* can not produce more than one *y* value. A quick way to determine if a relation is a function is to draw a series of vertical lines on the graph. If all of the vertical lines pass through no more than one *y* point on the graph, then we say that the relation passes the *vertical line test* (VLT) and it is a function. For each graph in the diagram below, determine if the relation is a function by using the VLT.



112. (2.610) Draw a right triangle that has a 15-cm hypotenuse and a 27-degree angle. To the nearest tenth of a cm, measure the side opposite the 27-degree angle, and then express your answer as a fraction of the length of the hypotenuse. Compare your answer with the value obtained from a calculator when you enter $\sin(27)$ in degree mode.

113. (2.611) (Continuation) Repeat the process on a right triangle that has a 10-cm hypotenuse and a 65-degree angle. Try an example of your choosing. Write a summary of your findings.

114. (II.17.12) A particle moves according to $(x, y) = (6 - t, -1 + 3t)$. For what value of *t* is the particle closest to the point (−2, 0)?

115. (II.17.14) What do the descriptions of motion defined by equations $P(t) = (-2 + t, 3 + 2t)$ and $Q(u) = (4 - 3u, -1 - 6u)$ have in common? How do they differ?

116. (II.19.6) Graph the line that is described parametrically by $(x, y) = (2t, 4 - t)$. Then
    a. find the point on the line that minimizes the distance to (3, 8);
    b. confirm that the point corresponding to $t = 0$ is exactly 5 units from (3, 8);
    c. find the other point on the line that is 5 units from (3, 8).

117. (2.617) If triangle ABC has a right angle at C, the ratio AC : AB is called the *sine ratio* of angle B, or simply the sine of B, and is usually written $\sin B$. What should the ratio BC : AB be called? Determine by hand the sine ratio for a 30-degree angle and for a 60-degree angle.

118. (II.20.1) Let $A = (7, 7)$, $B = (5, 1)$, and $P(t) = (6 + 3t, 4 - t)$. Plot *A* and *B*. Choose two values for *t* and plot the resulting points *P*(*t*), which should look equidistant from *A* and *B*. Make calculations to confirm the equidistance.

119. (II.23.6) Brett and Jordan are out driving in the coordinate plane, each on a separate straight road. The equations $B(t) = (-3, 4) + t\langle 1,2\rangle$ and $J(t) = (6, 1) + t\langle -1,1\rangle$ describe their respective travels, where *t* is the number of minutes after noon
    a. Make a sketch of the two roads, with arrows to indicate direction of travel.
    b. Where do the two roads intersect?
    c. How fast is Brett going? How fast is Jordan going?
    d. Do they collide? If not, who gets to the intersection first?

120. (2.623) To draw a right triangle that has a 1-degree angle and measure its sides accurately is difficult. To get the sine ratio for a 1-degree angle, however, there is an easy way — just use a calculator. Is the ratio a small or large number? How large can a sine ratio be?

121. (II.25.12) Find coordinates for the point where line $(x, y) = (3 + 2t, -1 + 3t)$ meets line $y = 2x - 5$.

122. (II.26.6) What are the axis intercepts of the line described by $P(t) = (5 + 3t, -2 + 4t)$?

123. (II.28.2) Find the point of intersection of the lines given by $P(t) = (-1 + 3t, 3 + 2t)$ and $Q(r) = (4 - r, 1 + 2r)$.

124. (2.624) Draw a right triangle that has an 18-cm hypotenuse and a 70-degree angle. To within 0.1 cm, measure the leg adjacent to the 70-degree angle, and express your answer as a fraction of the hypotenuse. Compare your answer with the value obtained from a calculator when you enter cos(70) in degree mode. This is an example of the *cosine ratio*.

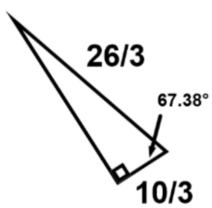
125. (3-4.100) A javelin lands with six feet of its length sticking out of the ground, making a 52-degree angle with the ground. The sun is directly overhead. The javelin's shadow on the ground is an example of a perpendicular projection. Find its length, to the nearest inch.

126. (2.657) Taylor lets out 120 meters of kite string, then wonders how high the kite has risen. Taylor is able to calculate the answer, after using a protractor to measure the 63-degree angle of elevation that the string makes with the ground. How high is the kite, to the nearest meter? What (unrealistic) assumptions did you make in answering this question?

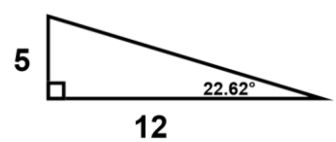
127. (2.677) Use a calculator (in degree mode) to find the sine of a 56-degree angle and the cosine of a 34-degree angle. Now find the sine of a 23-degree angle and the cosine of a 67-degree angle. The word cosine abbreviates "sine of the complement." Explain the terminology. The cosine function seems to be unnecessary. Explain.

128. For each set of triangles below, find any missing side lengths. Then find the sine, cosine, and tangent ratios for the acute angles in each set of triangles. Do this without a calculator, and then use your calculator to check your answers. What do you notice about your answers for each set? Try to explain why your results turned out as they did.

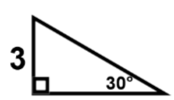
a. i.



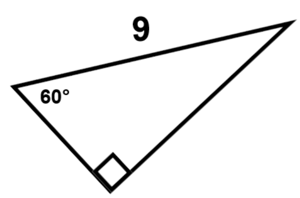
ii.



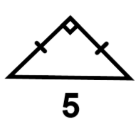
b. i.



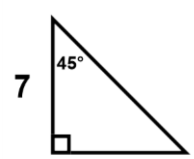
ii.



c. i.



ii.



129. (2.528) Let A = (0, 0), B = (4, 0), and C = (4, 3). Measure angle CAB. What is the slope of AC? Use this slope and the tangent function to check your angle measurement. Use a calculator to come as close as you can to the theoretically correct size of angle CAB.

130. (2.529) (Continuation) On a calculator, enter the expression $\tan^{-1}(0.75)$ (read as *inverse tangent of 0.75*). Record your result. How does this value compare with your answer from the previous problem? In your own words, explain how the tangent function is related to the inverse tangent function.

131. *Inverse sine* and *inverse cosine* are two functions that work similarly to how inverse tangent works for the tangent function. Specifically, if you know the sine of an angle, θ, (a Greek letter called "theta" often used to represent an unknown angle), you can use inverse sine to find the value of that angle. For example, if $\sin\theta = \frac{3}{5}$ and θ is an acute angle, then $\theta = \sin^{-1}\left(\frac{3}{5}\right) \approx 36.870°$. Using this information, solve each equation below for the indicated acute angle. Give your answers in exact form (in terms of one of the inverse functions) and in approximate form (decimal values rounded to the nearest thousandth).

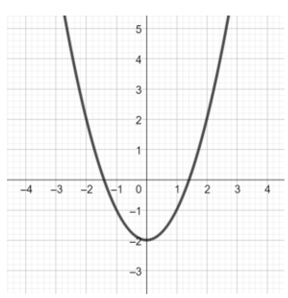
a) $\sin\theta = \frac{1}{2}$ b) $\cos\theta = \frac{4}{7}$ c) $\tan\theta = \frac{1}{5}$

d) $\cos\theta = \frac{12}{13}$ e) $\sin\theta = 0.8$ f) $\tan\theta = 1.13$

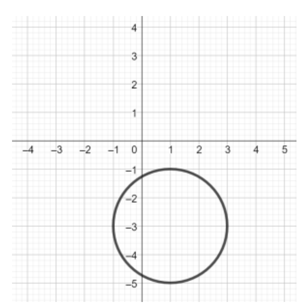
132. (2.648) The sides of a triangle are 12 cm, 35 cm, and 37 cm long.
   a. Show that this is a right triangle.
   b. Show that inverse tangent, inverse sine, and inverse cosine can all be used to find the size of the smallest angle of this triangle.

133. When analyzing relations and functions, it is often useful to discuss the relation's *domain* and *range*. The domain of a relation describes the values that can go into the relation and output a *real* value, and the range describes the outputs of the relation. In other words, the domain describes the inputs (usually denoted by the variable x) of a relation, and the range gives the outputs (typically y) of a relation. For the first graph below, the domain is all real numbers, and the range is $y \geq -2$. Estimate the domain and range for each relation whose graph is given below.

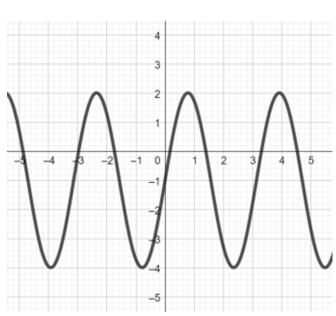
a.



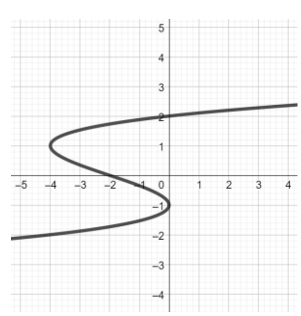
b.



c.



d.

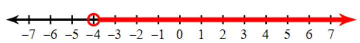


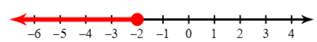
134. One way to describe the domain and range of a relation is to use inequalities, as you might have done for the previous problem. The domain and range of the types of graphs we look at are usually *intervals*, continuous pieces of the number line. Another way to write out an interval is with *interval notation*. Below is a chart providing examples of intervals as they would be graphed on a number line, how they can be written with inequalities, and how they can be written with interval notation. Fill out the missing parts of the table.

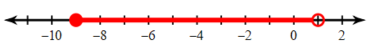
Number Line	Inequality	Interval Notation
		$(-\infty, -10)$
	$y \geq -19$	$[-19, \infty)$
		$(-5,5)$
	$r < -10$ or $r \geq -2$	$(-\infty, -10) \cup [-2, \infty)$

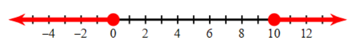
Problem 134

135. Below are four intervals graphed on separate number lines. Express each interval with inequalities and then with interval notation.

a. 

b. 

c. 

d. 

136. (I.31.3) Pat and Kim start hiking at a constant rate at 8 AM, traveling directly east. It takes them an hour to hike each mile. They pass a waterfall at noon and finish their hike at 3 PM. Let t be the number of hours since the hike began, and d be the corresponding distance (in miles) between them and the waterfall.

a. Draw a diagram of the situation.
b. Make a table of (t, d) pairs for $t = 0, 1, 2, 3, 4, 5, 6, 7$.
c. With t on the horizontal axis, draw a graph of d versus t.

137. (I.31.4) (Continuation) Graph the equation $y = |x - 4|$, and find its domain and range. Interpret the graph in this context.

138. (I.31.5) (Continuation) Let y be the distance between the Pat and Kim and Lake Omega, which they reach at 11 AM. Draw a graph that plots y versus t, for $t = 0, 1, 2, 3, 4, 5, 6, 7$. Write an equation that expresses y in terms of t, and state its domain and range. By the way, you have probably noticed that each of these absolute-value graphs has a corner point, which is called a *vertex*.

Math III

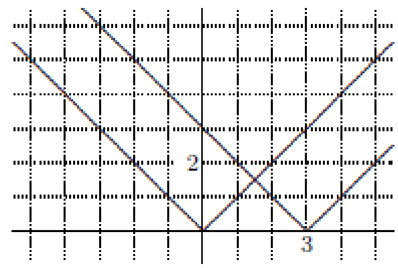
139. Recall that inverse operations are operations that undo each other, like adding and subtracting or multiplying and dividing. *Inverse functions* are two functions that undo each other. For example, $f(x) = x - 7$ is a function that subtracts 7 from whatever input it's given. The inverse function of f, f^{-1}, should undo whatever f does to its inputs, so $f^{-1}(x) = x + 7$. Write inverse functions for each function below.
   a. $f(x) = x + 2$ b. $g(x) = \frac{x}{9}$ c. $h(x) = 13 - x$ d. $j(x) = -5.7x$

140. (I.33.10) Graph $y = |x| + 3$ and $y = |x| - 5$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = |x| + k$. How does this transformation affect the domain and range of the *parent function* $y = |x|$? How can you tell from the graph whether *k* is positive or negative?

141. (III.88.9) The useful function $F(x) = 32 + 1.8x$ converts Celsius temperatures to Fahrenheit temperatures. Confirm this, then find a formula for the function *C* (which could also be called the *inverse function*, F^{-1}) that converts Fahrenheit temperatures to Celsius temperatures.

142. (I.32.12) Graph $y = |x - 5|$ and $y = |x + 3|$ then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = |x - h|$. How does this transformation affect the domain and range of $y = |x|$?

143. (I.32.13) Write an equation and state the domain and range for each of the graphs shown at right. Each graph goes through several *lattice points*.



Problem 143

144. (I.34.8) Sketch on the same axes the graphs of the given absolute value functions. Then state the domain and range of each.
   a. $y = |x|$ b. $y = 2|x|$ c. $y = 0.5|x|$ d. $y = -3|x|$

145. (I.34.9) What effect does the coefficient *a* have on the graph of the equation $y = a|x|$? Does this transformation change the domain and range of the parent function $y = |x|$? How can you tell whether *a* is positive or negative by looking at the graph?

146. (I.35.7) Compare the graphs of $y = x - 3$ and $y = |x - 3|$. How are they related?

147. (I.38.11) A hot-air balloon ride has been set up so that a paying customer is carried straight up at 50 feet per minute for ten minutes and then immediately brought back to the ground at the same rate. The whole ride lasts twenty minutes. Let *h* be the height of the balloon (in feet) and *t* be the number of minutes since the ride began. Draw a graph of *h* versus *t*. What are the domain and range of $h(t)$? What are the coordinates of the vertex? Find an equation that expresses *h* in terms of *t*.

148. Sometimes we can figure out a function's inverse just by thinking about it. Other times, it is helpful to have a procedure for finding an inverse function. One way of doing so is to do the following steps, shown in
    i. Set the function equal to y instead of $f(x)$.
    ii. Switch x and y in the formula.
    iii. Solve for the new y.

    This procedure allows us to see what operations we had to perform in order to undo whatever the given function does to its inputs. For example, for $f(x) = -\frac{1}{2}x + 6$, we would take the steps shown in the figure to the right in order to figure out that $f^{-1}(x) = -2x + 12$. Use this procedure to find the inverse for each function given below.

    a. $r(x) = 3x - 1$ b. $l(x) = \frac{x+8}{-2}$ c. $q(x) = \frac{x-1}{x+4}$

$$f(x) = -\frac{1}{2}x + 6$$
$$y = -\frac{1}{2}x + 6$$
$$x = -\frac{1}{2}y + 6$$
$$x - 6 = -\frac{1}{2}y$$
$$-2(x-6) = y$$
$$f^{-1}(x) = -2(x-6)$$
$$f^{-1}(x) = -2x + 12$$

Problem 148

149. (I.30.3) The equation $|x - 7| = 2$ is a translation of "the distance from *x* to 7 is 2."

150. Translate $|x - 7| \le 2$ into English, and graph its solutions on a number line.

151. Convert "the distance from −5 to *x* is at most 3" into symbolic form, and solve it. Write your answer as an inequality and in interval notation.

152. (I.45.4) What values of *x* satisfy the inequality $|x| > 12$? Graph this set on a number line, write it in interval notation, and describe it in words. Answer the same question for $|x - 2| \ge 12$.

153. (I.43.1) The fuel efficiency *m* (in miles per gallon) of a truck depends on the speed *r* (in miles per hour) at which it is driven. The relationship between *m* and *r* usually takes the form $m = a|r - h| + k$. For Sasha's truck, the optimal fuel efficiency is 24 miles per gallon attained when the truck is driven at 50 miles per hour. When Sasha drives at 60 miles per hour, however, the fuel efficiency drops to only 20 miles per gallon. Complete the chart and find the values for *a*, *h* and *k* that fit this model. What is the domain and range of $m = a|r - h| + k$? Within the context of the problem, what values of *r* are appropriate for this model? (This answer should be more restrictive than the domain you gave).

r	m
60	20
50	24
40	
30	
20	
10	

Problem 153

154. (I.39.3) My sleeping bag is advertised to be suitable for temperatures *T* between 20 degrees below zero and 20 degrees above zero (Celsius). Write an absolute-value inequality that describes these temperatures *T*.

Math III

155. Let $f(x) = 3x - 2$ and $g(x) = \frac{x}{3} + \frac{2}{3}$.
   a. Verify that f and g are inverses of each other.
   b. Fill out the table to the right. What do you notice?
   c. Sketch graphs of f and g. How are the graphs related?

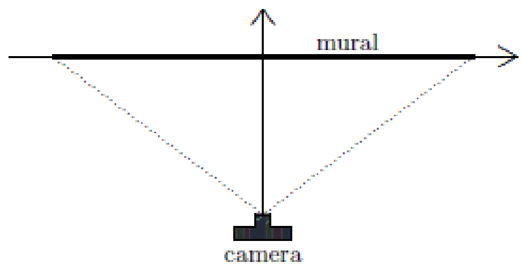
x	$f(x)$	x	$g(x)$
-2		-8	
-1		-5	
0		-2	
1		1	
2		4	

Problem 155

156. (I.43.6) Graph $y = 2|x + 1| - 3$, then describe how the graph of $y = |x|$ is transformed to produce the graph of $m = a|r - h| + k$. How does this transformation affect the domain and range?

157. (I.52.4) Using an absolute-value inequality, describe the set of numbers whose distance from 4 is greater than 5 units. Draw a graph of this set on a number line. Finally, describe this set of numbers using inequalities without absolute value signs and in interval notation.

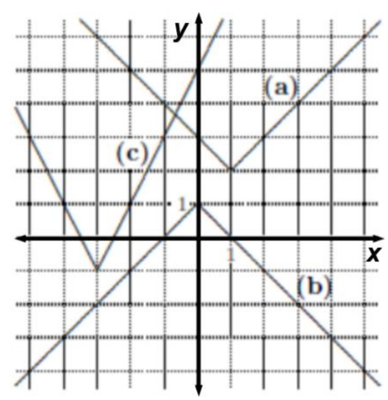
158. (I.45.10) Using the coordinate-axis system shown in the top view at right, the viewing area of a camera aimed at a mural placed on the x-axis is bordered by $y = \frac{7}{8}|x| - 42$. The dimensions are in feet. How far is the camera from the x-axis, and how wide a mural can be photographed?



Problem 158

159. (I.46.6) Write a formula that expresses the distance between p and 17. Describe all the possible values for p if this distance is to be greater than 29.

160. (I.44.7) Write an equation for each of the graphs at right. Find the domain and range for each graph.



Problem 160

161. (I.40.5) For each of the following inequalities, graph the solutions.
   a. $|x + 8| < 20$
   b. $|2x - 5| \le 7$
   c. $3|4 - x| \ge 12$

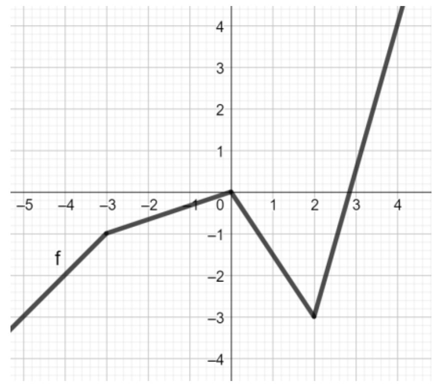
162. (I.47.5) Write and solve an inequality that describes all the points that are more than 3 units from 5.

163. (I.48.1) My car averages 29 miles per gallon of gasoline, but I know—after many years of fueling it—that the actual miles per gallon can vary by as much as 3 either way. Write an absolute-value inequality that describes the range of possible mpg figures for my car.

Math III

164. (I.50.3) Impeded by the current, the Outing Club took 4 hours and 24 minutes to paddle 11 km up the Allegheny River to their campsite last weekend. The next day, the current was with them, and it took only 2 hours to make the return trip to campus. Everyone paddled with the same intensity on both days. At what rate would the paddlers have traveled if there had been no current? What was the speed of the current?

165. The graph of f is given to the right. Sketch the graph of the function's inverse, and verify that it is not the graph of a function. Not all functions have inverses that are also functions. Functions whose inverses are also functions are said to be *invertible.*



Problem 165

166. (I.53.6) On the same axes, sketch the graphs of $y = |x - 3|$ and $y = 4 - |x - 3|$. Label the points of intersection with coordinates. Find the area enclosed.

167. (I.60.8) Pat and Kim are having an algebra argument. Kim is sure that $-x^2$ is equivalent to $(-x)^2$, but Pat thinks otherwise. How would you resolve this disagreement? What evidence does your calculator offer?

168. (I.60.5) When an object falls, it gains speed. Thus the number of feet *d* the object has fallen is not linearly related to the number of seconds *t* spent falling. In fact, for objects falling near the surface of the Earth, with negligible resistance from the air, $d = 16t^2$. How many seconds would it take for a cannonball to reach the ground if it were dropped from the top of the Eiffel Tower, which is 984 feet tall? How many seconds would it take for the cannonball to reach the ground if it were dropped from a point that is halfway to the top?

x	$\|x\|$	x^2
-2		
-1		
$-1/2$		
0		
$1/2$		
1		
2		

Problem 169

169. (I.62.2) Complete the table at right. Then graph by hand on separate axes $y = |x|$ and $y = x^2$. Check your graphs with your calculator. In what respects are the two graphs similar? In what respects do the two graphs differ?

170. (I.41.8) Graph $y = 3|x - 2| - 6$, and find coordinates for the vertex and the *x*-and *y*-intercepts. State the graph's domain and range. What is the parent function of $y = 3|x - 2| - 6$? How has $y = 3|x - 2| - 6$ been transformed from its parent function?

171. Sketch the graphs of $f(x) = |x|$ and its inverse. Is f an *invertible* function? Explain.

172. (I.77.5) The three functions $y = 2(x - 4) - 1$, $y = 2|x - 4| - 1$, and $y = 2(x - 4)^2 - 1$ look somewhat similar. Predict what the graph of each will look like, and then sketch them in your notebook (without using a calculator) by just plotting a few key points. In each case think about how the form of the equation can help provide information. Identify the domain and range for each.

173. (I.61.3) An avid gardener, Gerry Anium just bought 80 feet of decorative fencing to create a border around a new rectangular garden that is still being designed.
   a. If the width of the rectangle is 5 feet long, what is the length? What would be the total area? Write this data in the first row of the table to the right.
   b. Record data for the next five examples in the table.
   c. Let x be the width of the garden. In terms of x, fill in the last row of the table.
   d. Use your calculator to graph the rectangle's area versus x, for $0 \le x \le 40$. As a check, you can make a scatter plot using the table data. What is special about the values $x = 0$ and $x = 40$?
   e. Comment on the symmetric appearance of the graph. Why was it predictable?
   f. Find the point on the graph that corresponds to the largest rectangular area that Gerry can enclose using the 80 feet of available fencing. This point is called the *vertex*.

width	*length*	*area*
5		
9		
16		
22		
24		
35		
x		

Problem 173

174. (I.69.5) Quinn is four miles from the train station, from which a train is due to leave in 56 minutes. Quinn is walking along at 3 mph, and could run at 12 mph if it were necessary. If Quinn wants to be on that train, it will be necessary to do some running! What is the minimum number of miles she must run to catch the train?

175. (I.65.2) Sketch the graphs of $y = x^2 + 5$, $y = x^2 - 4$, and $y = x^2 + 1$ on the same axes. What is the effect of the value of c in equations of the form $y = x^2 + c$?

176. (I.62.4) A workman accidentally drops a hammer from the scaffolding of a tall building. The workman is 300 feet above the ground. As you answer the following, recall that an object falls $16t^2$ feet in t seconds (assuming negligible air resistance).

177. Write a formula that expresses the height h of the hammer after it has fallen for t seconds. How far above the ground is the hammer after falling for one second? for two seconds?

178. How many seconds does it take the hammer to reach the ground? How many seconds does it take for the hammer to fall until it is 100 feet above the ground?

179. By plotting some data points and connecting the dots, sketch a graph of h versus t. Notice that your graph is not a picture of the path followed by the falling hammer.

180. What is the domain of $h(t)$? How does this differ from the set of values of t that make sense to use in the context of this problem? This set is called the *feasible domain* for the word problem.

181. (I.65.5) Graph the equations on the same system of axes: $f(x) = x^2$, $g(x) = 0.5x^2$, $h(x) = 2x^2$, and $q(x) = -x^2$. What is the effect of a in equations of the form $y = ax^2$?

Math III

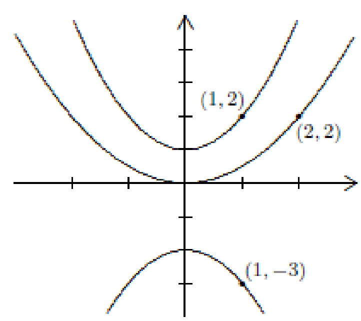
182. (I.62.6) Equations such as $A = 40x - x^2$ and $h = 300 - 16t^2$ define *quadratic functions*. Recall that the word *function* means that assigning a value to one of the variables (*x* or *t*) determines a *unique* value for the other (*A* or *h*). It is customary to say that "*A* is a function of *x*." In this example, however, it would be incorrect to say that "*x* is a function of *A*." Explain.

183. (I.62.7) The graph of a quadratic function is called a *parabola*. This shape is common to all graphs of equations of the form $f(x) = ax^2 + bx + c$ where *a* is nonzero. Confirm this by comparing the graph of $f(x) = x^2$ (the parent function), the graph of $g(x) = 40x - x^2$, and the graph of $h(x) = 300 - 16x^2$. How are the three graphs alike, and how are they different? What graphing window do you need to make sure all vertices and intercepts are displayed?

184. (I.66.1) Near the surface of the earth, assuming negligible resistance from the air, the height in feet of a falling object is modeled well by the equation $h(t) = h_0 - 16t^2$ where *h(t)* is the height of the object after it has been falling for *t* seconds, and h_0 is the height from which the object is dropped (ie, when *t* = 0).
    a. If an iron ball were dropped from the Washington Monument, which is 555 feet high, how far above the ground would the ball be after 2 seconds of falling? How long would it take for the ball to hit the ground?
    b. Due to air resistance, a falling bag of corn chips will not gain speed as rapidly as a falling iron ball. Cal Elayo, a student of science, found that the descent of a falling bag of chips is modeled well by the equation $h(t) = h_0 - 2.5t^2$ In an historic experiment, Cal dropped a bag of chips from a point halfway up the Monument, while a friend simultaneously dropped the iron ball from the top. After how many seconds did the ball overtake the bag of chips?
    c. Graph the equations $h(t) = 277.5 - 2.5t^2$ and $h(t) = 555 - 16t^2$ on the same system of axes. Calculate the *y*-and *t*-intercepts of both curves. What is the meaning of these numbers? Notice that the curves intersect. What is the meaning of the intersection point?

185. (I.66.3) For the point (4, 24) to be on the graph of $f(x) = ax^2$, what should the value of *a* be?

186. (I.68.2) Find a quadratic function for each of the graphs pictured at the right. Each curve has a designated point on it, and the *y*-intercepts are all at integer values. Also notice that for each parabola, the *y*-axis is the *axis of symmetry* – the line about which the parabola is symmetric.



Problem 186

187. (I.68.4) The point (4, 7) is on the graph of $g(x) = x^2 + c$. What is the value of *c*?

188. (SAT problem) If $x \uparrow y$ is defined as $x + y^2$ then find $3\uparrow5$. Is this equal to $5\uparrow3$? When does $x\uparrow y = y\uparrow x$?

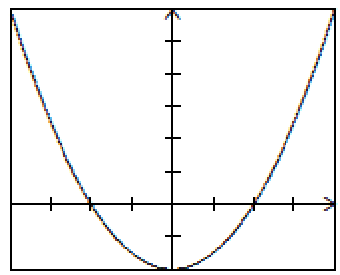
189. (I.71.3) Sketch the graphs of $f(x) = x^2$, $g(x) = (x-2)^2$, $h(x) = (x+3)^2$, and $m(x) = (x-5)^2$ on the same set of coordinate axes. Make a general statement as to how the graph of $y = (x-h)^2$ is related to the graph of the parent function $y = x^2$.

190. (I.71.4) (Continuation) Sketch the graphs $y = 2(x-3)^2$, $y = -3(x-3)^2$, and $y = 0.5(x-3)^2$. What do these graphs all have in common? How do they differ? What is the equation of a parabola whose vertex is at the point (−2, 0), is the same size as the graph $y = 2(x-3)^2$, and opens up?

191. (I.71.6) The axis of symmetry of a parabola is the line *x* = 4.
   a. Suppose that one *x*-intercept is 10; what is the other one?
   b. Suppose the point (12, 4) is on the graph; what other point also must be on the graph?

192. (I.72.4) Sketch the graphs of $y = (x-4)^2$ and *y* = 9 on your calculator screen. What are the coordinates of the point(s) of intersection? Now solve the equation $(x-4)^2 = 9$. Describe the connection between the points of intersection on the graph and the solution(s) to the equation.

193. (I.73.1) The graph of $f(x) = x^2 - 400$ is shown at right. Notice that no coordinates appear in the diagram. There are tick marks on the axes, however, which enable you, without using your graphing calculator, to figure out the actual window that was used for this graph. Find the high and low values for both the *x*-axis and the *y* axis. After you get your answer, check it on your calculator. To arrive at your answer, did you actually need to have tick marks on *both* axes?



Problem 193

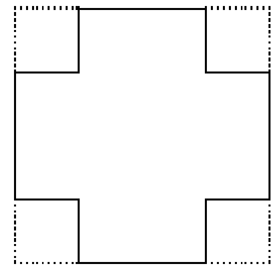
194. (I.73.2) Sketch the graph of $f(x) = x^2 + 3$ and $g(x) = |x| + 3$ on the same axis in your notebook. List three ways that the two graphs are alike and three ways in which they differ. Be sure your graph is large enough to clearly show these differences. On another axis, sketch the graph of $h(x) = 2(x-3)^2$ and $n(x) = 2|x-3|$. Also be prepared to explain how these two graphs compare.

195. (I.74.6) Graph the equations $y = (x-5)^2$, $y = (x-5)^2 - 4$, and $y = (x-5)^2 + 2$. Write the coordinates of the vertex for each curve. Describe how to transform the first parabola to obtain the other two. Next, a fourth parabola is created by shifting the first parabola so that its vertex is (5, −7). Write an equation for the fourth parabola.

196. (III.63.6) How does the graph of $y = f(x)$ compare to the graph of $y + 1 = f(x-3)$?

197. (I.76.2) Sketch the graphs of $y = (x-4)^2$ and $y = (4-x)^2$. What do you notice about the graphs? Explain why this is true.

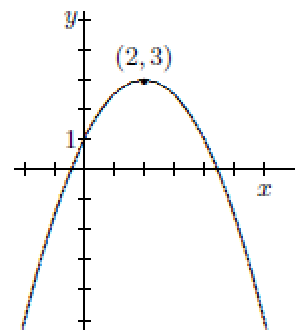
198. (I.76.5) The diagram to the right suggests an easy way of making a box with no top. Start with a square piece of cardboard, cut squares of equal sides from the four corners, and then fold up the sides. Here is the problem: To produce a box that is 8 cm deep and whose capacity is exactly one liter (1000 cc). How large a square must you start with?



Problem 198

199. (I.77.6) Without using a calculator, make a sketch of the parabola $f(x) = (x - 50)^2 - 100$, by finding the *x*-intercepts, the *y*-intercept, and the coordinates of the vertex. Label all four points with their coordinates on your graph. Identify the domain and range of this function.

200. (I.78.8) Find the *x*-intercepts of the following graphs, without expanding each squared term:
a. $y = (x - 4)^2 - 9$ b. $y = -2(x + 3)^2 + 8$
Check your work by sketching each parabola, incorporating the vertex and *x*-intercepts.



Problem 201

201. (I.80.1) When asked to find the equation of the parabola pictured at right, Ryan reasoned that the correct answer had to look like $y = a(x - 2)^2 + 3$, for some value of a. Justify Ryan's reasoning, then finish the problem by finding the correct value of a.

202. (I.80.2) (Continuation) Find an equation for the parabola whose symmetry axis is parallel to the *y*-axis, whose vertex is (−1, 4), and whose graph contains the point (1, 3). What is the range of this parabola?

203. (I.81.2) Graph the equation $g(x) = (x - 5)^2 - 7$ without a calculator by plotting its vertex and its *x*-intercepts (just estimate their positions between two consecutive integers). Then use your calculator to draw the parabola. Repeat the process on $h(x) = -2(x + 6)^2 + 10$.

204. (I.85.4) Find the *x*-intercepts in exact form of each of the following graphs.
a. $y = (x - 6)^2 - 10$ b. $y = 3(x - 7)^2 - 9$ c. $y = 120 - 3x^2$ d. $y = 4.2 - 0.7x^2$

205. (II.23.11) Find coordinates for a point on the line $4y = 3x$ that is 8 units from (0, 0).

206. (I.81.3) At most how many solutions can a quadratic equation have? Give an example of a quadratic equation that has two solutions. Give an example of a quadratic equation that has only one solution. Give an example of a quadratic equation that has no **real** solutions.

207. (I.64.9) Graph the following parabolas with the given windows.
a. $y = 0.001x^2$ in the window $-1000 \le x \le 1000$ and $0 \le y \le 1000$
b. $y = 0.01x^2$ in the window $-100 \le x \le 100$ and $0 \le y \le 100$.
What do you notice about these two graphs?

208. (I.64.9) (Continuation) Avery and Alden were comparing parabola graphs on their calculators. Avery had drawn $y = 0.001x^2$ in the window $-1000 \le x \le 1000$ and $0 \le y \le 1000$, and Alden had drawn $y = x^2$ in the window $-k \le x \le k$ and $0 \le y \le k$. Except for scale markings on the axes, the graphs looked exactly the same! What was the value of *k*?

209. (I.68.5) In your notebook, use one set of coordinate axes to graph the three curves $y = x^2 - x$, $y = x^2 + 2x$, and $y = x^2 - 4x$. Make three observations about graphs of the form $y = x^2 + bx$, where *b* is a nonzero number.

210. (I.63.2) Factor each of the following quadratic expressions.
a. $x^2 + 4x$ b. $2x^2 - 6x$ c. $3x^2 - 15x$ d. $-2x^2 - 7x$

211. (I.63.3) (Continuation) The *zero-product property* says that $a \cdot b = 0$ is true if $a = 0$ or $b = 0$ is true, and only if $a = 0$ or $b = 0$ is true. Explain this property in your own words (looking up the word or in the Reference section if necessary). Apply it to solve these equations.
a. $x^2 + 4x = 0$ b. $2x^2 - 6x = 0$ c. $3x^2 - 15x = 0$ d. $-2x^2 - 7x = 0$

212. (I.63.4) (Continuation) Find the *x*-intercepts of each of the following quadratic graphs.
a. $y = x^2 + 4x$ b. $y = 2x^2 - 6x$ c. $y = 3x^2 - 15x$ d. $y = -2x^2 - 7x$
Summarize by describing how to find the *x*-intercepts of any quadratic graph $y = ax^2 - bx$.

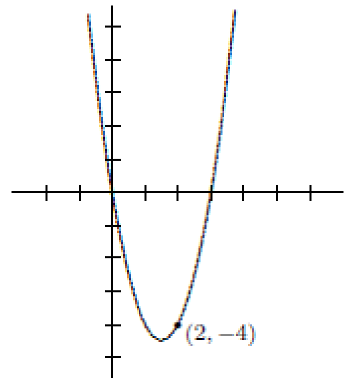
213. (I.70.1) The height *h* (in feet) above the ground of a baseball depends upon the time *t* (in seconds) it has been in flight. Cameron takes a mighty swing and hits a pop-up whose height is described **approximately** by the equation $h(t) = 80t - 16t^2$. (Note, this model does not take into account the height at which the bat hits the ball). Without resorting to graphing on your calculator, answer the following questions
a. How long is the ball in the air?
b. The ball reaches its maximum height after how many seconds of flight?
c. What is the maximum height?
d. It takes 0.92 seconds for the ball to reach a height of 60 feet. On its way back down, the ball is again 60 feet above the ground; what is the value of *t* when this happens?

214. (I.70.3) Solve the following equations for *x* without using a calculator.
a. $x^2 - 5x = 0$ b. $3x^2 + 6x = 0$ c. $ax^2 + bx = 0$

215. (I.70.6) Sketch the graphs of $f(x) = x^2 - 12x$, $g(x) = -2x^2 - 14x$, and $h(x) = 3x^2 + 18x$. Write an equation for the symmetry axis of each parabola. Devise a quick way to write an equation for the symmetry axis of any parabola $y = ax^2 + bx$. Test your method on the three given examples.

Math III

216. (I.73.6) There are several quadratic functions whose graphs intersect the x-axis at (0, 0) and (6, 0). Sketch graphs for a few of them, including the one that goes through (3, 9). Other than their axis of symmetry, what do all these graphs have in common? How do the graphs differ?



Problem 217

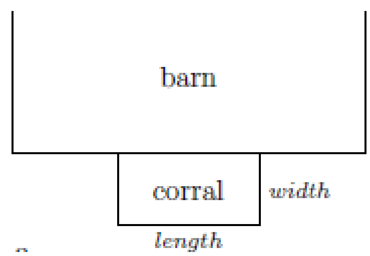
217. (I.73.4) When asked to find the equation of the parabola pictured at right, Ryan took one look at the x-intercepts and knew that the answer had to look like $y = ax(x - 3)$ for some value of a. Justify Ryan's reasoning, then finish the problem by finding the correct value of a.

218. (I.73.5) (Continuation) Find an equation for the parabola whose symmetry axis is parallel to the y-axis, whose x-intercepts are −2 and 3, and whose y-intercept is 4.

219. (I.77.8) The graph of a quadratic function intersects the x-axis at 0 and at 8. Draw two parabolas that fit this description and find equations for them. How many examples are possible? How can you say about the domains and ranges of these examples?

220. (I.77.9) Find an equation for the parabola whose x-intercepts are 0 and 8, whose axis of symmetry is parallel to the y-axis, and whose vertex is at
a. (4, −16) b. (4, −8) c. (4, −4) d. (4, 16)

221. (I.82.5) Sam breeds horses, and is planning to construct a rectangular corral next to the barn, using a side of the barn as one side of the corral. Sam has 240 feet of fencing available, and has to decide how much of it to use for the width of the corral.
   a. Suppose the width is 50 feet. What is the length? How much area would this corral enclose?
   b. Suppose the width is 80 feet. What is the enclosed area?
   c. Suppose the width is x feet. Express the length and the enclosed area in terms of x.



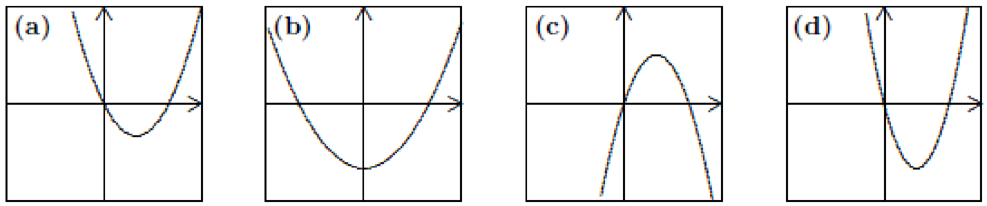
Problem 221

222. (I.82.6) (Continuation) Let y stand for the area of the corral that corresponds to width x. Notice that y is a quadratic function of x. Sketch a graph of y versus x. For what values of x does this graph make sense? (Recall that these sensible values are called the *feasible domain.*) For what value of x does y attain its largest value? What are the dimensions of the corresponding corral?

223. (II.14.12) What number is exactly midway between $23 - \sqrt{17}$ and $23 + \sqrt{17}$ What number is exactly midway between $\frac{-b-\sqrt{b^2-4ac}}{2a}$ and $\frac{-b+\sqrt{b^2-4ac}}{2a}$?

Math III

224. (I.82.8) Which of the following calculator screens could be displaying the graph of $y = x^2 - 2x$?



225. (I.78.3) For each of the following expressions, write down the factors. Then graph the function and record the *x*-intercepts.
   a. Factor $x^2 - 4x - 5 = 0$ and then graph $y = x^2 - 4x - 5$.
   b. Factor $x^2 + 12x + 35 = 0$ and then graph $y = x^2 + 12x + 35$.
   c. Factor $x^2 - 3x + 2 = 0$ and then graph $y = x^2 - 3x + 2$.

226. What is the connection between factoring a polynomial and finding the *x*-intercepts of its graph? Explain how the graph of a polynomial can help in the search for factors.

227. (I.78.4) (Continuation) Find the solutions to each of the following equations.
   a. $x^2 - 4x - 5 = 0$ b. $x^2 + 12x + 35 = 0$ c. $x^2 - 3x + 2 = 0$
   Explain the reasoning used to solve a polynomial equation that is in factored form.

228. (I.63.7) *Golf math I.* Using a driver on the 7th tee, Dale hits an excellent shot, right down the middle of the level fairway. The ball follows the parabolic path shown in the figure, described by the quadratic function $y = 0.5x - 0.002x^2$, which relates the height *y* of the ball above the ground to the ball's progress *x* down the fairway. Distances are measured in yards.

Problem 228

229. Use the distributive property to write this equation in factored form. Notice that $y = 0$ when $x = 0$. What is the significance of this data?

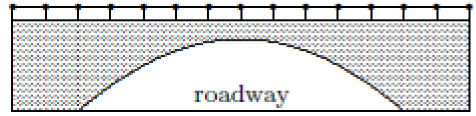
230. How far from the tee does the ball hit the ground?

231. At what distance *x* does the ball reach the highest point of its arc? What is the maximal height attained by the ball?

232. (SAT problem) What is the probability of flipping three coins and having two of them come up heads and one come up tails?

233. (I.91.3) On a single set of coordinate axes, graph several parabolas of the form $y = bx - x^2$. Mark the vertex on each curve. What do you notice about the configuration of all such vertices?

234. (I.86.4) The figure to the right shows a bridge arching over the Parkway East. To accommodate the road beneath, the arch is 100 feet wide at its base, 20 feet high in the center, and parabolic in shape.



Problem 234

235. The arch can be described by $y = kx(x - 100)$, if the origin is placed at the left end of the arch. Find the value of the coefficient k that makes the equation fit the arch.

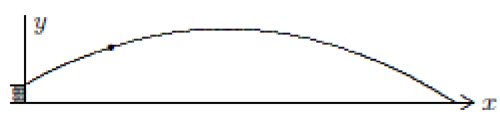
236. Is it possible to move a rectangular object that is 40 feet wide and 16.5 feet high (a wide trailer, for example) through the opening? Explain.

237. (I.82.7) In each of the following, supply the missing factor.
   a. $2x^2 - 5x - 12 = (2x + 3)(\qquad)$ b. $3x^2 - 2x - 1 = (3x + 1)(\qquad)$
   c. $4y^2 - 8y + 3 = (2y - 1)(\qquad)$ d. $6t^2 - 7t - 3 = (3t + 1)(\qquad)$

238. (I.71.10) Find the equation of the axis of symmetry for the graph of $f(x) = 2x^2 - 6x$. Sketch the graph, including the axis of symmetry. What are the coordinates of the vertex of the graph?

239. (I.71.11) (Continuation) Sketch the graph of $g(x) = 2x^2 - 6x - 3$ along with its axis of symmetry. Find the coordinates of the vertex of this parabola. How do these coordinates compare with the vertex of $f(x) = 2x^2 - 6x$? Find an equation for the graph of a quadratic curve that has the same axis of symmetry as $f(x) = 2x^2 - 6x$, but whose vertex is at (1.5, −2.5).

240. (I.65.3) *Golf math II.* Again using a driver on the 8th tee, which is on a plateau 10 yards above the level fairway, Dale hits another fine shot. Explain why the quadratic function $y = 10.0 + 0.5x - 0.002x^2$ describes this parabolic trajectory, shown in the figure above. Why should you expect this tee shot to go more than 250 yards? Estimate the length of this shot, then use your calculator to find a more accurate value. How does this trajectory relate to the trajectory for the drive on the previous hole?



Problem 240

241. Are quadratic functions invertible? Explain.

242. (II.60.4) Sketch the circle whose equation is $x^2 + y^2 = 100$. Using the same system of coordinate axes, graph the line $x + 3y = 10$, which should intersect the circle twice: at $A = (10, 0)$ and at another point B in the second quadrant. Estimate the coordinates of B. Now use algebra to find them exactly.

243. (I.66.2) Sketch a graph for each of the following quadratic functions. Identify the coordinates of each vertex and write an equation for each axis of symmetry.
   a. $y = 3x^2 + 6$ b. $y = x^2 + 6x$ c. $y = 64 - 4x^2$ d. $y = x^2 - 2x - 8$

Math III

244. (I.72.1) The table at right displays some values for a quadratic function $f(x) = ax^2 + bx + c$.
Explain how to use the table to show that $c = 0$.

x	0	1	2	3	4
$f(x)$	0	2	6	12	20

Problem 244

245. A point is on a curve only if the coordinates of the point satisfy the equation of the curve. Substitute the tabled coordinates (1, 2) into the given equation to obtain a linear equation in which a and b are the unknowns. Apply the same reasoning to the point (2, 6).
   a. Find values for a and b by solving these two linear equations.
   b. Use your values for a and b to identify the original quadratic equation. Check your result by substituting the other two tabled points (3, 12) and (4, 20) into the equation.

246. (I.74.1) In solving an equation such as $x^2 - 8x = 3$ by completing the square, we begin by taking half of -8, squaring that result and adding it to *both* sides of the equation. This step allows us to factor the left side of the equation as a perfect square binomial; this step is *completing the square*. Now we can find the solutions to $x^2 - 8x = 3$ by taking square roots of both sides and solving for x. Use this method to find the solutions to $x^2 - 8x = 3$.

247. (Continuation) When using completing the square to solve an equation such as $2x^2 + 12x = 18$, it is necessary to first divide each term by 2 so that the coefficient of is x^2 is 1. This transforms the equation into $x^2 + 6x = 9$. Now we are ready to complete the square on the left side of the equation by adding $\left(\frac{6}{2}\right)^2$ to both sides of the equation. Continue this process to find the solutions to the given equation.

248. Use completing the square to find the solutions for each equation.
   a. $k^2 - 18k + 48 = -9$
   b. $x^2 + 20x - 49 = 0$
   c. $7r^2 + 14r - 16 = 5$
   d. $3a^2 - 12a - 61 = 0$

249. Completing the square is a method that can be used to solve any quadratic equation. For some quadratic equations, though, it is easier to solve by factoring or using the quadratic formula. Try using completing the square to solve $3x^2 - 2x - 1 = 0$. What aspects of this equation make completing the square more difficult than it was for the equations given in the previous problem?

250. (I.74.2) *Completing the square*. Confirm that the equation $ax^2 + bx + c = 0$ can be converted into the form $x^2 + \frac{b}{a}x = -\frac{c}{a}$. Describe the steps. To achieve the goal suggested by the title, what should now be added to both sides of this equation?

251. (I.74.3) (Continuation) The left side of the equation $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$ can be factored as a perfect square trinomial. Show how. The right side of the equation can be combined over a common denominator. Show how. Finish the solution of the general quadratic equation by taking the square root of both sides of your most recent equation. The answer is called *the quadratic formula*. You may be able to check your formula using your calculator. Apply your formula: Solve $x^2 + 2x - 3 = 0$ by letting $a = 1$, $b = 2$, and $c = -3$.

Math III

252. (I.84.1) I am thinking of a right triangle, whose sides can be represented by $x - 5$, $2x$, and $2x + 1$. Find the lengths of the three sides. Note: Solving your equation leads to answers that do not fit the context of the problem and therefore should be discarded.

253. (I.74.4) As long as the coefficients *a* and *b* are nonzero, the parabolic graph $y = ax^2 + bx$ has two *x*-intercepts. What are they? Use them to find the axis of symmetry for this parabola. Explain why the axis of symmetry for $y = 2x^2 - 5x - 12$ is the same as the axis of symmetry for $y = 2x^2 - 5x$. In general, what is the symmetry axis for $y = ax^2 + bx + c$? Does your description make sense for $y = 2x^2 - 5x + 7$, even though the curve has no *x*-intercepts?

254. (I.74.5) (Continuation) If you know the axis of symmetry for a quadratic function, how do you find the coordinates of the vertex? Try your method on each of the following, by first finding the symmetry axis, then the coordinates of the vertex.
    a. $y = x^2 + 2x - 3$ b. $y = 3x^2 + 4x + 1$

255. (I.69.6) The work at right shows the step-by-step process used by a student to solve $x^2 + 6x - 5 = 0$ by the method of completing the square. Explain why the steps in this process are reversible. Apply this understanding to find a quadratic equation $ax^2 + bx + c = 0$ whose solutions are $x = 7 + \sqrt{6}$ and $x = 7 - \sqrt{6}$.

$$x^2 + 6x - 5 = 0$$
$$x^2 + 6x + 9 = 5 + 9$$
$$(x + 3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

Problem 255

256. (I.87.6) A mathematics teacher wants to make up a quadratic equation $ax^2 + bx + c = 0$, so that *a*, *b*, and *c* are integers, and the correct solutions are $x = \frac{1}{2}$ and $x = -3$. Find values for *a*, *b*, and *c* that will do the job. Is there more than one equation that will work?

257. (I.85.9) Expand and simplify each of the following expressions.
    a. $(x - 1)(x + 1)$ b. $(x - 1)(x^2 + x + 1)$ c. $(x - 1)(x^3 + x^2 + x + 1)$

258. (I.86.1) (Continuation) Write $x^5 - 1$ as the product of two factors.

259. (I.86.3) Factor each of the following as completely as you can.
    a. $p^4 - 4p^2$ b. $w^3 - 2w^2 - 15w$ c. $16y - 9yz^2$ d. $2x^2 + 20x + 50$

260. (I.77.11) Solve $x^2 + bx + c = 0$ by the method of completing the square. Apply your answer to the example $x^2 + 5x + 6 = 0$ by setting *b* = 5 and *c* = 6.

261. (I.70.2) Apply the zero-product property to solve the following equations.
    a. $(x - 2)(x + 3) = 0$ b. $x(2x + 5) = 0$ c. $5(x - 1)(x + 4)(2x - 3) = 0$

262. (II.49.4) A rectangle is 2 inches wide and more than 2 inches long. It so happens that this rectangle can be divided, by a single cut, into a 2-inch square and a small rectangle that has exactly the same shape as the large rectangle. What is the length of the large rectangle?

263. (I.69.2) A hose used by the fire department shoots water out in a parabolic arc. Let *x* be the horizontal distance from the hose's nozzle, and *y* be the corresponding height of the stream of water, both in feet. The quadratic function is $y = -0.016x^2 + 0.5x + 4.5$.
  a. What is the significance of the 4.5 that appears in the equation?
  b. Use your calculator to graph this function. Find the stream's greatest height.
  c. What is the horizontal distance from the nozzle to where the stream hits the ground?
  d. Will the stream go over a 6-foot high fence that is located 28 feet from the nozzle? Explain your reasoning.

264. (I.79.2) Find at least three integers (positive or negative) that, when put in the blank space, make the expression $x^2 + \_\_\_x - 36$ a factorable trinomial. Are there other examples? How many?

265. (I.79.3) (Continuation) Find at least three integers that, when put in the blank space, make the expression $x^2 + 4x - \_\_\_\_$ a factorable trinomial. Are there other examples? How many? What do all these integers have in common?

266. (II.61.6) The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ in two points. Call the third quadrant point *R* and the first-quadrant point *E*, and find their coordinates. Let *D* be the point where the line through *R* and the center of the circle intersects the circle again. Show that triangle *RED* is a right triangle.

267. (I.68.10) Without using a calculator, solve each of the following quadratic equations
  a. $(x+4)^2 = 23$ b. $7x^2 - 22x = 0$ c. $x^2 - 36x = 205$ d. $1415 - 16x^2 = 0$

268. (I.76.4) The SSA Ski Club is planning a ski trip for the upcoming long weekend. They have 40 skiers signed up to go, and the ski resort is charging \$120 for each person.
  a. Calculate how much money (revenue) the resort expects to take in.
  b. The resort manager offers to reduce the group rate of \$120 per person by \$2 for each additional registrant, up to a maximum number *m*. For example, if five more skiers were to sign up, all 45 would pay \$110 each, producing revenue \$4950 for the resort. Fill in the rest of the table at right, and you will discover the manager's value of *m*.
  c. Let *x* be the number of new registrants. In terms of *x*, write expressions for the total number of persons going, the cost to each, and the resulting revenue for the resort.
  d. Plot your revenue values versus *x*, for the relevant values of *x*. Because this is a *discrete* problem, it does not make sense to connect the dots.
  e. For the resort to take in at least \$4900, how many SSA skiers must go on trip?

extras	*persons*	*cost/person*	*revenue*
0			
1			
2			
3			
4			
5	45	110	4950
6			
7			
8			
9			
10			
11			
12			

Problem 268

Math III

269. (I.70.5) In the shot-put competition at the SSA-Freeport track meet, the trajectory of Blair's best put is given by the function $h(x) = -0.018x^2 + x + 5.0$, where *x* is the horizontal distance the shot travels, and *h* is the corresponding height of the shot above the ground, both measured in feet. Graph the function and calculate how far the shot went. What was the greatest height obtained? In the given context, what is the meaning of the "5.0" in the equation?

270. (I.80.5) A small calculator company is doing a study to determine how to price one of its new products. The theory is that the income *r* from a product is a function of the market price *p*, and one of the managers has proposed that the quadratic model $r(p) = p(3000 - 10p)$ provides a realistic approximation to this function.
    a. What was the manager's reasoning in devising this formula?
    b. What is the significance of the value *p* = 300 in this investigation?
    c. Assume that this model is valid and figure out the best price to charge for the calculator. How much income for the company will the sales of this calculator provide?
    d. If the management is going to be satisfied as long as the income from the new calculator is at least $190000, what range of prices *p* will be acceptable?

271. (I.75.1) The driver of a red sports car, moving at *r* feet per second, sees a pedestrian step out into the road. Let *d* be the number of feet that the car travels, from the moment when the driver *sees* the danger until the car has been brought to a complete stop. The equation $d(r) = 0.75r + 0.03r^2$ models the typical panic-stop relation between stopping distance and speed.
    a. Moving the foot from the accelerator pedal to the brake pedal takes a typical driver three fourths of a second. What does the term 0.75*r* represent in the stopping-distance equation? The term $0.03r^2$ comes from physics; what must it represent?
    b. How much distance is needed to bring a car from 30 miles per hour (which is equivalent to 44 feet per second) to a complete stop?
    c. How much distance is needed to bring a car from 60 miles per hour to a complete stop?
    d. Is it true that doubling the speed of the car doubles the distance needed to stop it?

272. (I.75.2) (Continuation) At the scene of a crash, an officer observed that a car had hit a wall 150 feet after the brakes were applied. The driver insisted that the speed limit of 45 mph had not been broken. What do you think of this evidence?

273. (I.75.5) The equation $y = 50x - 0.5x^2$ describes the trajectory of a toy rocket, in which *x* is the number of feet the rocket moves horizontally from the launch, and *y* is the corresponding number of feet from the rocket to the ground. The rocket has a sensor that causes a parachute to be deployed when activated by a laser beam.
    a. If the laser is aimed along the line $y = 20x$, at what altitude will the parachute open?
    b. At what slope could the laser be aimed to make the parachute open at 1050 feet?

274. (I.83.8) Graph the equation $y = -2x^2 + 5x + 33$. For what values of x...
    a. is *y* = 0? b. is *y* = 21? c. is $y \geq 0$?

Math III

275. (I.81.8) Graph the three points (−2, 1), (3, 1), and (0, 7). There is a quadratic function whose graph passes through these three points. Sketch the graph. Find its equation in two ways: First, begin with the equation $y = ax^2 + bx + c$ and use the three points to find the values of *a*, *b*, and *c*. (One of these values is essentially given to you.) Second, begin with the equation $y = a(x - h)^2 + k$ and use the three points to determine *a*, *h*, and *k*. (One of these values is almost given to you.) Your two equations do not look alike, but they should be equivalent. Check that they are.

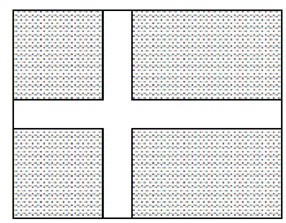
276. (I.88.1) The perimeter of a rectangular field is 80 meters and its area is 320 square meters. Find the dimensions of the field, correct to the nearest tenth of a meter.

277. (I.77.4) Solve the following quadratic equations.
a. $x^2 + 6x + 5 = 0$ b. $x^2 - 7x + 12 = 0$ c. $x^2 + 3x - 4 = 0$ d. $x^2 - x - 6 = 0$

278. (I.89.8) We know that the axis of symmetry for a parabola in the form $y = ax^2 + bx + c$ can be found from the formula $x = -\frac{b}{2a}$. The equation of the axis of symmetry can help us find the *y*-coordinate of the vertex. Make the appropriate substitution, using $x = -\frac{b}{2a}$ and find a formula for the *y*-coordinate of the vertex in terms of *a*, *b*, and *c*.

279. (II.10.5) A 9-by-12 rectangular picture is framed by a border of uniform width. Given that the combined area of picture plus frame is 180 square units, find the width of the border.

280. (I.90.4) The diagram at right shows the flag of Denmark, which consists of a white cross of uniform width against a solid red background. The flag measures 2 feet 11 inches by 3 feet 9 inches, and the area of the white cross is 5/21 of the area of the whole flag. Use this information to find the width of the white cross.


Problem 280

281. (I.82.1) Solve each of the following by completing the square.
a. $3x^2 - 6x = 1$ b. $2x^2 + 8x - 17 = 0$

282. (I.45.1) Wes walks from home to a friend's house to borrow a bicycle, and then rides the bicycle home along the same route. By walking at 4 mph and riding at 8 mph, Wes takes 45 minutes for the whole trip. Find the distance that Wes walked.

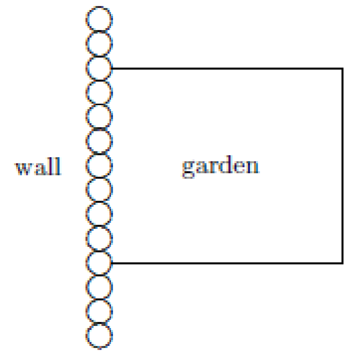
283. (I.90.1) Find both solutions to $3x^2 - 7x + 3 = 0$. As a check, you should find that the two answers are reciprocals of one another.

284. (I.86.5) There is a unique parabola whose symmetry axis is parallel to the *y*-axis, and that passes through the three points (1, 1), (−2, −2), and (0, −4). Write an equation for it. Given any three points, must there be a parabola that will pass through them? Explain.

285. (II.23.5) Let *A* = (0, 12) and *B* = (25, 12). If possible, find coordinates for a point *P* on the *x*-axis that makes angle *APB* a right angle.

286. (I.86.9) Sam is a guest on the TV show *Math Jeopardy*, and has just chosen the $300 question in the category “Quadratic Equations.” The answer is “The solutions are $x = 3$ and $x = -2$.” What question could Sam ask that would win the $300? Is there more than one possible correct question?

287. (I.72.2) Gerry Anium is designing another rectangular garden. It will sit next to a long, straight rock wall, thus leaving only three sides to be fenced. This time, Gerry has bought 150 feet of fencing in one-foot sections. Subdivision into shorter pieces is not possible. The garden is to be rectangular and the fencing (all of which must be used) will go along three of the sides as indicated in the picture.



Problem 287

a. If each of the two sides attached to the wall were 40 ft long, what would the length of the third side be?
b. Is it possible for the longest side of the rectangular garden to be 85 feet long? Explain.
c. Let x be the length of one of the sides attached to the wall. Find the lengths of the other two sides, in terms of x. Is the variable x continuous or *discrete*?
d. Express the area of the garden as a function of x, and graph this function. For what values of x does this graph have meaning?
e. Graph the line $y = 2752$. Find the coordinates of the points of intersection with this line and your graph. Explain what the coordinates mean with relation to the garden.
f. Gerry would like to enclose the largest possible area possible with this fencing. What dimensions for the garden accomplish this? What is the largest possible area?

288. (I.68.6) Add the first two odd positive numbers, the first three odd positive numbers, and the first four odd positive numbers. Do your answers show a pattern? What is the sum of the first n odd positive numbers?

289. (I.68.7) (Continuation) Copy the accompanying tables into your notebook and fill in the missing entries. Notice that the third column lists the differences between successive y-values. Is there a pattern to the column of differences? Do the values in this column describe a linear function? Explain. As a check, create a fourth column that tables the differences of the differences. How does this column help you with your thinking?

x	y
0	0
1	1
2	4
3	9
4	
5	

diff
1
3

Problem 289

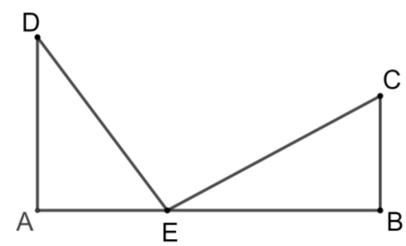
290. (I.68.8) (Continuation) Carry out the same calculations, but replace $y = x^2$ by a quadratic function of your own choosing. Is the new table of differences linear?

291. (I.90.6) Graph the nonlinear equation $y = 9 - x^2$, identifying all of the axis intercepts. On the same system of coordinate axes, graph the line $y = 3x - 5$, and identify its axis intercepts. You should see two points where the line intersects the parabola. First estimate their coordinates, then calculate the coordinates exactly by solving the system of simultaneous equations. Which methods of solution work best in this example?

292. (II.21.1) Is the following equation True or False for all values of x? $\sqrt{4x} + \sqrt{9x} = \sqrt{13x}$.

293. (I.83.10) Sketch the graphs of $f(x) = 3\sqrt{x}$ and $g(x) = x + 2$, and then find their points of intersection. Now solve the equation $3\sqrt{x} = x + 2$ by first squaring both sides of the equation. Do your answers agree with those obtained from the graph?

294. (II.1.10) In the diagram to the right, *AEB* is straight and angles *A* and *B* are right. *AD* = 15, *CB* = 10, and *AB* = 30. The length of *DE* + *EC* depends on the value chosen for *AE*. Another way to say this is that *DE* + *EC is a function of AE*.



Problem 294

   a. Letting *x* stand for *AE* (and 30 – *x* for *EB*), write a formula for the length *DE* + *EC*. What is the feasible domain of *x*?
   b. Enter this formula into your calculator and graph it. What is an appropriate viewing window for this graph?
   c. Find the value of *x* that produces the *shortest* path from *D* to *C* through *E*.
   d. Using this value of x, draw an accurate picture of the path from *D* to *C* through *E*.
   e. Make a conjecture about *angles AED* and *BEC*. To test your conjecture, find the measures of these angles.

295. (I.89.3) Sketch the graphs of $p(x) = \sqrt{x}$ and $f(x) = \sqrt{x - 3}$ on the same system of axes. Describe in words how the two graphs are related. Do they intersect?

296. (I.89.4) Sketch the graphs of $p(x) = \sqrt{x}$ and $g(x) = \sqrt{x} - 3$ on the same system of axes. (Note: this is *not* the same problem as above.) Describe in words how the two graphs are related. Do they intersect?

297. (I.91.4) Sketch the graphs of $h(x) = 2\sqrt{x}$ and $r(x) = x - 3$, and then find all points of intersection. Now solve the equation $2\sqrt{x} = x - 3$ by first squaring both sides of the equation. Do your answers agree with those obtained from the graph?

298. (Continuation) Solve the equations below by using the technique of squaring both sides as in the prior problem. By the way, extraneous roots sometimes occur when solving equations with square roots so be sure to check all of your answers in the original equation.
   a. $\sqrt{2n + 3} = n$
   b. $\sqrt{2x^2 - 7} = 5$

299. (Continuation) In the two previous problems, we used the technique of squaring both sides of the equations to "undo" the square root.
   a. Sketch the graphs of $f(x) = x^2$ and $g(x) = \sqrt{x}$ on the same coordinate axes.
   b. What are the domain and range of f and g?
   c. Why is it incorrect to say that f and g are inverses of each other? Use your graphs as evidence.
   d. How can we restrict (reduce) the domain of $f(x) = x^2$ so that it is the true inverse of g?

Math III

300. (SAT problem) Let **$x** be defined by $\$x = \frac{x+3}{x-1}$ for any *x* not equal to 1. What is the value of \$9? How about \$7? Is $\$(x-2)$ the same thing as $(\$x)\ -2$? Explain. Note that we usually use function notation to describe these types of definitions, as in $f(x) = \frac{x+3}{x-1}$. Find the domain of f.

301. Find a formula for *percent change*. Use it to find the percent change for the following scenarios.
   a. Kiara has been looking to buy a new bike. She found one she likes and has been waiting for it to go on sale. The bike costs \$548 but is on sale for \$411. What percent off will Kiara get on the bike?
   b. Justin is moving from 8th grade at the Middle School to 9th grade at the Senior School. His 8th grade class has 67 people and his 9th grade class will have 134. What is the percent change of Justin's class?

302. (I.59.4) It would take Tom 8 hours to paint the fence in the backyard. His friend Jerry would need 12 hours to do the same job by himself. They both start work at 9 in the morning, each at opposite ends of the fence. At what time in the afternoon is the task complete?

303. (I.47.2) Working alone, Jess can rake the leaves off a lawn in 50 minutes. Working alone, cousin Tate can do the same job in 30 minutes. Today they are going to work together, Jess starting at one end of the lawn and Tate starting simultaneously at the other end. In how many minutes will they meet and thus have the lawn completely raked?

304. (I.47.3) (Continuation) Suppose that Tate takes a ten-minute break after just five minutes of raking. Revise your prediction of how many minutes it will take to complete the job.

305. (I.2.4) When describing the growth of a population, the passage of time is sometimes described in generations, a generation being about 30 years. One generation ago, you had two ancestors (your parents). Two generations ago, you had four ancestors (your grandparents). Ninety years ago, you had eight ancestors (your great-grandparents). How many ancestors did you have 300 years ago? 900 years ago? Do your answers make sense?

306. Write the definition for a *rational number*, consulting additional resources if you need to. Using that definition, what do you anticipate a *rational expression* would be? *A rational function*?

307. (I.79.4) List any restrictions for x, where appropriate, and then combine into one fraction.
   a. $\frac{1}{3}+\frac{1}{7}$ b. $\frac{1}{15}+\frac{1}{19}$ c. $\frac{1}{x-2}+\frac{1}{x+2}$

   Evaluate your answer to part (c) with *x* = 5 and *x* = 17. How do these answers compare to your answers in parts (a) and (b)?

308. Find value(s) of *x* for each expression below that makes the expression undefined.
   a. $\frac{1}{x+1}$ b. $\frac{1}{x-4}$ c. $\frac{3}{5x}$ d. $\frac{4x}{x^2-6x-16}$

309. (I.57.7) Faced with the problem of dividing 5^{24} by 5^8, Brook is having trouble deciding which of these four answers is correct: $5^{16}, 5^3, 1^{16}$, and 1^3. Your help is needed. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the *common-base principle for division* that tells how to divide b^m by b^n to get another power of b. Then apply this principle to the following situations.
   a. Earth's human population is roughly 7×10^9, and its total land area, excluding the polar caps, is roughly 5×10^7 square miles. If the human population were distributed uniformly over all available land, approximately how many persons would be found per square mile?
   b. At the speed of light, which is 3×10^8 meters per second, how many seconds does it take for the Sun's light to travel the 1.5×10^{11} meters to Earth?

310. (I.61.1) What is the value of $\frac{5^7}{5^7}$? of $\frac{8^3}{8^3}$? of $\frac{c^{12}}{c^{12}}$? What is the value of any number divided by itself? If you apply the *common-base principle for division* dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what power? and $\frac{c^{12}}{c^{12}}$ should equal c raised to what power? It therefore makes sense to define c^0 to be what?

311. (I.80.7) For each of the following, list the restrictions for x, and then combine over a common denominator. **(a)** $\frac{1}{x-3}+\frac{2}{x}$ **(b)** $\frac{1}{x-3}+\frac{2}{x+3}$

312. (I.53.3) Faced with the problem of calculating $(5^4)^3$, Brook is having trouble deciding which of these three answers is correct: $5^{64}, 5^{12}$, or 5^7. Once you have answered Brook's question, experiment with other examples of this type until you are ready to formulate the principle that tells how to write $(b^m)^n$ as a power of b.

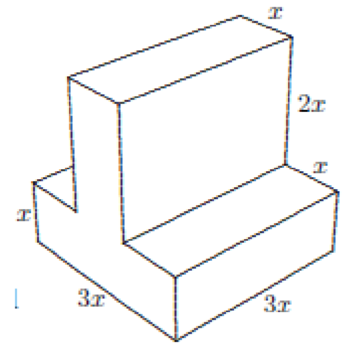
313. Write a *rational* expression that satisfies the conditions below.
   a. The expression is undefined when $x = 3$.
   b. The expression is undefined when $x = -6$.
   c. The expression is undefined when $x = 2$ and $x = -2$.
   d. The expression is undefined when $x = 7$ and $x = -5$.

314. (III.37.1) It is well known that multiplication can be *distributed* over addition or subtraction, meaning that $a \cdot (b + c)$ is equivalent to $a \cdot b + a \cdot c$, and that $a \cdot (b - c)$ is equivalent to $a \cdot b - a \cdot c$. It is *not* true that multiplication distributes over multiplication, however, for $a \cdot (b \cdot c)$ is not the same as $a \cdot b \cdot a \cdot c$. Now consider distributive questions about exponents: Is $(b + c)^n$ equivalent to $b^n + c^n$? Explore this question by choosing some numerical examples. Is $(b \cdot c)^n$ equivalent to $b^n \cdot c^n$? Look at more examples.

315. (I.62.1) Write the following monomials without using parentheses.
   a. $(ab)^2(ab^2)$ b. $(-2xy^4)(4x^2y^3)$ c. $(-w^3x^2)(-3w)$ d. $(7p^2q^3r)(7pqr^4)^2$

316. Equations involving fractions can be solved by first multiplying both sides of the equation by the least common denominator of all fractions. The transformed equation will then be free of fractions. Try solving the equations below using this method. List any restrictions for x.
   a. $\frac{3x}{5}-\frac{5}{6}=\frac{x}{10}$ LCD = ______ b. $\frac{5}{x}+\frac{1}{4}=\frac{1}{x}$ LCD = _______

317. (I.48.9) The diagram at the right represents a solid of uniform cross-section. All the lines of the figure meet at right angles. The dimensions are marked in the drawing in terms of x. Write simple formulas in terms of x for each of the following:
   a. the volume of the solid;
   b. the surface area you would have to cover in order to paint this solid;
   c. the length of decorative cord you would need if you wanted to outline all the edges of this solid.



Problem 317

318. (III.39.10) Explain your opinions of each of the following student responses.
   a. Asked to find an expression equivalent to $x^8 - x^5$, a student responded x^3.
   b. Asked to find an expression equivalent to $\frac{x^8-x^5}{x^2}$, a student responded $x^6 - x^3$.
   c. Another student said that $\frac{x^2}{x^8-x^5}$ is equivalent to $\frac{1}{x^6}-\frac{1}{x^3}$.

319. (I.86.8) Find the solution to each equation, and list any restrictions for x.
   a. $\frac{x}{3}+\frac{x}{5}=12$ b. $\frac{x-2}{-2}=\frac{4x-3}{4}$ c. $\frac{x+1}{3}+\frac{x-1}{x}=2$

320. Solving equations with fractions sometimes leads to transformed equations whose roots do not work in the original equation. Such roots are called *extraneous roots* and they should be indicated as such. Solve each of the equations below and determine if there are any extraneous roots. Can you offer a reasonable explanation as to why extraneous roots occur?
   a. $\frac{x}{x+3}+\frac{1}{x-3}=1$ b. $\frac{3}{x^2-7x+10}+2=\frac{x-4}{x-5}$

321. (I.90.5) Alex is making a 4-mile trip. The first two miles were at 30 mph. At what speed must Alex cover the remaining two miles so that the average speed for the entire trip will be each of the following (be careful!).
   a. 50 mph? b. 55 mph? c. 59.9mph? d. 60 mph?

322. (I.49.3) Most of Conservative Colby's money was put into a savings account that paid 5% interest a year, but some of it was put into a risky stock fund that paid 16% a year. Colby's total initial investment in the two accounts was \$10000. At the end of the first year, Colby received a total of \$775 in interest from the two accounts. Find the amount invested in each.

323. Given the function $p(x) = \frac{1}{x}$, fill out the table to the right. You may choose to put your *y* values as decimals.
   a. Sketch a graph of the values from the table.
   b. What do you notice about the *y* value when $x = 0$?
   c. There is a specific term used when the denominator of a rational function equals zero. Use your resources to find that term.
   d. What do you notice about the *y* value as *x* gets smaller and smaller?
   e. What do you notice about the *y* value as *x* gets bigger and bigger?

x	$p(x)$
-2	
-1	
0	
1	
2	
3	
4	

Problem 323

324. (Continued) Can $y = 0$? Why or why not? Justify your answer.

325. (Continued) Graph the equation from the problem above with your graphing calculator. Does your sketch match the graph your calculator produced? Use the table to check your answers for the parts above.

326. (III.28.9) Exponents are routinely encountered in scientific work, where they help investigators deal with large numbers:
   a. The human population of Earth is roughly 7000000000, which is usually expressed in *scientific notation* as 7×10^9. The average number of hairs on a human head is 5×10^5. Use scientific notation to estimate the number of human head hairs on Earth.
   b. Light moves very fast—approximately 3×10^8 meters every second. At that rate, how many meters does light travel in one year, which is about 3×10^7 seconds long? This so-called *light year* is used in astronomy as a yardstick for measuring even greater distances.

327. (III.30.12) The diameter of a typical atom is so small that it would take about 10^8 of them, arranged in a line, to reach just one centimeter. It is therefore a plausible estimate that a cubic centimeter could contain about $10^8 \times 10^8 \times 10^8 = (10^8)^3$ atoms. Write this huge number as a power of 10.

328. Suppose you are given two functions, $p(x) = \frac{1}{x}$ and $g(x) = \frac{4}{x}$. Fill out the table below and answer the following questions.
   a. How are the y values different for these two functions?
   b. Predict how the graph of g will compare to the graph of p based on your table. What will they have in common? What will be different?
   c. Sketch both graphs on the same set of axes and check your answer with your graphing calculator.

x	$p(x) = \frac{1}{x}$	$g(x) = \frac{4}{x}$
-2		
-1		
0		
1		
2		
3		
4		

Problem 328

Math III

329. (III.34.2) The common-base principle for multiplication predicts that $5^{1/2}$ times $5^{1/2}$ should be 5. Explain this logic, then conclude that $5^{1/2}$ is just another name for a familiar number. Use your calculator to check your prediction. How would you describe the number $6^{1/3}$, given that $6^{1/3} \cdot 6^{1/3} \cdot 6^{1/3}$ equals 6? Formulate a general meaning of expressions like $b^{1/n}$ and use a calculator to test your interpretation on simple examples like $8^{1/3}$ and $32^{1/5}$.

330. For each pair of functions below, predict how the graph of the second function will compare to the graph of the first. Check your answer with your graphing calculator.
    a. $p(x) = \frac{1}{x}$; $f(x) = \frac{2}{x}$
    b. $p(x) = \frac{1}{x}$; $g(x) = \frac{1}{x+3}$
    c. $p(x) = \frac{1}{x}$; $h(x) = \frac{1}{x-5}$
    d. $p(x) = \frac{1}{x}$; $q(x) = \frac{6}{x+1}$

331. (III.35.6) The result of dividing 5^7 by 5^3 is 5^4. What is the result of dividing 5^3 by 5^7, however? By considering such examples, decide what it means to put a *negative* exponent on a base.

332. (III.35.7) Exponents are routinely encountered in science, where they help to deal with small numbers. For example, the diameter of a proton is 0.0000000000003 cm. Explain why it is logical to express this number in scientific notation as 3×10^{-13}. Calculate the surface area and the volume of a proton.

333. (III.38.10) You have deposited \$1000 in a money-market account that earns 8 percent annual interest. Assuming no withdrawals or additional deposits are made, calculate how much money will be in the account one year later; two years later; three years later; four years later; *t* years later.

334. (III.40.5) The population of Grand Fenwick has been increasing at the rate of 2.4 percent per year. It has just reached 5280 (a milestone). What will the population be after ten years? after *t* years? After how many years will the population be 10560?

335. (III.38.1) Invent a division problem whose answer is b^0, and thereby discover the meaning of b^0.

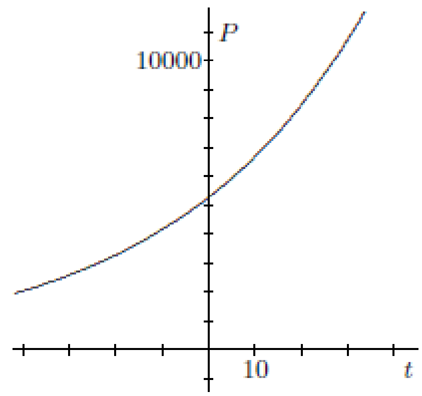
336. (SAT) Let b #x be defined as b^{2x}. Simplify (2#5)#*x* and 2#(5#*x*). Are these expressions the same? Try a few different values for *x* to check your answer.

337. (Continuation) Simplify the product (5#4)·(5#10) without using your calculator. If you write this result as (5#*a*) then what is the value of *a*? Is this surprising?

338. (III.40.7) Rewrite each equation so that it has the form "*x* = . . ." Please do not use your calculator's solver.
    a. $x^5 = a^3$
    b. $x^{\frac{1}{5}} = a^3$
    c. $(x+1)^{15.6} = 2.0$
    d. $x^{-2} = a$

Math III

339. (III.40.6) The figure at right shows part of the graph of $P(t) = 5280(1.024)^t$, a function that describes a small town whose population has been growing at an annual rate of 2.4 percent.
   a. Identify the domain and range of $P(t)$ using interval notation.
   b. What is $P(0)$, and what is its meaning?
   c. Use the graph to estimate the solution of the equation $P(t) = 10560$.
   d. Calculate $P(-30)$. What does this number mean?
   e. Comment on the part of the graph that lies outside the borders of the illustration. How would it look if you could see it, and what does it mean?



Problem 339

340. (III.41.3) Show that $P + Pr + (P + Pr)r = P(1 + r)^2$. Based on your work with exponential growth, interpret the three individual terms on the left side of this equation, and explain why their sum should equal the expression on the right side.

341. (III.41.6) A helium-filled balloon is slowly deflating. During any 24-hour period, it loses 5 percent of the helium it had at the beginning of that period. The balloon held 8000 cc of helium at noon on Monday. How much helium did it contain 3 days later? 4.5 days later? 20 days later? *n* days later? 12 hours later? *k* hours later? Approximately how much time is needed for the balloon to lose half its helium? This time is called the *half-life*. Be as accurate as you can.

342. (III.42.4) Convert the following to equivalent forms in which no negative exponents appear.
   a. $\left(\frac{2}{5}\right)^{-1}$ b. $\frac{6}{x^{-2}}$ c. $\left(-\frac{5}{2}\right)^{-3}$ d. $\frac{6xy}{3x^{-1}y^{-2}}$ e. $\left(\frac{2x^2}{3x^{-1}}\right)^{-2}$

343. (II.25.3) Working against a 1-km-per-hour current, some members of the Outing Club paddled 7 km up the Allegheny River one Saturday last spring and made camp. The next day, they returned downstream to their starting point, aided by the same one-km-per-hour current. They paddled for a total of 6 hours and 40 minutes during the round trip. Use this information to figure out how much time the group would have needed to make the trip if there had been no current.

344. (III.42.6) In order that a \$10000 investment grow to \$20000 in seven years, what must be the annual rate of interest? Seven years is thus called the *doubling time* for the investment.

345. (III.43.1) On one system of coordinate axes, graph the equations $y = 3^x$, $y = 2^x$, $y = 1.024^x$, and $y = \left(\frac{1}{2}\right)^x$. What do graphs of the form $y = b^x$ have in common? How do they differ?

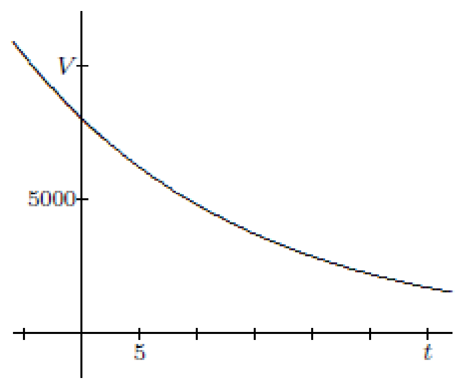
Math III

346. Kelli works as a lifeguard at her local pool in the summer. Her first summer she makes \$11.20 per hour. Her second summer she makes \$12.10 an hour.
   a. What is the *percent change* from her first to second summers? Round your answer to three decimal places.
   b. Kelli is told that her hourly wage will continue to increase the same percent each year she works at the pool. What will she make her third summer? Her fourth?
   c. Write an equation in function notation to describe Kelli's hourly wage after x summers.

347. (Continued) Using your graphing calculator and your answer in part c, find the number of years Kelli would have to work at the pool for her hourly wage to be greater than \$24.

348. (Continued) How did you use the *percent change* of Kelli's wages to write the equation for her wage after x summers? Explain how *percent change* is related to an *exponential function*.

349. (III.42.1) The figure to the right shows part of the graph of $V(t) = 8000(0.95)^t$. This function tells the story of a shrinking balloon that loses 5 percent of its helium each day.
   a. Identify the domain and range of V.
   b. What is $V(0)$, and what is its significance?
   c. Use the graph to estimate the t-value that solves the equation $V(t) = 4000$.
   d. Calculate $V(-3)$. What does this value mean?
   e. Comment on the part of the graph that lies outside the borders of the illustration. How would it look if you could see it, and what does it mean?



Problem 349

350. (III.43.2) Make up a context for the expression $4000(1.005)^{12}$, in which the "12" counts months. In this context, what do the expressions $4000((1.005)^{12})^n$ and $4000(1.0617)^n$ mean?

351. (III.45.1) Convert the following to simpler equivalent forms.

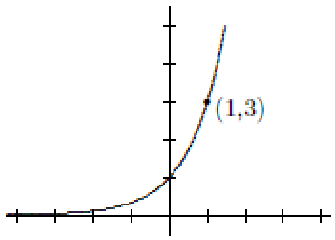
   a. x^6x^{-6} b. $(8a^{-3}b^6)^{\frac{1}{2}}$ c. $\left(\frac{x^{\frac{1}{2}}}{y^{\frac{2}{3}}}\right)^6\left(\frac{x^{\frac{1}{2}}}{y^{\frac{2}{3}}}\right)^{-6}$

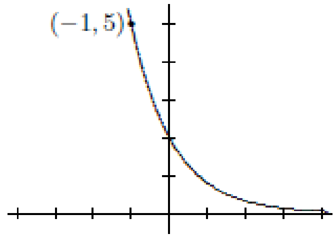
352. (III.45.9) Explain why calculating $z^{2.5}$ is a square-root problem. What does $z^{0.3}$ mean?

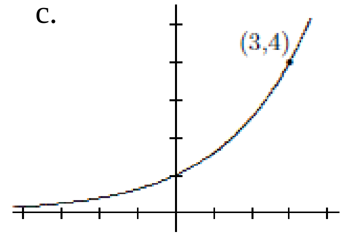
353. (III.46.2) Verify that $(-8)^{\frac{1}{3}}$ can be evaluated, but that $(-8)^{\frac{1}{4}}$ cannot, and explain why $(-8)^{\frac{2}{6}}$ is ambiguous. To avoid difficulties like these, it is customary to restrict the base of an exponential expression to be a positive number when the exponent is not an integer.

Math III

354. (III.46.5) Find a plausible equation $y = a \cdot b^x$ for each of the exponential graphs shown below.

a. 

b. 

c. 

355. (III.47.6) Make up a context for the equation $y = 5000(1.005)^x$.

356. (III.47.7) (Continuation) Find the value of x (in fraction form) that makes y = 12500. Find the value of x (in fraction form) that makes y = 2000. Interpret these answers in the context you chose.

357. (III.37.7) Find equivalent ways to rewrite (without using a calculator) the following expressions.

a. $\frac{6a^8}{3a^4}$ b. $(3p^3q^4)^2$ c. $b^{\frac{1}{2}}b^{\frac{1}{3}}b^{\frac{1}{6}}$ d. $\left(\frac{2x^3}{3y^2}\right)^2$ e. $\left(d^{\frac{1}{2}}\right)^6$

358. (III.47.9) A constant monthly interest rate of 1.4% is equivalent to what annual interest rate?

359. (III.48.5) Write each of the following numbers as a power of 10. You should not need a calculator.

a. 1000 b. 1000000 c. 0.01 d. $\sqrt{10}$ e. $100\sqrt{10}$ f. $\frac{1}{\sqrt[3]{100}}$

360. (III.48.6) Using your calculator, write 2018 as a power of 10. Using this answer (and no calculator), write $\frac{1}{2018}$ as a power of 10.

361. (III.48.7) (Continuation) Turn your calculator on, type LOG, (or press the LOG key), type 2018, and press ENTER. Compare the displayed value with the first of your two previous answers.

362. (III.48.8) For each of the following, type LOG followed by the given number, and press ENTER. Interpret the results. By the way, “log” is short for *logarithm,* to be discussed soon.

a. 1000 b. 1000000 c. 0.01 d. $\sqrt{10}$ e. $100\sqrt{10}$ f. $\frac{1}{\sqrt[3]{100}}$

363. (III.66.4) A rectangular sheet of paper (such as the one shown in the figure at right) has thickness 0.003 inches. Suppose that it is folded in half, then folded in half again, then folded in half again—fifty times in all. How thick is the resulting wad of paper? How many folds would it take to reach the Sun?

364. Let's talk about logarithms! A logarithms is defined as follows: $\log a = b$ is equivalent to the statement $10^b = a$. In other words, logarithms are the *inverse functions* of exponetial functions. $\log a = b$ is called the *logarithmic form* of this relationship, and $10^b = a$ is the *exponential form*. Use this information to identify the form of each equation below and rewrite the equation in the other form.
   a. $\log s = k$ b. $10^d = g$ c. $\log_5 h = j$ d. $7^m = q$

365. (III.49.2) Using the LOG function of your calculator, solve each of the following for *x*:
   a. $10^x = 3$ b. $10^x = 300$ c. $10^x = 9$ d. $10^x = 3^{-1}$ e. $10^x = \sqrt{3}$
   You should see a few patterns in your answers; try to explain them.

366. (III.49.7) Rewrite
   a. the logarithmic equation 4 = log 10000 as an exponential equation;
   b. the exponential equation $10^{3.30} = 2018$ as a logarithmic equation.

367. (III.49.8) The function $P(t) = 3960(1.02)^t$ describes the population of Dilcue, North Dakota *t* years after it was founded.
   a. Find the doman and range of P.
   b. What was the founding population of Dilcue?
   c. At what annual rate has the population of Dilcue been growing?
   d. Calculate $P(65)/P(64)$.

368. (III.49.9) (Continuation) Try to solve the equation $P(t) = 77218$. What are you solving for? Can you do it with the tools you have? Explain.

369. (III.50.6) Without using your calculator, solve each of the following equations.
   a. $1000^x = 100000$ b. $27^x = 243$ c. $8^x = 32$
   Explain why all three equations have the same solution.

370. (III.95.10) Simplify $(\log_a b)(\log_b a)$.

371. (III.50.7) Given a positive number *p*, the solution to $10^x = p$ is called the *base-10 logarithm of p*, expressed as $x = \log_{10} p$, or simply $x = \log p$. For example, $10^4 = 10000$ means that 4 is the base-10 logarithm of 10000, or 4 = log 10000. The LOG function on your calculator provides immediate access to such numerical information. Using your calculator for confirmation, and remembering that *logarithms are exponents*, explain why it is predictable that
   a. log 64 is three times log 4;
   b. log 12 is the sum of log 3 and log 4;
   c. log 0.02 and log 50 differ only in sign.

372. (III.51.1) For each description of an exponential function $f(x) = k \cdot b^x$, find *k* and *b*.
   a. $f(0) = 3$ and $f(1) = 12$ b. $f(0) = 4$ and $f(2) = 1$

Math III

373. (III.51.5) *What if the base of an exponential equation isn't 10*? One way of solving an equation like $1.02^x = 3$ is to use your calculator's LOG function to rewrite the equation in the form $(10^{0.0086})^x = 10^{0.4771}$. First justify this conversion, then solve $10^{0.0086x} = 10^{0.4771}$.

374. (Continuation) You have now calculated *the logarithm of 3 using base 1.02*, for which $x = \log_{1.02} 3$ is the usual notation. There are several different ways of reading $\log_{1.02} 3$, such as "log base 1.02 of 3," "log 3, base 1.02," "the base-1.02 logarithm of 3," or "log to the base 1.02 of 3." There are two ways to approach solving this problem. We'll take a look at both.
    a. Using your calculator, find the value of log(1.02).
    b. Use the information above to write a general rule for inputting $\log_{10} N$ in the calculator.
    c. You derived another method to solve the problem in the previous problem. You found the answer using two base-10 logarithms. Explain.
    d. Use the method you used in the previous problem to write a general rule for solving $\log_a b$ with base-10 logarithms.

375. (III.49.1) Explain your opinions of each of the following student responses.
    a. Asked for an expression equivalent to $x^3 + x^{-3}$, a student responded x^0.
    b. Asked for an expression equivalent to $(x^{-1} + y^{-1})^{-2}$, a student responded $x^2 + y^2$.

376. (III.52.4) For the first 31 days of your new job, your boss offers you two salary options. The first option pays you \$1000 on the first day, \$2000 on the second day, \$3000 on the third day, and so on—in other words, \$1000*n* on the *nth* day. The second option pays you one penny on the first day, two pennies on the second day, four pennies on the third day—the amount doubling from one day to the next. Which option do you prefer, and why?

377. (III.52.5) (Continuation) You have chosen the second payment option, and—on the thirty-first day—your boss pays you the wages for that day in pennies. You wonder whether all these coins are going to fit into your dormitory room, which measures 12 feet by 15 feet by 8 feet. Verify that a penny is 0.75 inch in diameter, and that seventeen of them make a stack that is one inch tall. Use this information to decide whether the pennies will all fit.

378. (III.61.5) Without using a calculator, simplify the following expressions.
    a. $\log(10^{-4})$ b. $10^{\log 1000}$ c. $\log(10^{-2.48})$ d. $10^{\log 4.8}$

379. (III.52.6) Solve $2^x = 1000$. In other words, find $\log_2 1000$, the base-2 logarithm of 1000.

380. (III.52.7) You now know how to calculate logarithms by using 10 as a *common base*. Use this method to evaluate the following. Notice those for which a calculator is not necessary.
    a. $\log_8 5$ b. $\log_5 8$ c. $\log_5 \sqrt{5}$ d. $\log_{1.005} 2.5$ e. $\log_3 \frac{1}{9}$

381. (III.52.9) There is an exponential function $f(t) = k \cdot b^t$, for which the graph $y = f(t)$ contains the points (1, 6) and (3, 24). Find *k* and *b*.

382. (III.52.10) You have come to associate a function such as $p(t) = 450 \cdot (1.08)^t$ with the size of something that is growing (exponentially) at a fixed rate. Could such an interpretation be made for the function $d(t) = 450 \cdot 2^t$? Explain.

383. (III.53.2) Given $10^{0.301} \approx 2$ and $10^{0.477} \approx 3$, solve without a calculator.
 a. $10^x = 6$ b. $10^x = 8$ c. $10^x = \frac{2}{3}$ d. $10^x = 1$

384. Use your understanding of logarithms to evaluate the expressions below.
 a. $\log_3 9$ b. $\log_2 \frac{1}{8}$ c. $\log_4 64$
 d. $\log_7 \frac{1}{343}$ e. $\log_5 \sqrt{125}$ f. $\log_9 \frac{1}{\sqrt{81}}$

385. (III.53.3) Given that 0.301 ≈ log 2 and that 0.477 ≈ log 3, evaluate the following expressions without using a calculator.
 a. log 6 (b) log 8 (c) $\log\left(\frac{2}{3}\right)$ (d) log 1

386. (III.53.5) Given that $m = \log a$ and $n = \log b$,
 a. express *a* as a power of 10, and express *b* as a power of 10;
 b. use your knowledge of exponents to express *ab* as a power of 10;
 c. conclude that $\log(ab) = \log a + \log b$.

387. (III.53.6) (Continuation) Justify the following rules.
 a. $\log(a^r) = r \log a$ b. $\log\left(\frac{a}{b}\right) = \log a - \log b$

388. For the function $f(x) = \left(1 + \frac{1}{x}\right)^x$, complete the following.
 a. Fill out the table to the right.
 b. Sketch a graph of f.
 c. As *x* decreases, what value does *y* approach?
 d. As *x* increases, what value does *y* approach? You should get the same value for parts c. and d. This number is called *e* and is also known as *Euler's number*.

x	$f(x) = \left(1 + \frac{1}{x}\right)^x$
$-150{,}000$	
$-1{,}000$	
-500	
-70	
-10	
-1	
0	
4	
30	
260	
2,400	
99,000	

Problems 388

389. (Continuation) Although Leonhard Euler is often credited for discovering the number *e*, mathematicians John Napier and Jacob Bernoulli are also involved in its origin story. Similarly to how 10 is often used as a logarithm base (recall the *common logarithm*), the number *e* is also regularly used as a logarithm base. $\log_e x$ is referred to as the *natural logarithm* and is denoted as $\ln x$. Use what you know about logarithms to evaluate the expressions below. (Hint: it might help you to rewrite $\ln a$ as $\log_e a$.)
 a. $\ln e$ b. $\ln e^2$ c. $\ln \frac{1}{e}$ d. $\ln \sqrt{e}$

390. (III.82.5) An *arithmetic sequence* is a list in which each term is obtained by adding a constant amount to its predecessor. For example, the list 4.0 , 5.2 , 6.4 , 7.6 , . . . is arithmetic. The first term is 4.0; what is the fiftieth? What is the millionth term? What is the n^{th} term?

391. (III.82.6) Suppose that a_1, a_2, a_3, . . . is an arithmetic sequence, in which $a_3 = 19$ and $a_{14} = 96$. Find a_1.

392. (III.93.4) Explain how $5^{\frac{1}{256}}$ can be calculated using only the square root key on your calculator.

393. (III.53.7) The function $F(x) = 31416(1.24)^x$ describes the number of mold spores found growing on a pumpkin pie *x* days after the mold was discovered.
   a. How many spores were on the pie when the mold was first discovered?
   b. How many spores were on the pie two days before the mold was discovered?
   c. What is the daily rate of growth of this population?
   d. What is the hourly rate of growth?
   e. Describe the spore count on the same pie by the function $G(x)$, where *x* counts the number of hours since the mold was discovered on the pie.

394. (III.54.5) Another approach to solving an equation like $5^x = 20$ is to *calculate base-10 logarithms of both sides of the equation*. Justify the equation $x \log 5 = \log 20$, then obtain the desired answer in the form $x = \frac{\log 20}{\log 5}$. Evaluate this expression. Notice that $\log_5 20 = \frac{\log 20}{\log 5}$.

395. (III.54.6) Write an expression for $\log_a N$ that refers only to base-10 logarithms, and explain.

396. (III.54.7) Asked to simplify $\frac{\log 20}{\log 5}$, Brett replied "log 4." What do you think of this answer?

397. (III.87.12) Given the function $P(x) = 3960(1.06)^x$, find a formula for the *inverse function* $P^{-1}(x)$. In particular, calculate $P^{-1}(5280)$, and invent a context for this question. Graph $y = P^{-1}(x)$.

398. (III.56.8) Ryan spills some soda and neglects to clean it up. When leaving for spring break, Ryan notices some ants on the sticky mess but ignores them. Upon returning seventeen days later, Ryan counts 3960 ants in the same place. The next day there are 5280 ants. Assuming that the size of the ant population can be described by a function of the form $F(t) = a \cdot b^t$, calculate the number of ants that Ryan saw when leaving for spring break.

399. (III.60.1) What is half of 2^{40}? What is one third of 3^{18}?

400. The rules that apply to logarithms also apply to natural logarithms. Rewrite each rule listed below in terms of natural logs.
   a. Product Rule b. Quotient Rule c. Power Rule:

401. (III.57.3) A *geometric sequence* is a list in which each term is obtained by multiplying its predecessor by a constant. For example, 81, 54, 36, 24, 16, . . . is geometric, with constant multiplier $\frac{2}{3}$. The first term of this sequence is 81; what is the 40^{th} term? the millionth term? the n^{th} term? Check your formula for $n = 1$, $n = 2$, and $n = 3$.

402. (III.78.3) Write down the first few terms of any geometric sequence of positive terms. Make a new list by writing down the logarithms of these terms. This new list is an example of what is called an *arithmetic sequence*. What special property does it have?

403. (III.57.5) Given that $\log_c 8 = 2.27$ and $\log_c 5 = 1.76$, evaluate the following expressions *without a calculator*.
   a. $\log_c 40$ b. $\log_c \left(\frac{5}{8}\right)$ c. $\log_c 2$ d. $\log_c(5^m)$ e. $\log_c 0.04$

404. (III.58.3) Without calculator, find x.
   a. $\log_4 x = -1.5$ b. $\log_x 8 = 16$ c. $27 = 8(x-2)^3$

405. (III.58.8) When $10^{3.5623}$ is evaluated, how many digits are found to the left of the decimal point? You should be able to answer this question without using your calculator.

406. (III.59.8) Calculate **(a)** $\log_5 2018$ **(b)** $\log_{1.005} 3$ **(c)** $\log_{0.125} 64$ **(d)** $\frac{\log 2018}{\log 5}$

407. (III.60.2) Let $R(t) = 55(1.02)^t$ describe the size of the rabbit population in the SSA woods t days after the first of June. Use your calculator to make a graph of this function inside the window $-50 \le t \le 100$, $0 \le R(t) \le 500$. (You will need to work with the variables x and y instead of t and R, of course.) What is the y-intercept of the graph, and what does it signify? Does your calculator show an x-intercept? Would it show an x-intercept if the window were enlarged?

408. (III.60.3) (Continuation) Choose a point on the graph that is very close to the y-intercept, then find the slope between this point and the y-intercept. This will give you an estimate for the rate (in rabbits per day) at which the population is growing on June 1^{st}. In the same way, estimate the rate at which the population is growing on September 1^{st}, which is 92 days after June 1^{st}. Explain how your two answers are both consistent with the given 2% growth rate.

409. (III.62.5) Draw the graph of the equation $f(x) = \log_2 x$. How does this graph compare to the graph of the equation $g(x) = 2^x$? How are these two functions related to each other?

Math III

x	10^x	x	$\log x$
-3		0.001	
	1		0
0.5	3.162	3.162	
	100		2
3		1000	
	1996	1996	3.300

Problems 410, 411, 412

410. (III.61.3) Fill in the missing entries in the two tables shown at right. Do this without a calculator.

411. (III.61.4) (Continuation) What do your results tell you about the graphs of $f(x) = 10^x$ and $g(x) = \log x$? How are these two functions related to each other?

412. (Continuation) Show algebraically that $f(x) = \log x$ and $g(x) = 10^x$ are inverses of each other.

413. (III.61.11) Compare the graph of $f(x) = \log x$ and the graph of $g(x) = \log(10x)$. How are they related?

414. It was stated previously that when logarithms are calculated using *e* as the base, they are called *natural*, and the natural logarithm function is denoted as $y = \ln x$. Graph the functions $f(x) = e^x$ and $g(x) = \ln x$ on the same set of axes and answer the following.
    a. What is the slope of the logarithm curve at its *x*-intercept?
    b. What is the relationship between the points on the graph of f and the points on the graph of g?
    c. What can you say about the slopes of the curves at any pair of corresponding points? By the way, the *slope of a curve at a point* means the slope of the tangent line at that point.

415. (III.56.7) What happens when you ask your calculator to evaluate $\log(-7)$? Why?

416. (III.66.6) Let $f(x) = 5^x$. Calculate $\frac{f(1.003)-f(1.000)}{0.003}$ and then explain its significance.

417. (III.64.3) Simplify each of the following expressions.
    a. $(3^{-1}+4^{-1})^{-1}$ b. $\frac{6^{4000}}{12^{2000}}$ c. $7^u 7^u$ d. $\sqrt{64x^{16}}$ e. $2^m 3^{-m}$

418. (III.64.1) What is the *x*-intercept of the graph of $y = \log(x-3) - 1$? How does the graph of $y = \log(x-3) - 1$ compare with the graph of $y = \log x$?

419. (III.64.2) Write 10^{2018} as a power of 2.

420. (III.64.4) Explain how to use your calculator's base-10 log function to obtain base-16 logarithms.

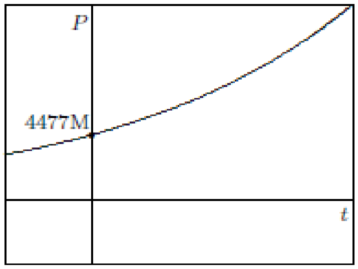
421. (III.64.8) A function of the form $H(x) = a \cdot b^x$ has the property that $H(1) = 112$ and $H(3) = 63$. Find the values $H(0)$ and $H(4)$.

422. (III.64.9) Given that $\log_4 x$ is somewhere between -1.0 and 0.5, what can be said about *x*?

423. (III.73.1) Explain why the expression $\log(a) - \log(b)$ should not be confused with $\log(a-b)$. Rewrite $\log(a) - \log(b)$ in an equivalent logarithmic form.

424. (III.66.7) How does the graph of $f(x) = \left(\frac{1}{2}\right)^x$ compare with the graph of $g(x) = 2^x$? What features do these curves have in common? At their common *y*-intercept, how are the slopes of the tangent lines to these curves related?

425. (III.72.1) The size of the Earth's human population at the beginning of the year 1980 + *t* is described by the function $P(t) = 4477000000(1.0176)^t$, whose graph is shown at right. At what rate is *P*(*t*) increasing when *t* = 10? There are at least two ways to interpret this question; give an answer for each of your interpretations.



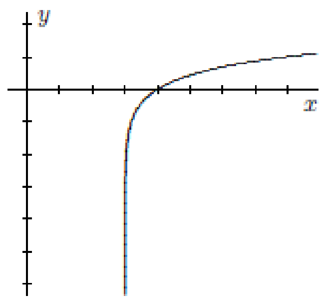
Problems 425 and 426

426. (III.72.2) (Continuation) At what rate is *P*(*t*) increasing when *t* = 20? There are at least two ways to interpret this question, so give an answer for each of your interpretations. Does it make sense to say that the Earth's human population is growing at a *constant rate*? Discuss.

427. (III.56.6) Use exponential notation to rewrite the following.

a. $x^2\sqrt{x}$ b. $\frac{x}{\sqrt{x}}$ c. $\frac{\sqrt{x}}{\sqrt[3]{x}}$ d. $\sqrt[3]{x^3y^4z^5}$

428. (III.65.6) The point (3, 8) is on the graph of $y = 2^x$. What is the corresponding point on the graph of the inverse function $y = \log_2 x$? Find four more pairs of points like these.



Problems 429

429. (III.62.9) The equation graphed at right is $g(x) = \log_5(x-3)$. What is the *x*-intercept of this graph? There are many vertical lines that do not intersect this graph; which one of them is farthest to the right? For what *x*-values does the equation make sense? What *x*-value corresponds to *y* = 1? to *y* = 2? to *y* = 3?

430. (III.62.10) (Continuation) How does the given graph compare to the graph of $p(x) = \log_5 x$? How does the given graph compare to the graph of $f(x) = \log_5(x+2)$?

431. (III.91.5) Compare the *domains* and *ranges* of the functions $f(x) = 2\log x$ and $g(x) = \log(x^2)$.

432. (III.73.6) What else can be said about the positive number *p*, given that

a. $0.0 < \log_7 p < 1.0$? b. $0.0 < \log_{1/2} p < 1.0$? c. $0.0 < \log_b p < 1.0$?

433. (III.63.4) Write an equation for the curve obtained by shifting the curve $p(x) = 2^x$…

a. three units to the right; b. five units down.

For each, identify *x*- and *y*-intercepts and other significant features.

434. (III.63.5) Find the *y*-intercept of the graph $y + 1 = 2^{x-3}$. How does the graph of $y + 1 = 2^{x-3}$ compare with the graph of $y = 2^x$ How about the graph of $y = 2^{x-3} - 1$?

Math III Reference

arithmetic sequence: A list in which each term is obtained by adding a constant amount to the preceding term.

axis of symmetry: A line that separates a figure into two parts that are equivalent by reflection across the line. Every *parabola* has an axis of symmetry.

collinear: Two points that are on the same line are said to be *collinear*.

combination: A grouping of objects for which the order does not matter. The number of combinations of k objects that can be made if selecting from n total objects is denoted ${}_nC_k$.

common logarithm: The base-10 logarithm, $\log_{10} x$, often denoted simply as $\log x$.

completing the square: Adding a quantity to a trinomial so that the new trinomial can be factored as a perfect square.

components describe how to move from one unspecified point to another. They are obtained by subtracting coordinates.

cosine ratio: Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse. The word cosine is a combination of complement and sine, so named because the cosine of an angle is the same as the sine of the complementary angle.

dihedral: An angle that is formed by two intersecting planes. To measure its size, choose a point that is common to both planes, then through this point draw the line in each plane that is perpendicular to their line of intersection.

directed line segment: A line segment with an indicated direction. When drawn in the coordinate plane, they look like arrows. Directed line segments are used to graph *vectors.*

discrete: A variable that is restricted to integer values.

domain: The domain of a relation consists of all numbers for which the function gives a value. For example, the domain of a linear function is all real numbers, while the domain of a logarithm function consists of positive numbers only ($x > 0$).

doubling time: When a quantity is described by an increasing *exponential function of t*, this is the time needed for the current amount to double.

Euler's number (e) is approximately 2.71828. This irrational number frequently appears in scientific investigations. One of the many ways of defining it is $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

Math III Reference

exponent: An integer that indicates the number of equal factors in a product. For example, the exponent is 3 in the expression w^3, which means $w \cdot w \cdot w$.

exponential form looks like the equation $B^E = N$. The *logarithmic form* of this relationship is the equation $\log_B N = E$.

exponential functions look like $f(x) = a \cdot b^x$, with a constant base and a variable exponent. Do not confuse these with power functions!

exponential growth/decay: a type of exponential function used to model consistent change. Exponential growth/decay can be modeled by a function of the form $f(x) = a(1 \pm r)^x$, where *a* is the initial value or population and *r* is the rate of change. If the function is growing, inside the parentheses is +*r*, if it is decaying, inside the parentheses is -*r*.

exponents, rules of: These apply when there is a *common base*: $a^m \cdot a^n = a^{m+n}$ and $\dfrac{a^m}{a^n} = a^{m-n}$; when there is a *common exponent*: $a^m \cdot b^m = (ab)^m$ and $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$; or when an exponential expression is raised to a power: $\left(a^m\right)^n = a^{mn}$. Notice the special case of the common-base rules: $a^0 = 1$. [1, 2] Two rules for negative and rational exponents: $a^{-m} = \dfrac{1}{a^m}$ and $a^{\frac{1}{m}} = \sqrt[m]{a}$.

extraneous roots: Solutions of a transformed equation that do not work in the original equation.

feasible domain: When using a function to model a real-world scenario, it is sometimes the case that the domain of the mathematical function is larger than the set of values that would make sense to use as inputs within the context of the problem. For example, if your input variable t represents time, it might not make sense to have negative time. In that case, the feasible domain for that function would be $t \geq 0$.

function: A function is a rule that describes how the value of one quantity (the dependent variable) is determined uniquely by the value of another quantity (the independent variable). A function can be defined by a formula, a graph, a table, or a text.

geometric sequence: A list in which each term is obtained by applying a constant multiplier to the preceding term.

half-life: When a quantity is described by a *decreasing exponential function of t*, this is the time needed for half of the current amount to disappear.

head: Vector terminology for the second vertex of a directed segment.

Math III Reference

head-to-tail: Describes a way of graphing vectors in which one vector's tail (starting point) is drawn at the position of another vector's head (ending point).

integer: The whole numbers and their opposites, that is, the set $\{...-3, -2, -1, 0, 1, 2, 3, ...\}$.

interval: A connected piece of a number line. It might extend infinitely far in the positive direction (as in $-1 < x$), extend infinitely far in the negative direction (as in $t \leq 7$), or be confined between two endpoints (as in $2 < m \leq 7$).

interval notation: A way of writing intervals. The symbol "(" or ")" indicates that an endpoint is not included in an interval, and the symbol "[" or "]" indicates that an endpoint is included. For example, $[4,7)$ describes the interval $4 \leq x < 7$. If an interval is made up of two or more disjoint (non-overlapping) intervals, then one can use the union symbol, "$\cup$". For example, the set of numbers $y < -3$ or $y \geq 2$ can be expressed in interval notation as $(-\infty, -3) \cup [2, \infty)$.

inverse cosine function ($\cos^{-1} x$): The inverse function of cosine, i.e. the function/operation that undoes cosine. Whereas the cosine function takes in an angle and outputs the ratio of the lengths of the adjacent leg to the hypotenuse, the inverse cosine function takes in that ratio and outputs that angle.

inverse function: Any function f processes input values to obtain output values. A function that reverses this process is said to be *inverse* to f, and is often denoted f^{-1}. In other words, $f(a) = b$ must hold whenever $f^{-1}(b) = a$ does. For some functions ($f(x) = x^2$, for example), it is necessary to restrict the domain in order to define an inverse that is also a function. Notice that f^{-1} does not mean reciprocal.

inverse sine function ($\sin^{-1} x$): The inverse function of sine takes in a sine ratio and outputs the corresponding acute angle. If $\frac{\text{opposite leg}}{\text{hypotenuse}} = \sin\theta$ and $0 \leq \theta \leq 90°$ is , then $\theta = \sin^{-1}\left(\frac{\text{opposite leg}}{\text{hypotenuse}}\right)$.

inverse tangent function ($\tan^{-1} x$): The inverse function of tangent takes in a tangent ratio and outputs the corresponding acute angle. If $\frac{\text{opposite leg}}{\text{adjacent leg}} = \tan\theta$ and $0 \leq \theta \leq 90°$ is , then $\theta = \tan^{-1}\left(\frac{\text{opposite leg}}{\text{adjacent leg}}\right)$.

invertible: A function is *invertible* if its inverse is also a function. Most functions are not invertible.

irrational number: A number that cannot be expressed exactly as the ratio of two integers. Two familiar examples are π and $\sqrt{2}$. See *rational number*.

linear: A polynomial, equation, or function of the first degree. For example, $y = 2x - 3$ defines a linear function, and $2x + a = 3(x - c)$ is a linear equation.

Math III Reference

logarithm: Another name for an exponent; specifically, the exponent needed to express a given positive number as a power of a given positive base. Using 4 as the base, the logarithm of 64 is 3, because $64 = 4^3$.

logarithmic form looks like the equation $\log_B N = E$. The *exponential form* of this relationship looks like $B^E = N$.

logarithms, rules of: These are exponential rules in disguise, because logarithms are exponents: $\log ab = \log a + \log b$ and $\log \frac{a}{b} = \log a - \log b$ and $\log(a^k) = k \log a$ hold for any base, any positive numbers *a* and *b*, and any number *k* ; the change-of-base formula $\log_c a = \frac{\log a}{\log c}$ holds for any base, and any positive numbers *a* and *c*.

magnitude: The magnitude of a vector **u** is its length, denoted by the absolute-value signs |**u**|.

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (*a, b*) and (*c, d*), the midpoint is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

model: An equation (or equations) that describe a context quantitatively.

natural logarithm: The base-*e* logarithm, $\log_e x$, often denoted $\ln x$.

parabola: The shape of a graph of the form $y = ax^2 + bx + c$. All parabolas have a *vertex* and an *axis of symmetry*.

parameter: See *parametric equations.*

parametric equations: Equations for *x* and *y* that determine the ordered pairs (*x, y*) on a graph based on a *parameter* typically denoted by *t*. Parametric equations often model scenarios for which the position of an object depends on time.

parent function: The simplest form of a given type of function. For example, $y = x$ is the parent function of $y = -x + 2$, and $y = |x|$ is the parent function for $y = 3|x| + 7$. Knowing the parent function of a transformed function allows one to know the general shape and behavior of the function.

percent change: The amount (expressed as a percent) by which a quantity has increased or decreased. Percent change is found with the formula $\frac{\text{final value - initial value}}{\text{initial value}} \cdot 100$.

permutation: An arrangement of objects for which the order matters. The number of permutations of *k* objects that can be made if selecting from *n* total objects is denoted ${}_nP_k$.

Math III Reference

point of intersection: A point where one line or curve meets another. The coordinates of a point of intersection must satisfy the equations of the intersecting curves.

point-slope form: The line with slope m that passes through the point (h, k) can be described in point-slope form by either $y - k = m(x - h)$ or $y = m(x - h) + k$.

probability: A number between 0 and 1, often expressed as a percent, that expresses the likelihood that a given event will occur. For example, the probability that two coins will *both* fall showing heads is 25%.

quadratic equation: A polynomial equation of degree 2.

quadratic formula: The solution to the quadratic equation $ax^2 + bx + c = 0$, which can be written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

quadratic function: A function defined by an equation of the form $y = ax^2 + bx + c$, where y is the dependent variable. The word quadratic comes from a Latin word that means "to make square", and it refers to the presence of a squared variable in the equation.

range: The range of a relation consists of all possible values the function can return. For example, the range of $y = x^2$ is $y \geq 0$.

rational expression: A fraction whose numerator and denominator are polynomials.

rational number: A number that can be written as the ratio of two integers. For example, 5, $\frac{7}{13}$, and 0.631 are rational numbers. See also *irrational number*.

real numbers: The real numbers are those that we can plot on a standard number line. Real numbers can be categorized as either irrational or rational. Rational numbers include the integers, which include whole numbers. Real numbers are a subset (type) of the complex numbers.

resultant vector: When two or more vectors are combined in a way that creates another vector, that vector is called the *resultant vector*.

rise: The vertical change or change in y-values when calculating *slope*.

run: The horizontal change or change in x-values when calculating *slope*.

sample space: The collection of all possible outcomes of a random experiment. For example, if you were to flip two coins, the sample space would be the set {HH, HT, TH, TT}, where "HT" denotes the first coin landing on heads and the second coin landing on tails.

scalar: In the context of vectors, this is just another name for a number.

scalar multiple (of a vector): Multiplying a vector by a scalar (number) c results in a vector that points in the same direction as the original vector and whose magnitude is now c times as large as it was originally.

simultaneous solution: A solution to a system of equations must satisfy every equation in the system.

sine ratio: Given a right triangle, the sine of one of the acute angles is the ratio of the length of the side opposite the angle to the length of the hypotenuse.

slope: The slope of a line is a measure of its steepness. It is computed by the ratio $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in y}}{\text{change in x}}$. A line with positive slope rises as the value of x increases. If the slope is negative, the line drops as the value of x increases.

standard position (vectors): A vector is drawn in standard position when it is drawn starting at the origin.

tail: The first vertex of a *vector*, indicating the starting position.

tail-to-tail: Vector terminology for directed segments with a common first vertex.

tangent ratio: Given a right triangle, the tangent of one of the acute angles is the ratio of the side opposite the angle to the side adjacent to the angle.

unit vector: A *vector* that is one unit in length.

vectors: Vectors are drawn as directed segments (arrows). They represent a change in position by a combination of horizontal and vertical translation. Vectors are described by *components*, just as points are described by coordinates. Vectors have *magnitude* (size) and direction. The vector from point A to point B is often denoted $\overrightarrow{AB}$ or abbreviated by a boldface letter such as $\boldsymbol{u}$, and its magnitude is often denoted $\left|\overrightarrow{AB}\right|$ or $|\boldsymbol{u}|$.

vertical line test: If a relation is a function, one should be able to draw a vertical line through any point on its graph, intersecting the graph just once. This procedure is known as the *vertical line test* (VLT for short).

whole numbers: The numbers $\{0, 1, 2, 3, \ldots\}$.

zero-product property: If the product of a set of factors is zero, then at least one of the factors must be zero. In symbols, if $ab = 0$ then either $a = 0$ or $b = 0$.

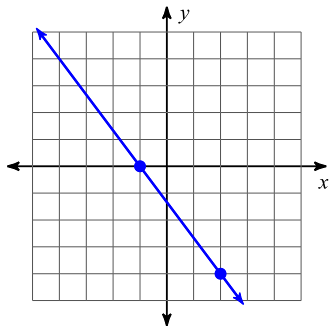
 Name________________________

1. Slope

Date________________

Find the slope of each line.

1)



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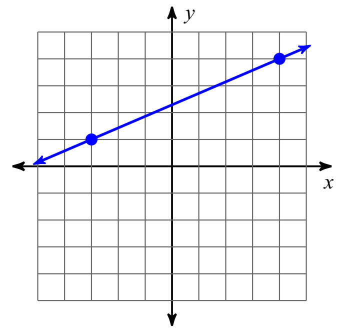


Find the slope of the line through each pair of points.

3) $(8, -11), (1, -17)$

4) $(-12, 2), (-11, -20)$

Find the slope of each line.

5) $6x = 10y - 20$

6) $-y + 3x = 0$

7) $\frac{1}{5}y = 1 + \frac{3}{5}x$

8) $0 = -4x - 2 + 2y$

Find the slope of a line parallel to each given line.

9) $y = 3$

10) $6x = -8y - 8$

11) $0 = 3x + 5 + y$

12) $-x = 1$

Find the slope of a line perpendicular to each given line.

13) $2y = -9x - 10$

14) $2 = x$

15) $1 = y$

16) $0 = y - 3x - 1$

Math III Extra Practice Name________________________________

2. Linear Equations

Date________________

Write the slope-intercept form of the equation of the line through the given points.

1) through: $(-1, -2)$ and $(1, 3)$

2) through: $(5, -3)$ and $(-5, 4)$

Write the slope-intercept form of the equation of the line through the given point with the given slope.

3) through: $(-3, 4)$, slope $= -\frac{5}{3}$

4) through: $(1, 4)$, slope $= -4$

Write the slope-intercept form of the equation of the line described.

5) through: $(0, -5)$, parallel to $y = 3x - 2$

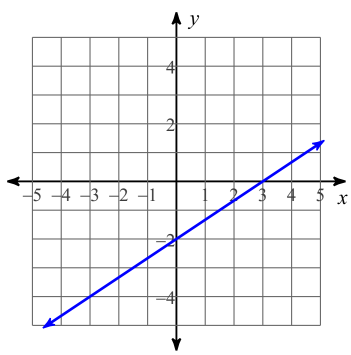
6) through: $(-2, 1)$, parallel to $y = -2x + 1$

7) through: $(-5, -2)$, perp. to $y = -5x$

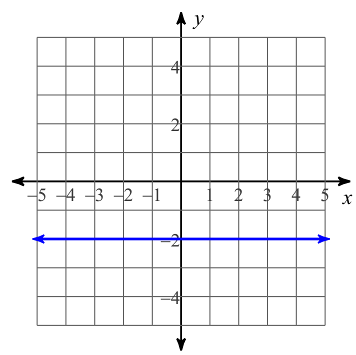
8) through: $(-1, -2)$, perp. to $y = \frac{1}{2}x + 3$

Write the slope-intercept form of the equation of each line.

9)

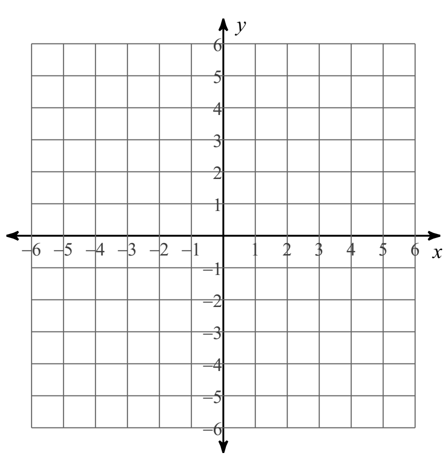


10)

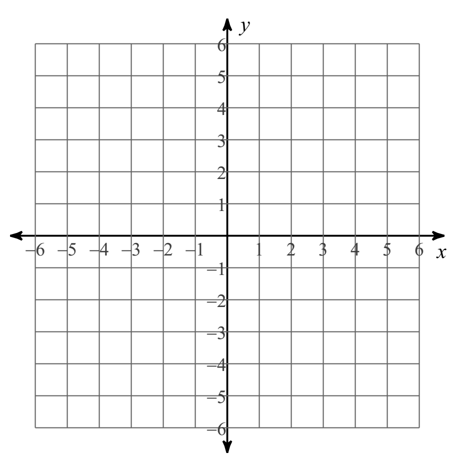


Sketch the graph of each line.

11) $2x - 5y = -25$



12) $y = -7x - 2$



3. Systems of Linear Equations

Date________________

Solve each system:

1) $y = -2$
$y = -4x - 18$

2) $y = -7x + 15$
$y = 2x + 6$

3) $-8x + 4y = -12$
$y = 4x - 11$

4) $y = -4x - 12$
$-8x - 2y = 24$

5) $-7x - 5y = 13$
$3x + y = -1$

6) $x + y = -5$
$-6x + 4y = 0$

7) $-4x - 4y = -7$
$-2x - 2y = 1$

8) $-8x - 7y = -3$
$-x + 4y = 24$

9) The senior classes at High School A and High School B planned separate trips to Yellowstone National Park. The senior class at High School A rented and filled 10 vans and 12 buses with 734 students. High School B rented and filled 9 vans and 11 buses with 670 students. Every van had the same number of students in it as did the buses. How many students can a van carry? How many students can a bus carry?

10) Alberto and Aliyah each improved their yards by planting daylilies and ornamental grass. They bought their supplies from the same store. Alberto spent $68 on 8 daylilies and 4 bunches of ornamental grass. Aliyah spent $108 on 14 daylilies and 6 bunches of ornamental grass. What is the cost of one daylily and the cost of one bunch of ornamental grass?

11) Adam's school is selling tickets to a play. On the first day of ticket sales the school sold 12 senior citizen tickets and 6 student tickets for a total of $114. The school took in $87 on the second day by selling 3 senior citizen tickets and 6 student tickets. What is the price each of one senior citizen ticket and one student ticket?

4. Circles

Identify the center and radius of each.

1) $x^2 + y^2 = 256$

2) $x^2 + y^2 = 144$

3) $(x - 15)^2 + (y + 13)^2 = 1$

4) $(x + 12)^2 + (y - 7)^2 = 1$

5) $(x + 8)^2 + (y - \sqrt{91})^2 = 31$

6) $(x - 3)^2 + (y + 1)^2 = 196$

Use the information provided to write the equation of each circle.

7) Center: $(15, 7)$
Radius: 1

8) Center: $(-5, -11)$
Radius: $3\sqrt{7}$

9) Center: $(14, -5)$
Area: 16π

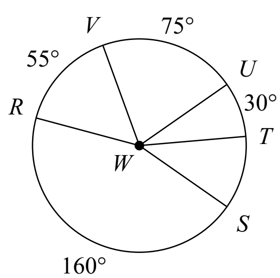
10) Center: $(3, -8)$
Area: 49π

11) Center lies in the second quadrant
Tangent to $x = -15$, $x = -17$, and the x-axis

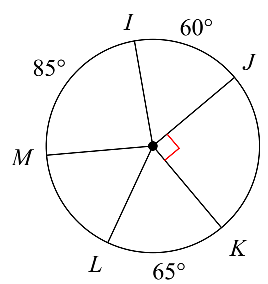
12) Center lies in the second quadrant
Tangent to $x = -1$, $y = 9$, and $y = 5$

Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

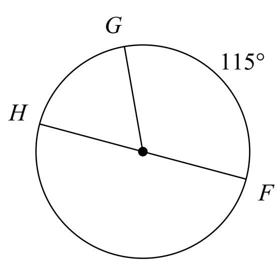
13) $m\angle RWT$



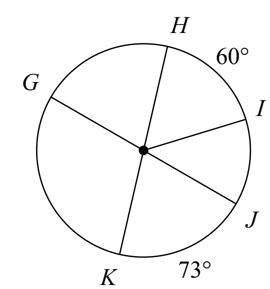
14) $m\overset{\frown}{IJL}$



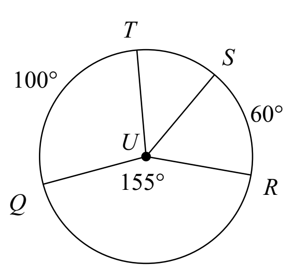
15) $m\overset{\frown}{GFH}$



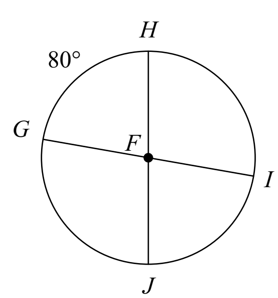
16) $m\overset{\frown}{HJG}$



17) $m\angle TUR$

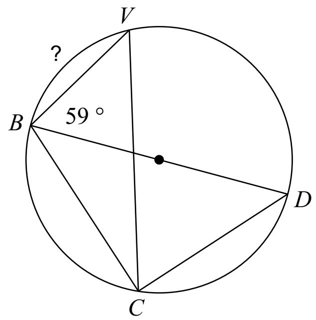


18) $m\angle IFJ$



Find the measure of the arc or angle indicated.

19)



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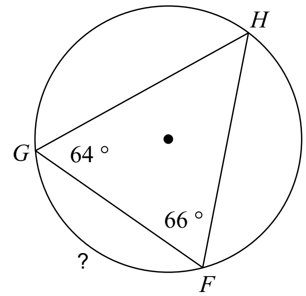

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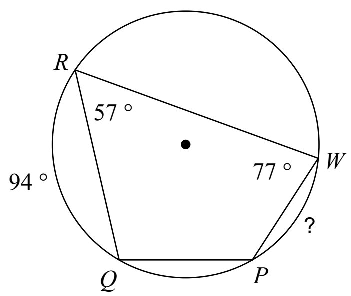

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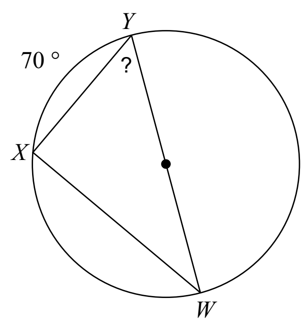
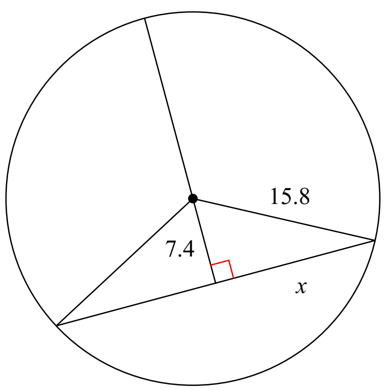


Find the length of the segment indicated. Round your answer to the nearest tenth if necessary.

23)



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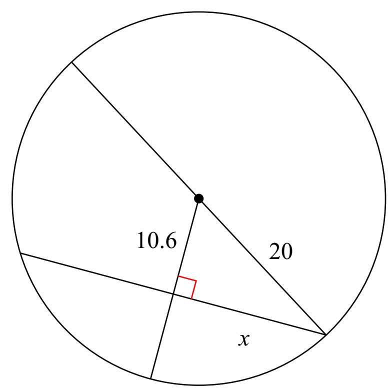

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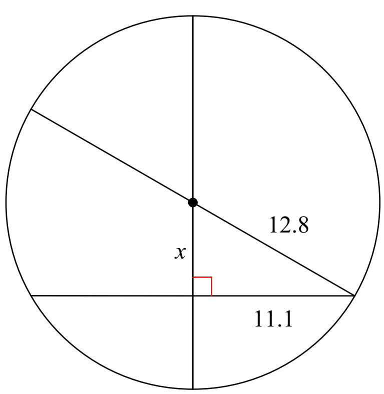

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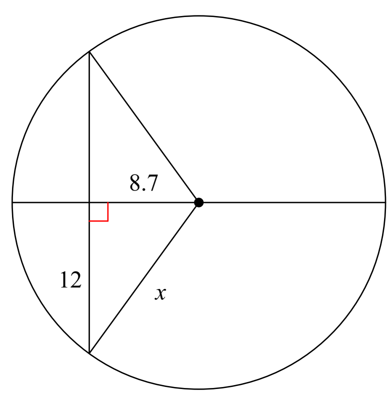


Name________________________________

5. Probability Sample/Outcome Space

Date________________

Represent the sample space using set notation.

1) When a button is pressed, a computer program outputs a random odd number greater than 1 and less than 13. You press the button once.

2) You roll a six-sided die.

3) A sandwich shop has three types of sandwiches: ham, turkey, and chicken. Each sandwich can be ordered with white bread or multi-grain bread.

4) A coffee shop offers small, medium, and large sizes. Customers can choose between French roast, Italian roast, and American roast.

Find the number of possible outcomes in the sample space.

5) When a button is pressed, a computer program outputs a random odd number greater than 1 and less than 11. You press the button once.

6) A pizza stand offers hand-tossed, pan, and thin-crust pizza.

7) The chess club must decide when to meet for a practice. The possible days are Tuesday, Wednesday, or Thursday. The possible times are 3 or 4 p.m.

8) An ice cream stand offers single-scoop waffle-cones or bowls. Four flavors are available: strawberry, chocolate, vanilla, and mint chocolate chip.

9) A sandwich shop has three types of sandwiches: ham, turkey, and chicken. Each sandwich can be ordered with white bread or multi-grain bread. Customers can add any combination of the five available toppings

10) A new car is available in a sedan model and a hatchback model. It is available in five different colors. Customers can choose to add any combination of five optional features.

 Name________________________________ ID: 1

6. Permutations and Combinations

Date________________

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

1) The student body of 135 students wants to elect two representatives.

2) Stefan and Darryl are planning trips to two countries this year. There are 4 countries they would like to visit. One trip will be one week long and the other two weeks.

3) There are 300 politicians at a meeting. They each shake hands with everyone else. How many handshakes were there?

4) There are 50 applicants for two jobs: computer programmer and software tester.

5) A group of 50 people are going to run a race. The top 7 finishers advance to the finals.

6) There are 10 people at a meeting. They each give a Valentine's Day card to everyone else. How many cards were given?

7) A group of 32 people need to take an elevator to the top floor. They will go in groups of eight. They are deciding who will take the elevator on its second trip.

8) There are 70 students at a meeting. They each give a Valentine's Day card to everyone else. How many cards were given?

9) A group of 18 people need to take an elevator to the top floor. They will go in groups of six. They are deciding who will take the elevator on its second trip.

10) A group of 20 people are going to run a race. The top 11 finishers advance to the finals.

11) There are 30 people at a meeting. They each give a Valentine's Day card to everyone else. How many cards were given?

12) A team of 17 softball players needs to choose a captain and co-captain.

 Name________________________________

7. Vectors

Date________________

Sketch a graph of each vector.

1) $\overrightarrow{PQ}$ where $P = (0, 3)$ $Q = (-9, -7)$

2) $\overrightarrow{AB}$ where $A = (6, 5)$ $B = (-3, 8)$

3) $\overrightarrow{CD}$ where $C = (4, 5)$ $D = (-3, -4)$

4) $\overrightarrow{CD}$ where $C = (1, 10)$ $D = (1, -3)$

Write each vector in component form.

5) $\overrightarrow{PQ}$ where $P = (-10, -6)$ $Q = (7, -7)$

6) $\overrightarrow{RS}$ where $R = (0, 4)$ $S = (-4, -8)$

7) $\overrightarrow{RS}$ where $R = (-7, -6)$ $S = (-4, 8)$

8) $\overrightarrow{CD}$ where $C = (-1, 5)$ $D = (-8, -6)$

Find the magnitude for each vector.

9) $\overrightarrow{PQ}$ where $P = (-3, -8)$ $Q = (1, 6)$

10) $\overrightarrow{AB}$ where $A = (8, 6)$ $B = (6, 6)$

11) $\overrightarrow{CD}$ where $C = (9, 9)$ $D = (-2, -10)$

12) $\overrightarrow{AB}$ where $A = (-2, -3)$ $B = (0, -7)$

Find the component form of the specified vector operation.

13) $\mathbf{f} = \langle 0, 8 \rangle$
$\mathbf{g} = \langle -9, -1 \rangle$
Find: $-\mathbf{f} + \mathbf{g}$

14) $\mathbf{u} = \langle -3, -9 \rangle$
$\mathbf{v} = \langle 8, -9 \rangle$
Find: $\mathbf{u} - \mathbf{v}$

15) $\mathbf{f} = \langle -2, -8 \rangle$
$\mathbf{b} = \langle 5, -10 \rangle$
Find: $\mathbf{f} - \mathbf{b}$

16) $\mathbf{a} = \langle 6, -5 \rangle$
$\mathbf{v} = \langle 0, -8 \rangle$
Find: $-\mathbf{a} - \mathbf{v}$

17) $\mathbf{f} = \langle -18, 24 \rangle$
Find: $-7\mathbf{f}$

18) $\mathbf{a} = \langle -5, 11 \rangle$
Find: $8\mathbf{a}$

19) $\mathbf{f} = \langle -21, -\sqrt{43} \rangle$
Find: $\sqrt{2} \cdot \mathbf{f}$

20) $\mathbf{f} = \langle 4, -11 \rangle$
Find: $2\mathbf{f}$

21) $\mathbf{f} = \langle 30, -40 \rangle$
Find the vector opposite **f**

22) $\mathbf{a} = \langle -11, 1 \rangle$
Find the vector opposite **a**

23) $\mathbf{a} = \langle 15, -36 \rangle$
Find the vector opposite **a**

24) $\mathbf{f} = \langle -3, \sqrt{91} \rangle$
Find the vector opposite **f**

25) $\mathbf{u} = \langle -3, 3 \rangle$
$\mathbf{v} = \langle 3, -10 \rangle$
Find: $5\mathbf{u} + 4\mathbf{v}$

26) $\mathbf{a} = \langle -7, 1 \rangle$
$\mathbf{v} = \langle -12, 2 \rangle$
Find: $7\mathbf{a} - 5\mathbf{v}$

27) $\mathbf{u} = \langle -2, 5 \rangle$
$\mathbf{g} = \langle -9, -12 \rangle$
Find: $-4\mathbf{u} - 8\mathbf{g}$

28) $\mathbf{f} = \langle 12, 1 \rangle$
$\mathbf{b} = \langle -11, 2 \rangle$
Find: $-2\mathbf{f} - 3\mathbf{b}$

29) $\mathbf{u} = \langle -18, 24 \rangle$
Unit vector in the direction of **u**

30) $\mathbf{a} = \langle 2\sqrt{3}, 5 \rangle$
Unit vector in the direction of **a**

31) $\mathbf{u} = \langle -11, 7 \rangle$
Unit vector in the opposite direction of **u**

32) $\mathbf{f} = \langle 5, 10 \rangle$
Unit vector in the opposite direction of **f**

Draw a vector diagram to find the sum of each pair of vectors. Then state the magnitude of the resultant. (Ignore the angle degree value in the answers)

33) $\mathbf{t} = \langle 9, 12 \rangle$ $\mathbf{u} = \langle -20, 10 \rangle$

34) $\mathbf{t} = \langle 4, 1 \rangle$ $\mathbf{u} = \langle -5, -12 \rangle$

35) $\mathbf{m} = \langle 5, 12 \rangle$ $\mathbf{n} = \langle -11, 7 \rangle$

36) $\mathbf{a} = \langle 12, 16 \rangle$ $\mathbf{b} = \langle 7, -20 \rangle$

Find the dot product of the given vectors.

37) $\mathbf{u} = \langle 3, 0 \rangle$
$\mathbf{v} = \langle 9, 3 \rangle$

38) $\mathbf{u} = \langle 8, 0 \rangle$
$\mathbf{v} = \langle 0, 7 \rangle$

39) $\mathbf{u} = \langle -2, 2 \rangle$
$\mathbf{v} = \langle -6, -9 \rangle$

40) $\mathbf{u} = \langle -6, 9 \rangle$
$\mathbf{v} = \langle 5, 3 \rangle$

41) $\mathbf{u} = \langle -1, -1 \rangle$
$\mathbf{v} = \langle 8, -8 \rangle$

42) $\mathbf{u} = \langle 4, 2 \rangle$
$\mathbf{v} = \langle 2, 1 \rangle$

8. Parametric Equations

Date________________

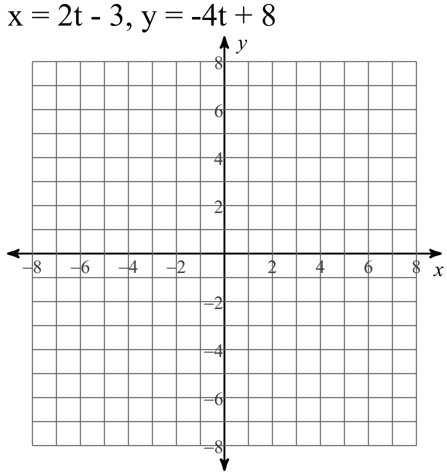
Determine a pair of parametric equations for each situation

1) A bug at (2, -6) when t = 0 seconds and (-8, 5) when t = 1.

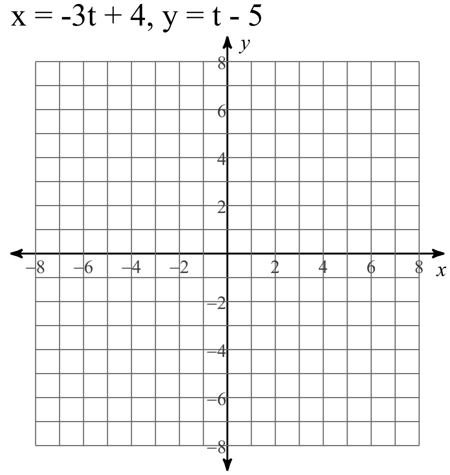
2) A bug at (-1, 1) when t = 0 seconds and (3, 4) when t = 2.

For each pair of parametric equations describing a bug's movement (t is in seconds),
a. Determine the bug's position when t = 0, t = 1, and t = 2
b. Determine the when and where the bug will pass through the x and y intercepts
c. Sketch a graph of the curve

3) $x = 2t - 3$, $y = -4t + 8$



4) $x = -3t + 4$, $y = t - 5$



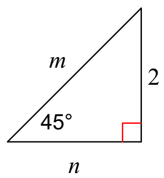
Math III Extra Practice Name______________________________

9. Special Right Triangles

Date________________

Find the missing side lengths. Leave your answers as radicals in simplest form.

1)



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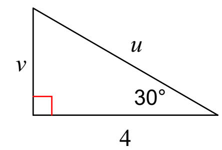
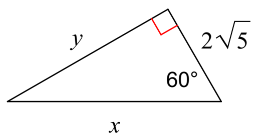


3)



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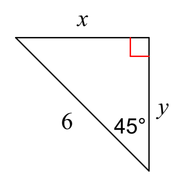

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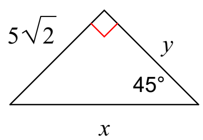

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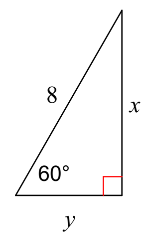

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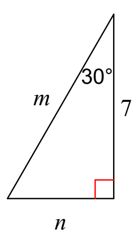

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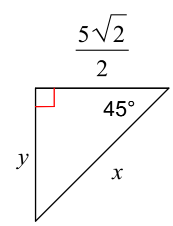

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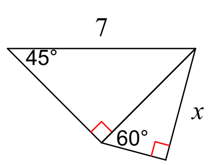

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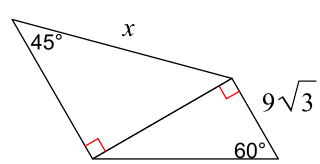

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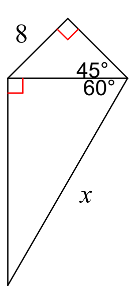

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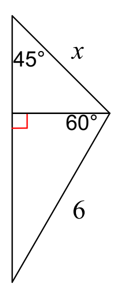
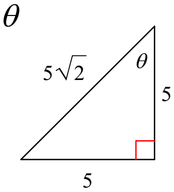


10. Trig Function Ratios

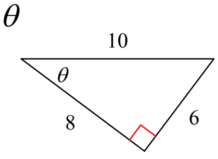
Date________________

Find the value of the trig function indicated.

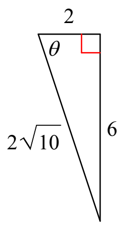
1) $\sin\theta$



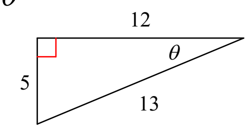
2) $\sin\theta$



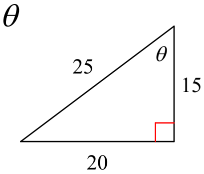
3) $\cos\theta$



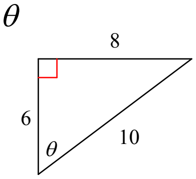
4) $\cos\theta$



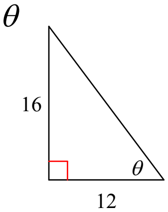
5) $\tan\theta$



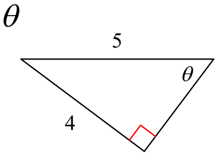
6) $\tan\theta$



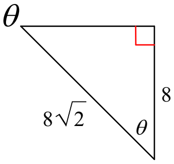
7) $\sin\theta$



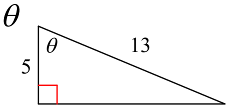
8) $\cos\theta$



9) $\tan\theta$



10) $\sin\theta$



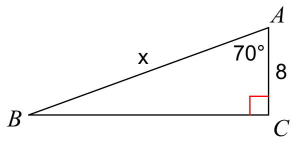
Math III Extra Practice Name________________________ ID: 1

11. Finding Missing Sides with Trig Functions

Date________________

Find the measure of each side indicated. Round to the nearest tenth.

1)



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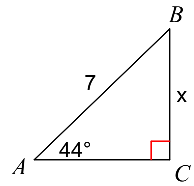

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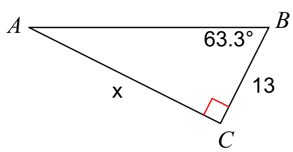

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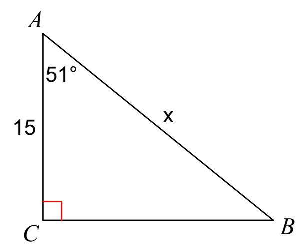
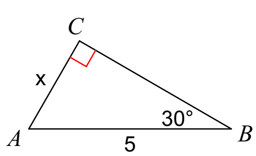


5)



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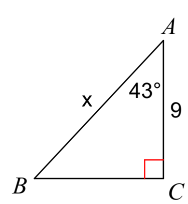
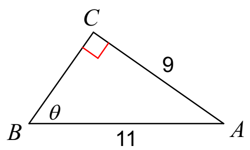
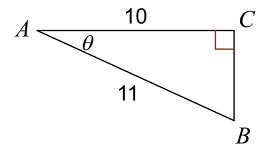


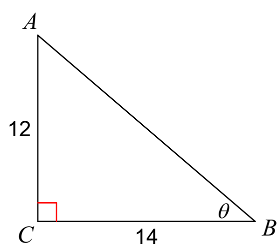
12. Finding Missing Angles with Inverse Trig Functions

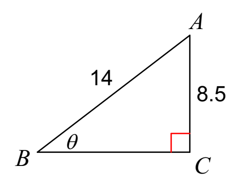
Date________________

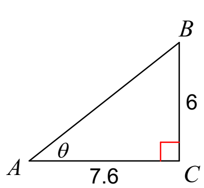
Find the measure of each angle indicated. Round to the nearest tenth.

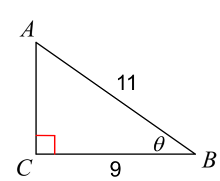
1) 

2) 

3) 

4) 

5) 

6) 

Math III Extra Practice Name________________________________

13. Absolute Value

Date________________

Graph each equation. Include the vertex and at least two other points on the graph.

1) $y = |x + 3| - 4$

2) $y = |x - 2| + 1$

3) $y = 2|x + 1| - 1$

4) $y = -2|x - 3| + 1$

Solve each equation.

5) $|n + 5| = 5$

6) $|p - 6| = 2$

7) $|-10 + b| + 4 = 21$

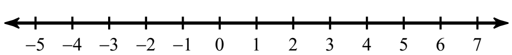
8) $|-8n| - 7 = -39$

9) $\left|\frac{m}{2}\right| - 4 = -4$

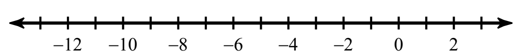
10) $\frac{|-3x|}{8} = 4$

Solve each inequality and graph its solution.

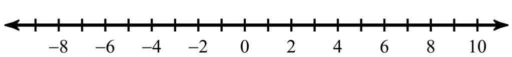
11) $|9x| < 27$



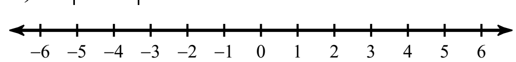
12) $|x + 5| > 5$



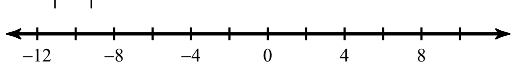
13) $-2 + \left|\frac{x}{4}\right| < 0$



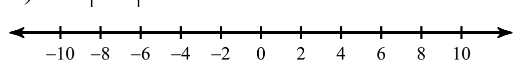
14) $8|-9a| \geq 72$



15) $\left|\frac{x}{7}\right| - 4 \geq -3$



16) $-2|7a| < -112$



14. Inverse Functions

Date________________

Find the inverse of each function.

1) $g(n) = n + 2$

2) $f(x) = 2x + 8$

3) $f(x) = \frac{5x + 20}{8}$

4) $h(x) = -4 + \frac{3}{2}x$

5) $f(x) = \frac{2}{x}$

6) $h(n) = \frac{1}{n - 2}$

7) $f(x) = \frac{4}{-x + 2}$

8) $g(n) = \frac{n + 2}{n + 1}$

9) $h(x) = (x - 1)^5 + 1$

10) $f(n) = \sqrt[3]{n + 3} - 1$

Name________________________________

15. Distance = Rate*Time Problems

Date________________

1) An Air Force plane flew to Jakarta and back. On the trip there it flew 445 mph and on the return trip it went 356 mph. How long did the trip there take if the return trip took ten hours?

2) Sumalee left the science museum traveling toward the dump one hour before Heather. Heather traveled in the opposite direction going 10 km/h slower than Sumalee for four hours after which time they were 590 km apart. How fast did Sumalee travel?

3) John made a trip to his cabin on the lake and back. The trip there took six hours and the trip back took four hours. He averaged 75 km/h on the return trip. Find the average speed of the trip there.

4) A cargo plane flew to Istanbul and back. It took four hours longer to go there than it did to come back. The average speed on the trip there was 252 km/h. The average speed on the way back was 420 km/h. How many hours did the trip there take?

5) A container ship left Port 48 and traveled toward a navigational buoy at an average speed of 20 mph. Sometime later a cruise ship left traveling in the same direction but at an average speed of 24 mph. After traveling for five hours the cruise ship caught up with the container ship. Find the number of hours the container ship traveled before the cruise ship caught up.

6) A diesel train left Washington and traveled north. A cattle train left 11 hours later traveling 55 km/h faster in an effort to catch up to it. After six hours the cattle train finally caught up. What was the diesel train's average speed?

7) Chelsea left the hospital and drove south at an average speed of 31 mph. Aliyah left at the same time and drove in the opposite direction with an average speed of 46.5 mph. How long does Aliyah need to drive before they are 170.5 mi. apart?

8) Dan left the mall and drove toward the train station at an average speed of 26.4 km/h. Mary left 2.5 hours later and drove in the same direction but with an average speed of 59.4 km/h. Find the number of hours Dan drove before Mary caught up.

 Name________________________________ ID: 1

16. Factoring Quadratic Trinomials

Date________________

Factor each completely.

1) $n^2 + 13n + 42$

2) $n^2 + 3n - 40$

3) $m^2 - 13m + 30$

4) $n^2 + 11n + 10$

5) $5b^2 + 9b - 2$

6) $-5k^2 + 27k - 28$

7) $2p^2 + 19p + 45$

8) $7a^2 + 20a - 32$

9) $-5k^2 + 33k + 56$

10) $7n^2 - 26n - 45$

11) $2r^2 + 5r - 18$

12) $-5a^2 - 2a + 7$

13) $8r^2 - 41r - 42$

14) $9a^2 + 21a + 10$

15) $-9x^2 - 24x + 20$

16) $6m^2 - 25m + 24$

17) $10n^2 + 23n - 5$

18) $-6x^2 + 55x + 50$

19) $4v^2 + 45v + 81$

20) $-9n^2 - 17n + 30$

17. Solving Quadratic Equations

Date________________

Solve each equation by factoring.

1) $b(b+8)=0$

2) $7(7r-2)(r-2)=0$

3) $7p^2-28p=0$

4) $b^2-3b=0$

5) $b^2-11b+24=-6$

6) $x^2-14=-5$

7) $6x^2-36x+30=0$

8) $x^2-36=0$

9) $k^2+7k=0$

10) $5b^2+15b-200=0$

11) $4p^2=4p+24$

12) $n^2-3n=4$

13) $7n^2=448$

14) $4x^2=-40x-64$

Solve each equation by taking square roots.

15) $7n^2+9=247$

16) $5b^2+1=321$

17) $9n^2-8=397$

18) $49v^2-5=95$

Solve each equation by completing the square.

19) $a^2-2a-15=0$

20) $5m^2-10m-15=0$

21) $n^2+20n+19=0$

22) $3k^2+12k-15=0$

Solve each equation with the quadratic formula.

23) $k^2+10k+18=0$

24) $-4n^2+6n+130=0$

25) $6r^2+7r+4=2$

26) $2r^2-12r-25=-5$

18. Parabolas

Date________________

Identify the vertex, axis of symmetry, and direction of opening of each. Then sketch the graph.

1) $y = 2x^2$

2) $y = -\frac{1}{2}(x - 1)^2 - 3$

3) $y = (x + 1)^2$

4) $y = (x + 1)^2 + 3$

Identify the vertex, axis of symmetry, direction of opening, and y-intercept of each. Then sketch the graph.

5) $y = x^2 + 4x + 5$

6) $y = -\frac{1}{4}x^2 + 2x - 6$

7) $y = \frac{1}{4}x^2 - \frac{3}{2}x - \frac{3}{4}$

8) $y = -2x^2 - 16x - 34$

Use the information provided to write the vertex form equation of each parabola.

9) Vertex: $(-5, 10)$, y-intercept: -40

10) Vertex: $(-1, 1)$, y-intercept: 0

11) Opens up or down, Vertex: $(2, -9)$, Passes through: $(1, -11)$

12) Opens up or down, Vertex: $(0, 4)$, Passes through: $(2, 5)$

Use the information provided to write the standard form equation of each parabola.

13) Opens up or down, and passes through $\left(-3, \frac{13}{2}\right)$, $(-2, 7)$, and $(0, 5)$

14) Opens up or down, and passes through $(4, 9)$, $(2, 9)$, and $(3, 7)$

19. Solving Radical Equations (Square Roots Only)

Date________________

Solve each equation. Remember to check for extraneous solutions.

1) $\sqrt{\frac{x}{9}} = 0$

2) $\sqrt{b - 10} = 4$

3) $-4\sqrt{2x + 23} = -12$

4) $4 = -1 + \sqrt{15 - m}$

5) $x = \sqrt{5x}$

6) $\sqrt{-1 - 2b} = b$

7) $a - 1 = \sqrt{6a - 11}$

8) $\sqrt{69 - 6x} = x - 7$

9) $r - 4 = \sqrt{4r - 19}$

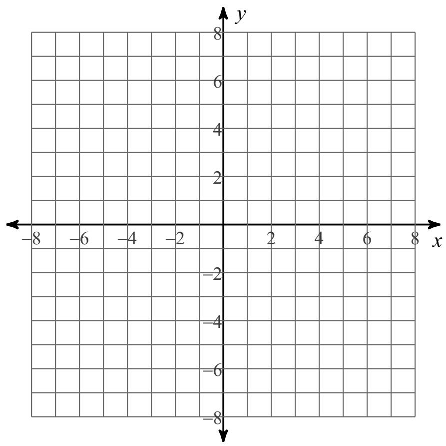
10) $7 + \sqrt{2x - 15} = x$

20. Graphing Square Root Functions

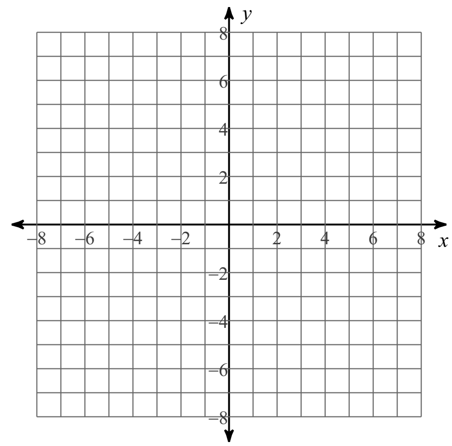
Date________________

Identify the domain and range of each. Then sketch the graph.

1) $y = -2\sqrt{x}$

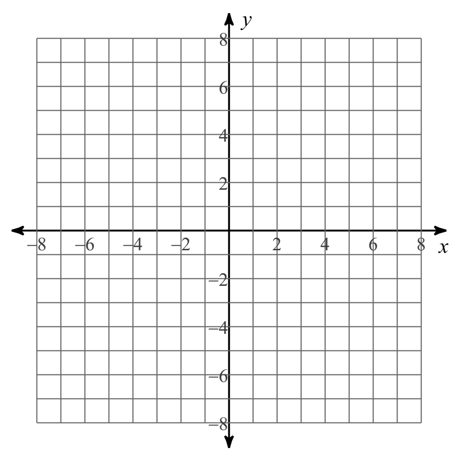


2) $y = \sqrt{x - 2}$

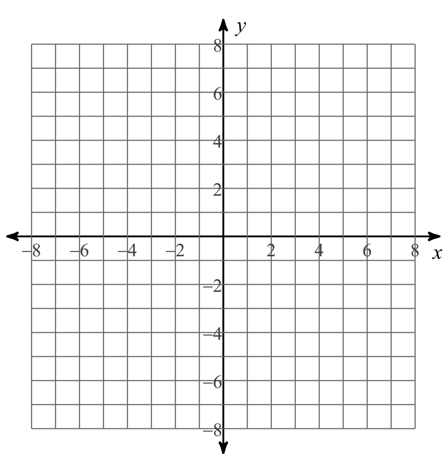


3) $y = \sqrt{x} - 2$

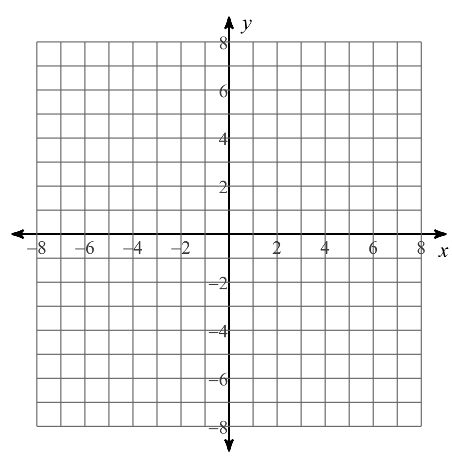
4) $y = \sqrt{x + 6} - 3$



5) $y = -\frac{3}{4}\sqrt{x + 4} + 2$



6) $y = \sqrt{16x} - 5$



Name________________________________

21. Percent Change

Date________________

Find each percent change. State if it is an increase or a decrease.

1) From 89 to 27

2) From 42 to 41

3) From 72 to 28

4) From 58 to 31

5) From 81 to 30

6) From 36 to 20.4

7) From 94 to 63

8) From 74 to 49

9) From 23 to 42

10) From 33 to 38

11) From 89 to 3

12) From 44 to 8

13) From 45 to 24

14) From 27 to 84

15) From 18 to 85

16) From 66.2 to 28

17) From 88 to 70.9

18) From 31 to 51

19) From 50 to 22

20) From 16 to 2

22. Work Problems

Date________________

Solve each question. Round your answer to the nearest hundredth.

1) Working alone, it takes Shanice ten hours to mop a warehouse. Mary can mop the same warehouse in six hours. If they worked together how long would it take them?

2) Working alone, Jennifer can sweep a porch in ten minutes. Sumalee can sweep the same porch in five minutes. If they worked together how long would it take them?

3) Working together, Molly and Dan can pick forty bushels of apples in 3.94 hours. Had he done it alone it would have taken Dan nine hours. How long would it take Molly to do it alone?

4) Working together, Jennifer and Kathryn can harvest a field in 4.74 hours. Had she done it alone it would have taken Kathryn nine hours. Find how long it would take Jennifer to do it alone.

5) Working alone, Natalie can pour a large concrete driveway in 7.4 hours. Scott can pour the same driveway in 5.6 hours. Find how long it would take them if they worked together.

6) Dan can paint a fence in 8.5 hours. Darryl can paint the same fence in 6.4 hours. If they worked together how long would it take them?

7) Working alone, Jennifer can oil the lanes in a bowling alley in 6.4 hours. One day her friend Kayla helped her and it only took 2.87 hours. Find how long it would take Kayla to do it alone.

8) Mei can dig a 10 ft by 10 ft hole in 6.9 hours. One day her friend Ryan helped her and it only took 2.93 hours. Find how long it would take Ryan to do it alone.

Name________________________________

23. Rational Expressions and Equations

Date________________

Simplify each expression.

1) $\frac{3y}{2} + \frac{2x}{3y}$

2) $\frac{6x}{12y^3} + 4$

3) $\frac{2p}{p+6} - \frac{2}{p-2}$

4) $\frac{3}{3r} - \frac{r+5}{3r-18}$

5) $\frac{6}{m-4} - \frac{3}{3m}$

6) $\frac{5r}{6r+15} + \frac{3}{2}$

Solve each equation. Remember to check for extraneous solutions.

7) $\frac{1}{5x} = \frac{1}{5x^2} - \frac{x-6}{5x^2}$

8) $\frac{1}{6n} - \frac{1}{6} = \frac{3n-1}{2n}$

9) $\frac{5}{4x^2} = 1 - \frac{1}{4x}$

10) $\frac{1}{3} + \frac{1}{a^2-3a} = \frac{1}{3a^2-9a}$

11) $\frac{n+5}{n^2-5n} = 1 - \frac{1}{n}$

12) $\frac{p-6}{p} = 1 + \frac{p-5}{p-1}$

24. Exponenent Rules and Exponential Functions

Date________________

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

1) $a^3b^4 \cdot a^2b^3$

2) $y^4 \cdot 2xy^2 \cdot 4x^4y^0$

3) $(4y^2)^3$

4) $(2x^2y^2)^0$

5) $\dfrac{2y^3}{2y^2}$

6) $\dfrac{xy^3}{4y^0}$

7) $4x^{\frac{1}{2}}y^2 \cdot 3x^{\frac{1}{2}}y^2$

8) $yx^{\frac{4}{3}} \cdot 3x^{\frac{5}{3}}$

9) $\left(yx^{\frac{5}{3}}\right)^2$

10) $\left(x^3y^{\frac{2}{3}}\right)^{\frac{1}{2}}$

11) $\dfrac{3u^{-4}v^4}{u^4v^2}$

12) $\dfrac{y^3}{x^4y^3}$

13) $\dfrac{x^3}{3y^{-1}}$

14) $\dfrac{4y^{-4}}{2x^4y^4}$

15) $(4x^4)^{-1}$

16) $(m^{-2}n^{-1})^4$

17) $\left(\dfrac{yx^4 \cdot x^{-2}y^0 \cdot x^{-3}y^4}{2yx^3}\right)^2$

18) $\dfrac{(2xy^{-3})^{-4} \cdot (2yx^{-4})^2}{x^3}$

Sketch the graph of each function.

19) $f(x) = 5 \cdot 2^x$

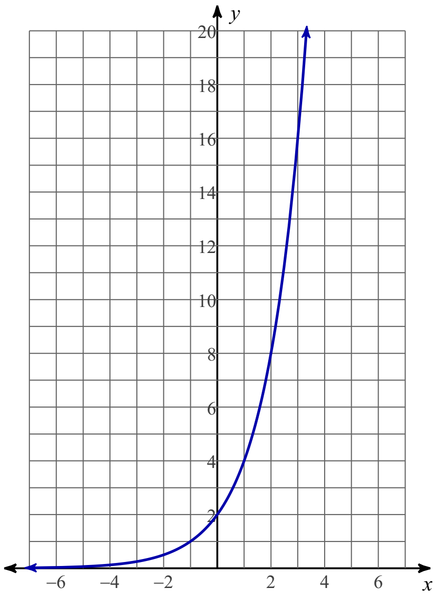
20) $f(x) = 2 \cdot 3^x$

21) $f(x) = 4 \cdot \left(\dfrac{1}{2}\right)^x$

22) $f(x) = -5 \cdot \left(\dfrac{1}{2}\right)^x$

Write an equation for each graph.

23)



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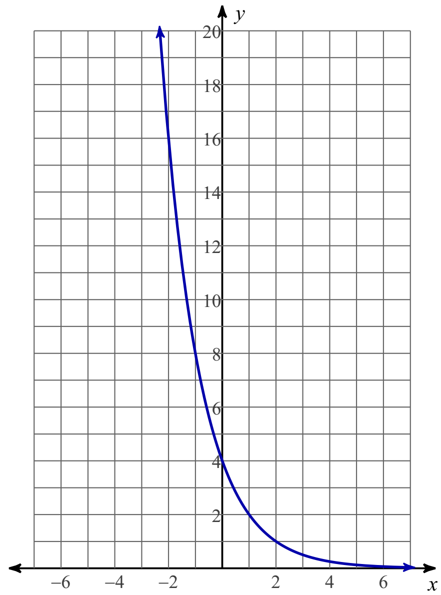

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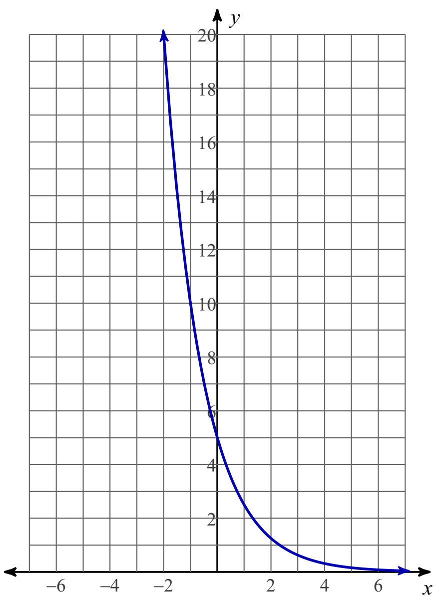

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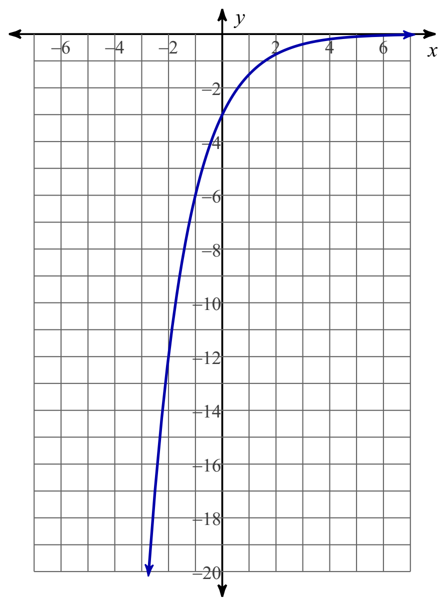


Sketch the graph of each function.

27) $y = 2 \cdot 2^{x+2}$

28) $y = 3 \cdot \left(\frac{1}{2}\right)^{x-1}$

29) $y = 3 \cdot \left(\frac{1}{2}\right)^{x} + 1$

30) $y = 2 \cdot 3^{x} - 2$

Math III Extra Practice Name________________________________

25. Logarithms

Date________________

Rewrite each equation in exponential form.

1) $\log_{9} 81 = 2$

2) $\log_{b} a = 18$

3) $\log_{4} \frac{1}{64} = -3$

4) $\log_{\frac{3}{2}} p = -1$

5) $\log_{144} \frac{1}{12} = -\frac{1}{2}$

6) $\log_{5} 184 = n$

Evaluate each expression.

7) $\log_{5} 25$

8) $\log_{3} \frac{1}{27}$

9) $\log_{2} 8$

10) $\log_{5} \frac{1}{125}$

11) $\log_{4} 1$

12) $\log_{7} 343$

13) $\log_{4} -16$

14) $\log_{64} \frac{1}{4}$

15) $\log_{49} \frac{1}{7}$

16) $\log_{243} 3$

17) $\log_{4} 4$

18) $\log_{\frac{1}{6}} \frac{1}{36}$

Rewrite each equation in logarithmic form.

19) $16^{\frac{1}{2}} = 4$

20) $8^2 = 64$

21) $6^y = x$

22) $15^2 = 225$

23) $v^{20} = u$

24) $17^{-6} = m$

Use the properties of logarithms and the values below to find the logarithm indicated. Do not use a calculator to evaluate the logs.

25) $\log_3 7 \approx 1.8$
$\log_3 10 \approx 2.1$
$\log_3 4 \approx 1.3$
Find $\log_3 \frac{7}{4}$

26) $\log_7 3 \approx 0.6$
$\log_7 10 \approx 1.2$
$\log_7 4 \approx 0.7$
Find $\log_7 40$

27) $\log_4 10 \approx 1.7$
$\log_4 6 \approx 1.3$
$\log_4 7 \approx 1.4$
Find $\log_4 \frac{1}{7}$

28) $\log_9 4 \approx 0.6$
$\log_9 7 \approx 0.9$
$\log_9 6 \approx 0.8$
Find $\log_9 \frac{7}{9}$

29) $\log_5 4 \approx 0.9$
$\log_5 6 \approx 1.1$
$\log_5 7 \approx 1.2$
Find $\log_5 \frac{7}{24}$

30) $\log_7 8 \approx 1.1$
$\log_7 3 \approx 0.6$
$\log_7 10 \approx 1.2$
Find $\log_7 \frac{9}{7}$

Sketch the graph of each function.

31) $y = \log_6 (x + 6)$

32) $y = \log_3 (x + 2) - 2$

33) $y = \log_4 (x - 1) - 5$

34) $y = \log_5 (x - 1) + 3$

 Name________________________________

26. Solving Exponential Equations without Logarithms

Date________________

Solve each equation.

1) $5^{-3a} = 125$

2) $2^{2 - x} = 64$

3) $4^{-2a} = 16$

4) $5^{-3n} = 25$

5) $5^{v + 1} = 5^{3 - 2v}$

6) $3^{-3x - 1} = 3^{-2x}$

7) $5^{2x - 2} = 125$

8) $2^{3 - 3a} = 2^{6}$

9) $2^{-2x} \cdot 2^{1 - 2x} = 8$

10) $6^{-k} \cdot 6^{1 - k} = 6^{3}$

11) $16 \cdot 4^{-3b - 2} = 4^{2b}$

12) $5^{-3r} \cdot 5^{-r} = 5^{4}$

13) $2^{2x} \cdot 2^{3x} = 2^{-3x}$

14) $4 \cdot 2^{-n} = 2^{3}$

15) $5^{-p} = 125$

16) $5^{2b + 1} = 5^{3b + 1}$

17) $6^{-p + 2} = 36$

18) $3^{-2r + 3} = 3^{-3r}$

19) $64^{-2n} = 32^{3n}$

20) $6^{3m - 2} = 36$

21) $2^{-3p} = 4$

22) $3^{3p} = 9$

Math III Extra Practice Name________________

27. Solving Exponential Equations with Logarithms

Date________________

Solve each equation. Round your answers to the nearest ten-thousandth.

1) $7^{x} = 74$

2) $9^{x} = 81$

3) $12^{x} = 91$

4) $17^{x} = 63$

5) $19^{2n} = 15$

6) $10^{p-4} = 23$

7) $7^{-5n} = 71$

8) $6^{3x} = 7$

9) $7 \cdot 10^{n} = 69$

10) $-2 \cdot 10^{k} = -60$

11) $10 \cdot 4^{n} = 47$

12) $5^{v} - 1 = 99$

13) $2e^{p-6} = 14$

14) $11^{v+1} + 1 = 69$

15) $-6 \cdot 20^{-7n} = -96$

16) $2 \cdot 6^{9a} = 18$

17) $5^{-8x-4} = 66$

18) $20^{6v+5} = 89$

19) $19^{9x-9} = 70$

20) $7^{5x-5} = 48$

21) $-17^{x+8} - 1 = -8$

22) $-7 \cdot 3^{9x} + 8 = -34$

23) $-5 \cdot 3^{-4n} - 6 = -24$

24) $-3^{10x} + 8 = 7$

Math III Extra Practice Name________________________________

28. Sequences

Date________________

Find the next three terms in each sequence.

1) $-25,\ -17,\ -9,\ -1,\ 7, ...$

2) $1,\ 1,\ \frac{4}{3},\ 2,\ \frac{16}{5}, ...$

3) $-2,\ -4,\ -12,\ -48,\ -240, ...$

4) $-1,\ -2,\ -6,\ -24,\ -120, ...$

For each sequence, state if it is arithmetic, geometric, or neither.

5) $-2,\ 12,\ -72,\ 432,\ -2592, ...$

6) $4,\ 16,\ 36,\ 64,\ 100, ...$

7) $23,\ 31,\ 39,\ 47,\ 55, ...$

8) $5,\ -\frac{5}{3},\ \frac{5}{9},\ -\frac{5}{27},\ \frac{5}{81}, ...$

9) $-2,\ -4,\ -12,\ -48,\ -240, ...$

10) $\frac{1}{3},\ \frac{2}{3},\ 1,\ \frac{4}{3},\ \frac{5}{3}, ...$

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, and the explicit formula.

11) $-7,\ -107,\ -207,\ -307, \ldots$
Find a_{28}

12) $5,\ -2,\ -9,\ -16, \ldots$
Find a_{39}

13) $29,\ 32,\ 35,\ 38, \ldots$
Find a_{40}

14) $-38,\ -34,\ -30,\ -26, \ldots$
Find a_{40}

Determine if the sequence is geometric. If it is, find the common ratio, the term named in the problem, and the explicit formula.

15) $4,\ 8,\ 16,\ 32, \ldots$
Find a_{10}

16) $-4,\ -8,\ -16,\ -32, \ldots$
Find a_{12}

17) $6,\ 26,\ 126,\ 626, \ldots$
Find a_{11}

18) $3,\ 6,\ 12,\ 24, \ldots$
Find a_{11}

Write the explicit formula for each sequence.

19) $-0.6,\ 3,\ -15,\ 75,\ -375, \ldots$

20) $1,\ \frac{4}{3},\ \frac{5}{3},\ 2,\ \frac{7}{3}, \ldots$

21) $-0.25,\ -1,\ -4,\ -16,\ -64, \ldots$

22) $37,\ 30,\ 23,\ 16,\ 9, \ldots$

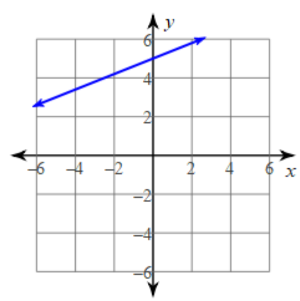
Extra Practice Answers

Answers to 1. Slope

1) $-\frac{4}{3}$
2) $\frac{3}{7}$
3) $\frac{6}{7}$
4) -22
5) $\frac{3}{5}$
6) 3
7) 3
8) 2
9) 0
10) $-\frac{3}{4}$
11) -3
12) Undefined
13) $\frac{2}{9}$
14) 0
15) Undefined
16) $-\frac{1}{3}$

Answers to 2. Linear Equations

1) $y = \frac{5}{2}x + \frac{1}{2}$
2) $y = -\frac{7}{10}x + \frac{1}{2}$
3) $y = -\frac{5}{3}x - 1$
4) $y = -4x + 8$
5) $y = 3x - 5$
6) $y = -2x - 3$
7) $y = \frac{1}{5}x - 1$
8) $y = -2x - 4$
9) $y = \frac{2}{3}x - 2$
10) $y = -2$
11)



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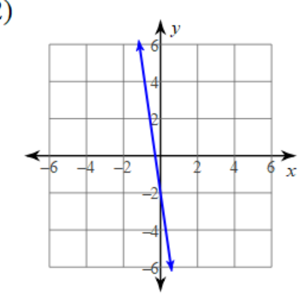


Answers to 3. Systems of Linear Equations

1) $(-4, -2)$
2) $(1, 8)$
3) $(4, 5)$
4) Infinite number of solutions
5) $(1, -4)$
6) $(-2, -3)$
7) No solution
8) $(-4, 5)$
9) Van: 17, Bus: 47
10) daylily: \$3, bunch of ornamental grass: \$11
11) senior citizen ticket: \$3, student ticket: \$13

Answers to 4. Circles

1) Center: $(0, 0)$ Radius: 16
2) Center: $(0, 0)$ Radius: 12
3) Center: $(15, -13)$ Radius: 1
4) Center: $(-12, 7)$ Radius: 1
5) Center: $(-8, \sqrt{91})$ Radius: $\sqrt{31}$
6) Center: $(3, -1)$ Radius: 14
7) $(x - 15)^2 + (y - 7)^2 = 1$
8) $(x + 5)^2 + (y + 11)^2 = 63$
9) $(x - 14)^2 + (y + 5)^2 = 16$
10) $(x - 3)^2 + (y + 8)^2 = 49$
11) $(x + 16)^2 + (y - 1)^2 = 1$
12) $(x + 3)^2 + (y - 7)^2 = 4$
13) 160°
14) 215°
15) 295°
16) 287°
17) 105°
18) 80°
19) 62°
20) 100°
21) 54°
22) 55°
23) 14
24) 17
25) 6.4
26) 14.8

Answers to 5. Probability Sample/Outcome Space

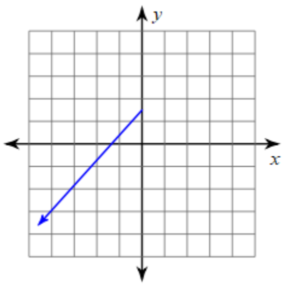
1) {3, 5, 7, 9, 11}
2) {1, 2, 3, 4, 5, 6}
3) {(H, W), (H, M), (T, W), (T, M), (C, W), (C, M)}
4) {(S, F), (S, I), (S, A), (M, F), (M, I), (M, A), (L, F), (L, I), (L, A)}
5) 4
6) 3
7) 6
8) 8
9) 192
10) 320

Answers to 6. Permutations and Combinations

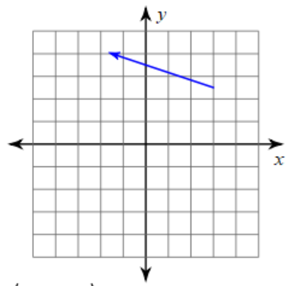
1) Combination; 9,045
2) Permutation; 12
3) Combination; 44,850
4) Permutation; 2,450
5) Combination; 99,884,400
6) Permutation; 90
7) Combination; 10,518,300
8) Permutation; 4,830
9) Combination; 18,564
10) Combination; 167,960
11) Permutation; 870
12) Permutation; 272

Answers to 7. Vectors

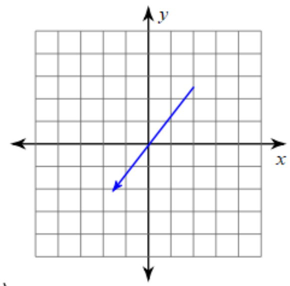
1)



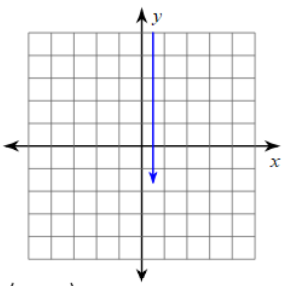
2)



3)



4)



5) $\langle 17, -1\rangle$

6) $\langle -4, -12\rangle$

7) $\langle 3, 14\rangle$

8) $\langle -7, -11\rangle$

9) $2\sqrt{53} \approx 14.56$

10) 2

11) $\sqrt{482} \approx 21.954$

12) $2\sqrt{5} \approx 4.472$

13) $\langle -9, -9\rangle$

14) $\langle -11, 0\rangle$

15) $\langle -7, 2\rangle$

16) $\langle -6, 13\rangle$

17) $\langle 126, -168\rangle$

18) $\langle -40, 88\rangle$

19) $\langle -21\sqrt{2}, -\sqrt{86}\rangle$

20) $\langle 8, -22\rangle$

21) $\langle -30, 40\rangle$

22) $\langle 11, -1\rangle$

23) $\langle -15, 36\rangle$

24) $\langle 3, -\sqrt{91}\rangle$

25) $\langle -3, -25\rangle$

26) $\langle 11, -3\rangle$

27) $\langle 80, 76\rangle$

28) $\langle 9, -8\rangle$

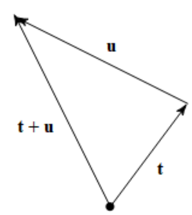
29) $\left\langle -\frac{3}{5}, \frac{4}{5}\right\rangle$

30) $\left\langle \frac{2\sqrt{111}}{37}, \frac{5\sqrt{37}}{37}\right\rangle$

31) $\left\langle \frac{11\sqrt{170}}{170}, -\frac{7\sqrt{170}}{170}\right\rangle$

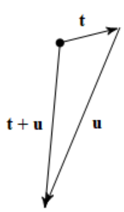
32) $\left\langle -\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right\rangle$

33)



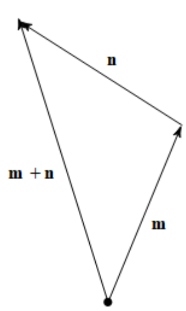
24.6; 116.57°

34)



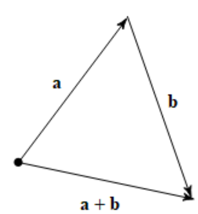
11.05; 264.81°

35)



19.92; 107.53°

36)



19.42; 348.11°

37) 27

38) 0

39) −6

40) −3

41) 0

42) 10

Answers to 8. Parametric Equations

1) x = -10t + 2, y = 11t - 6
2) x = 2t - 1, y = 3/2t + 1
3) (-3,8), (-1,4), (1,0)
x-int when y = 0, t = 2 sec, x = 1
y-int when x = 0, t = 3/2 sec, y = 2
4) (4,-5), (1,-4), (-2,-3)
x-int when y = 0, t = 5 sec, x = -11
y-int when x = 0, t = 4/3 sec, y = -11/3

Answers to 9. Special Right Triangles

1) $m = 2\sqrt{2},\ n = 2$
2) $u = \dfrac{8\sqrt{3}}{3},\ v = \dfrac{4\sqrt{3}}{3}$
3) $x = 4\sqrt{5},\ y = 2\sqrt{15}$
4) $x = 3\sqrt{2},\ y = 3\sqrt{2}$
5) $x = 10,\ y = 5\sqrt{2}$
6) $x = 4\sqrt{3},\ y = 4$
7) $m = \dfrac{14\sqrt{3}}{3},\ n = \dfrac{7\sqrt{3}}{3}$
8) $x = 5,\ y = \dfrac{5\sqrt{2}}{2}$
9) $\dfrac{7\sqrt{6}}{4}$
10) $27\sqrt{2}$
11) $16\sqrt{2}$
12) $3\sqrt{2}$

Answers to 10. Trig Function Ratios

1) $\dfrac{\sqrt{2}}{2}$
2) $\dfrac{3}{5}$
3) $\dfrac{\sqrt{10}}{10}$
4) $\dfrac{12}{13}$
5) $\dfrac{4}{3}$
6) $\dfrac{4}{3}$
7) $\dfrac{4}{5}$
8) $\dfrac{3}{5}$
9) 1
10) $\dfrac{12}{13}$

Answers to 11. Finding Missing Sides with Trig Functions

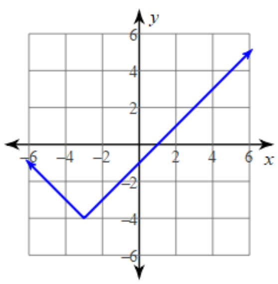
1) 23.4
2) 4.9
3) 25.8
4) 23.8
5) 2.5
6) 12.3

Answers to 12. Finding Missing Angles with Inverse Trig Functions

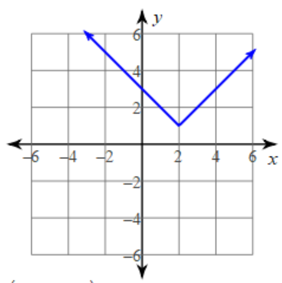
1) 54.9°
2) 24.6°
3) 40.6°
4) 37.4°
5) 38.3°
6) 35.1°

Answers to 13. Absolute Value

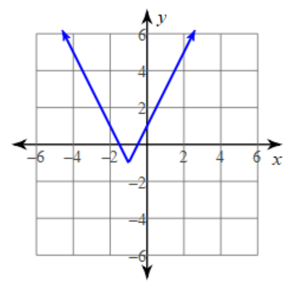
1)



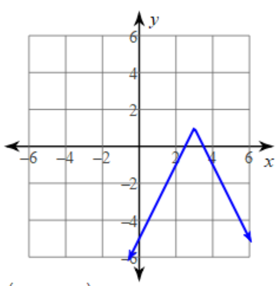
2)



3)



4)



5) $\{0, -10\}$

6) $\{8, 4\}$

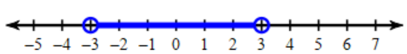
7) $\{27, -7\}$

8) No solution.

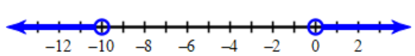
9) $\{0\}$

10) $\left\{-\frac{32}{3}, \frac{32}{3}\right\}$

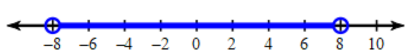
11) $-3 < x < 3$:



12) $x > 0$ or $x < -10$:

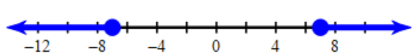


13) $-8 < x < 8$:

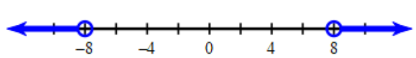


14) $a \le -1$ or $a \ge 1$:

15) $x \ge 7$ or $x \le -7$:



16) $a > 8$ or $a < -8$:



Answers to 14. Inverse Functions

1) $g^{-1}(n) = n - 2$
2) $f^{-1}(x) = \frac{-8 + x}{2}$
3) $f^{-1}(x) = \frac{-20 + 8x}{5}$
4) $h^{-1}(x) = \frac{2}{3}x + \frac{8}{3}$
5) $f^{-1}(x) = \frac{2}{x}$
6) $h^{-1}(n) = \frac{1}{n} + 2$
7) $f^{-1}(x) = -\frac{4}{x} + 2$
8) $g^{-1}(n) = \frac{2 - n}{n - 1}$
9) $h^{-1}(x) = \sqrt[5]{x - 1} + 1$
10) $f^{-1}(n) = -3 + (n + 1)^3$

Answers to 15. Distance = Rate*Time Problems

1) 8 hours
2) 70 km/h
3) 50 km/h
4) 10 hours
5) 6 hours
6) 30 km/h
7) 2.2 hours
8) 4.5 hours

Answers to 16. Factoring Quadratic Trinomials

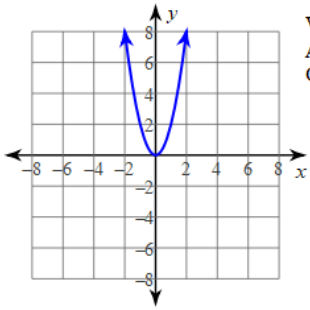
1) $(n+7)(n+6)$
2) $(n+8)(n-5)$
3) $(m-3)(m-10)$
4) $(n+10)(n+1)$
5) $(5b-1)(b+2)$
6) $-(5k-7)(k-4)$
7) $(2p+9)(p+5)$
8) $(7a-8)(a+4)$
9) $-(5k+7)(k-8)$
10) $(7n+9)(n-5)$
11) $(2r+9)(r-2)$
12) $-(5a+7)(a-1)$
13) $(r-6)(8r+7)$
14) $(3a+2)(3a+5)$
15) $-(3x-2)(3x+10)$
16) $(2m-3)(3m-8)$
17) $(2n+5)(5n-1)$
18) $-(x-10)(6x+5)$
19) $(v+9)(4v+9)$
20) $-(n+3)(9n-10)$

Answers to 17. Solving Quadratic Equations

1) $\{-8, 0\}$
2) $\left\{\frac{2}{7}, 2\right\}$
3) $\{4, 0\}$
4) $\{3, 0\}$
5) $\{6, 5\}$
6) $\{3, -3\}$
7) $\{5, 1\}$
8) $\{6, -6\}$
9) $\{-7, 0\}$
10) $\{5, -8\}$
11) $\{3, -2\}$
12) $\{-1, 4\}$
13) $\{8, -8\}$
14) $\{-8, -2\}$
15) $\{\sqrt{34}, -\sqrt{34}\}$
16) $\{8, -8\}$
17) $\{3\sqrt{5}, -3\sqrt{5}\}$
18) $\left\{\frac{10}{7}, -\frac{10}{7}\right\}$
19) $\{5, -3\}$
20) $\{3, -1\}$
21) $\{-1, -19\}$
22) $\{1, -5\}$
23) $\{-5 + \sqrt{7}, -5 - \sqrt{7}\}$
24) $\left\{-5, \frac{13}{2}\right\}$
25) $\left\{-\frac{1}{2}, -\frac{2}{3}\right\}$
26) $\{3 + \sqrt{19}, 3 - \sqrt{19}\}$

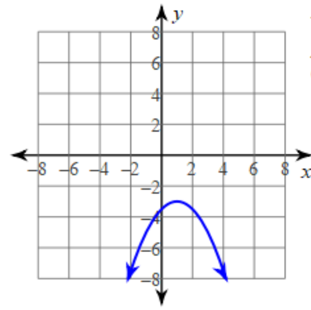
Answers to 18. Parabolas

1)



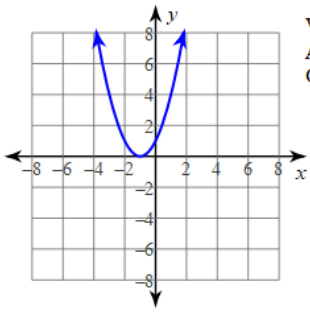
Vertex: $(0, 0)$
Axis of Sym.: $x = 0$
Opens: Up

2)



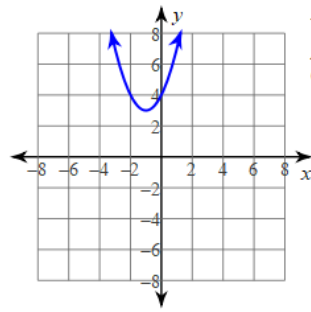
Vertex: $(1, -3)$
Axis of Sym.: $x = 1$
Opens: Down

3)



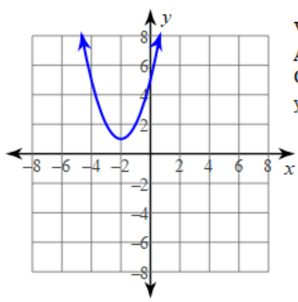
Vertex: $(-1, 0)$
Axis of Sym.: $x = -1$
Opens: Up

4)



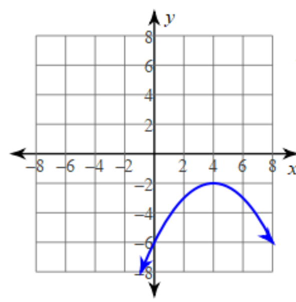
Vertex: $(-1, 3)$
Axis of Sym.: $x = -1$
Opens: Up

5)



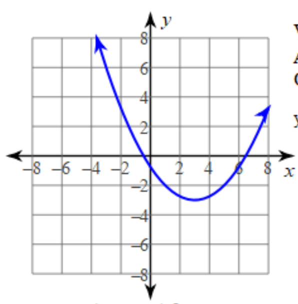
Vertex: $(-2, 1)$
Axis of Sym.: $x = -2$
Opens: Up
y-int: 5

6)



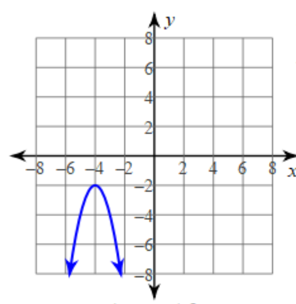
Vertex: $(4, -2)$
Axis of Sym.: $x = 4$
Opens: Down
y-int: -6

7)



Vertex: $(3, -3)$
Axis of Sym.: $x = 3$
Opens: Up
y-int: $-\frac{3}{4}$

8)



Vertex: $(-4, -2)$
Axis of Sym.: $x = -4$
Opens: Down
y-int: -34

9) $y = -2(x + 5)^2 + 10$

10) $y = -(x + 1)^2 + 1$

11) $y = -2(x - 2)^2 - 9$

12) $y = \frac{1}{4}x^2 + 4$

13) $y = -\frac{1}{2}x^2 - 2x + 5$

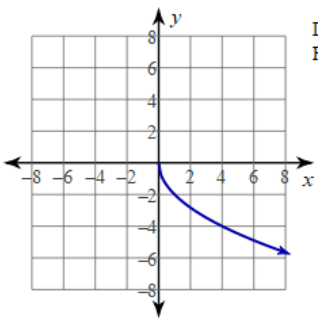
14) $y = 2x^2 - 12x + 25$

Answers to 19. Solving Radical Equations (Square Roots Only)

1) $\{0\}$
2) $\{26\}$
3) $\{-7\}$
4) $\{-10\}$
5) $\{0, 5\}$
6) No solution.
7) $\{2, 6\}$
8) $\{10\}$
9) $\{5, 7\}$
10) $\{8\}$

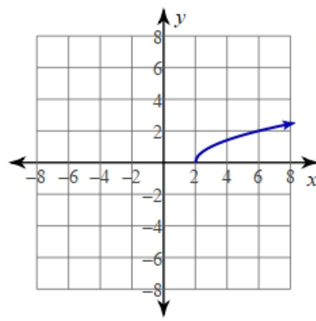
Answers to 20. Graphing Square Root Functions

1)



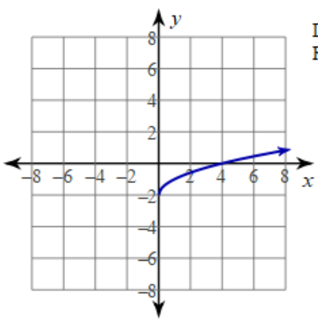
Domain: $x \geq 0$
Range: $y \leq 0$

2)



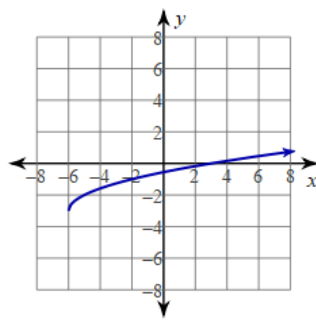
Domain: $x \geq 2$
Range: $y \geq 0$

3)



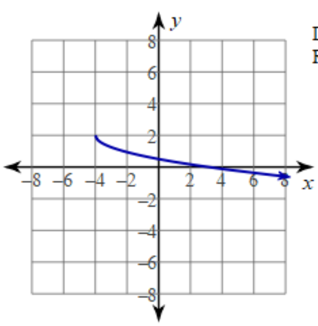
Domain: $x \geq 0$
Range: $y \geq -2$

4)



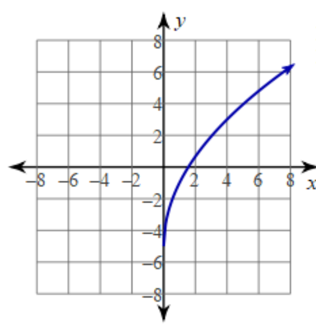
Domain: $x \geq -6$
Range: $y \geq -3$

5)



Domain: $x \geq -4$
Range: $y \leq 2$

6)



Domain: $x \geq 0$
Range: $y \geq -5$

Answers to 21. Percent Change

1) 70% decrease
2) 2% decrease
3) 61% decrease
4) 47% decrease
5) 63% decrease
6) 43% decrease
7) 33% decrease
8) 34% decrease
9) 83% increase
10) 15% increase
11) 97% decrease
12) 82% decrease
13) 47% decrease
14) 211% increase
15) 372% increase
16) 58% decrease
17) 19% decrease
18) 65% increase
19) 56% decrease
20) 88% decrease

Answers to 22. Work Problems

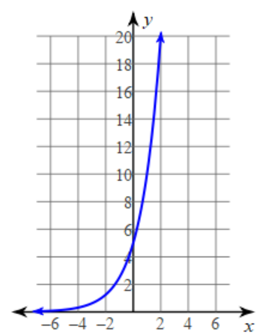
1) 3.75 hours
2) 3.33 minutes
3) 7.01 hours
4) 10.01 hours
5) 3.19 hours
6) 3.65 hours
7) 5.2 hours
8) 5.09 hours

Answers to 23. Rational Expressions and Equations

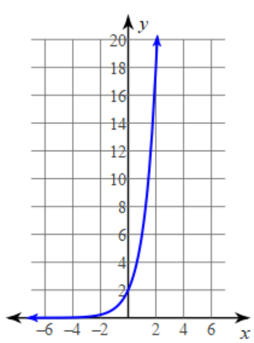
1) $\dfrac{9y^2 + 4x}{6y}$
2) $\dfrac{8y^3 + x}{2y^3}$
3) $\dfrac{2p^2 - 6p - 12}{(p-2)(p+6)}$
4) $\dfrac{-2r - 18 - r^2}{3r(r-6)}$
5) $\dfrac{5m + 4}{m(m-4)}$
6) $\dfrac{28r + 45}{6(2r+5)}$
7) $\left\{\dfrac{7}{2}\right\}$
8) $\left\{\dfrac{2}{5}\right\}$
9) $\left\{\dfrac{5}{4}, -1\right\}$
10) $\{2, 1\}$
11) $\{7\}$
12) $\{2, -3\}$

Answers to 24. Exponenent Rules and Exponential Functions

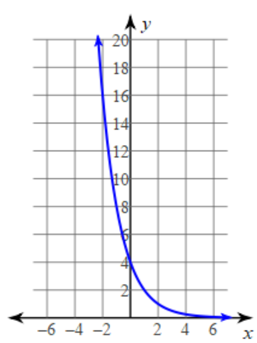
1) a^5b^7
2) $8y^6x^5$
3) $64y^6$
4) 1
5) y
6) $\dfrac{xy^3}{4}$
7) $12y^4 \cdot x$
8) $3y \cdot x^3$
9) $y^2x^{\frac{10}{3}}$
10) $x^{\frac{3}{2}}y^{\frac{1}{3}}$
11) $\dfrac{3v^2}{u^8}$
12) $\dfrac{1}{x^4}$
13) $\dfrac{yx^3}{3}$
14) $\dfrac{2}{y^8x^4}$
15) $\dfrac{1}{4x^4}$
16) $\dfrac{1}{m^8n^4}$
17) $\dfrac{y^8}{4x^8}$
18) $\dfrac{y^{14}}{4x^{15}}$
19)



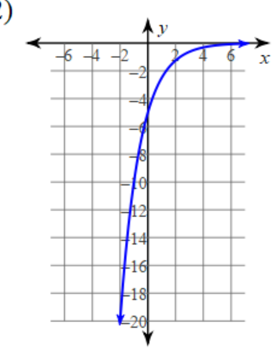
20)



21)



22)



Answers to 24. Exponenent Rules and Exponential Functions (Continued)

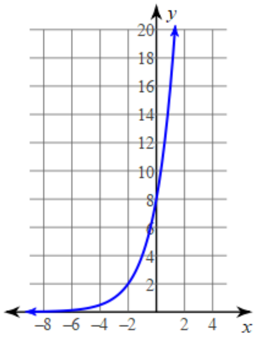
23) $f(x) = 2 \cdot 2^x$

24) $f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$

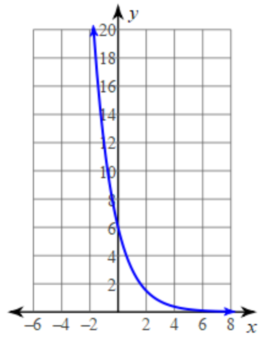
25) $f(x) = 5 \cdot \left(\frac{1}{2}\right)^x$

26) $f(x) = -3 \cdot \left(\frac{1}{2}\right)^x$

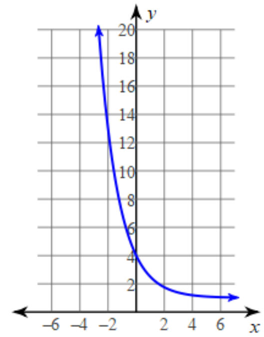
27)



28)



29)



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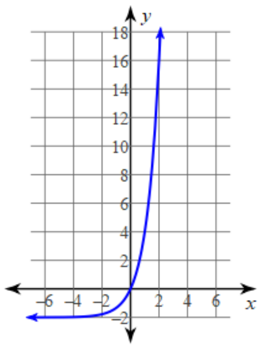


Answers to 25. Logarithms

1) $9^2 = 81$

2) $b^{18} = a$

3) $4^{-3} = \frac{1}{64}$

4) $\left(\frac{3}{2}\right)^{-1} = p$

5) $144^{-\frac{1}{2}} = \frac{1}{12}$

6) $5^n = 184$

7) 2

8) -3

9) 3

10) -3

11) 0

12) 3

13) Undefined

14) $-\frac{1}{3}$

15) $-\frac{1}{2}$

16) $\frac{1}{5}$

17) 1

18) 2

19) $\log_{16} 4 = \frac{1}{2}$

20) $\log_8 64 = 2$

21) $\log_6 x = y$

22) $\log_{15} 225 = 2$

23) $\log_v u = 20$

24) $\log_{17} m = -6$

25) 0.5

26) 1.9

27) -1.4

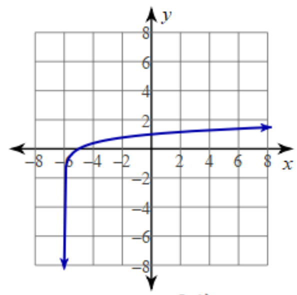
28) -0.1

Answers to 25. Logarithms (Continued)

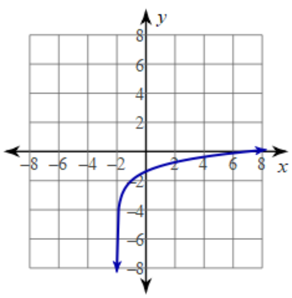
29) -0.8

30) 0.2

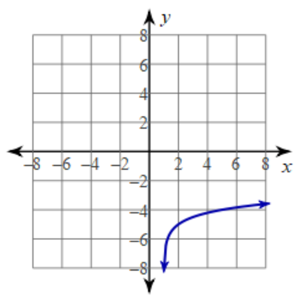
31)



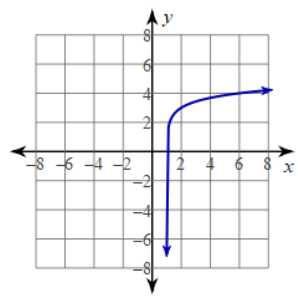
32)



33)



34)



Answers to 26. Solving Exponential Equations without Logarithms

1) $\{-1\}$

2) $\{-4\}$

3) $\{-1\}$

4) $\left\{-\frac{2}{3}\right\}$

5) $\left\{\frac{2}{3}\right\}$

6) $\{-1\}$

7) $\left\{\frac{5}{2}\right\}$

8) $\{-1\}$

9) $\left\{-\frac{1}{2}\right\}$

10) $\{-1\}$

11) $\{0\}$

12) $\{-1\}$

13) $\{0\}$

14) $\{-1\}$

15) $\{-3\}$

16) $\{0\}$

17) $\{0\}$

18) $\{-3\}$

19) $\{0\}$

20) $\left\{\frac{4}{3}\right\}$

21) $\left\{-\frac{2}{3}\right\}$

22) $\left\{\frac{2}{3}\right\}$

Answers to 27. Solving Exponential Equations with Logarithms

1) 2.2119	2) 2	3) 1.8153	4) 1.4623
5) 0.4599	6) 5.3617	7) −0.4381	8) 0.362
9) 0.9938	10) 1.4771	11) 1.1163	12) 2.8614
13) 7.9459	14) 0.7597	15) −0.1322	16) 0.1363
17) −0.8254	18) −0.5836	19) 1.1603	20) 1.3979
21) −7.3132	22) 0.1812	23) −0.2915	24) 0

Answers to 28. Sequences

1) 15, 23, 31

2) $\frac{16}{3}, \frac{64}{7}, 16$

3) −1440, −10080, −80640

4) −720, −5040, −40320

5) Geometric

6) Neither

7) Arithmetic

8) Geometric

9) Neither

10) Arithmetic

11) Common Difference: $d = -100$
$a_{28} = -2707$
Explicit: $a_n = 93 - 100n$

12) Common Difference: $d = -7$
$a_{39} = -261$
Explicit: $a_n = 12 - 7n$

13) Common Difference: $d = 3$
$a_{40} = 146$
Explicit: $a_n = 26 + 3n$

14) Common Difference: $d = 4$
$a_{40} = 118$
Explicit: $a_n = -42 + 4n$

15) Common Ratio: $r = 2$
$a_{10} = 2048$
Explicit: $a_n = 4 \cdot 2^{n-1}$

16) Common Ratio: $r = 2$
$a_{12} = -8192$
Explicit: $a_n = -4 \cdot 2^{n-1}$

17) Not geometric

18) Common Ratio: $r = 2$
$a_{11} = 3072$
Explicit: $a_n = 3 \cdot 2^{n-1}$

19) $a_n = -0.6 \cdot (-5)^{n-1}$

20) $a_n = \frac{2}{3} + \frac{1}{3}n$

21) $a_n = -0.25 \cdot 4^{n-1}$

22) $a_n = 44 - 7n$