MATH II

Mathematics Department
Shady Side Academy
Pittsburgh, PA
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[Problems originated with the Mathematics Department at Phillips Exeter Academy, NH.]

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SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

MATH II

MISSION/HISTORY: As part of an on-going curriculum review the Mathematics Department of Shady Side Academy sent two members of the senior school math faculty to visit Phillips Exeter Academy [PEA] in 2008 to observe their classes. After this observation and much reflection, the department decided to adopt this problem-based curriculum. The materials used in the Mathematics I and II courses are taken directly from PEA. We thank the teachers at PEA for the use of their materials.

RATIONALE: The Shady Side Academy Mathematics Department Goals are as follows:

Students will develop the habit of using mathematical reasoning based on logical thinking.
Students will develop adequate skills necessary to solve problems mathematically.
Students will recognize that the structure and order of mathematics can be discovered in the world around us.
Students will recognize the connections of mathematics to other disciplines.
Students will express themselves clearly in mathematical discourse.
Students will be familiar with and proficient in appropriate technology.
Students will achieve their highest mathematical goals.
Students will gain an appreciation for the study of mathematics.

In addition, the teachers in the Department of Mathematics want you to be an articulate student of mathematics. We want you to be able to speak and write mathematics well. We want you to be a fearless problem solver so that you approach problems with curiosity and not trepidation. The Mathematics II classroom is student-centered. The curriculum is problem- based with an integrated design. You will continually learn new material while reviewing prior topics.

EXPECTATIONS: In order for you to be successful in this course, the Mathematics Department has the following suggestions and expectations. First, we expect you to attempt every problem. More than merely writing the problem number, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. Next, seek help wherever you can find it. We expect you to cooperate with your peers and teachers. The Mathematics Department is a team of teachers striving to help all students reach their potential. You are encouraged to ask any teacher for help if your own is not available. Finally, as stated in the Student Handbook on page 13: “Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets.” We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

To the Student

Contents: Members of the PEA Mathematics Department have written the material in this book. As you work through it, you will discover that algebra and geometry have been integrated into a mathematical whole. There is no Chapter 5, nor is there a section on tangents to circles. The curriculum is problem-centered, rather than topic-centered. Techniques and theorems will become apparent as you work through the problems, and you will need to keep appropriate notes for your records — there are no boxes containing important theorems. You will begin the course with this binder of problems, graph paper, and a protractor. All of your solutions are to be kept in this binder. It will be periodically collected and will factor into your term grade. There is no index in your binder but the reference section at the end should help you recall the meanings of key words that are defined in the problems (where they usually appear italicized).

Comments on problem-solving: You should approach each problem as an exploration. Reading each question carefully is essential, especially since definitions, highlighted in italics, are routinely inserted into the problem texts. It is important to make accurate diagrams whenever appropriate. Useful strategies to keep in mind are: create an easier problem, guess and check, work backwards, and recall a similar problem. It is important that you work on each problem when assigned, since the questions you may have about a problem will likely motivate class discussion the next day.

Problem-solving requires persistence as much as it requires ingenuity. When you get stuck, or solve a problem incorrectly, back up and start over. Keep in mind that you're probably not the only one who is stuck, and that may even include your teacher. If you have taken the time to think about a problem, you should bring to class a written record of your efforts, not just a blank space in your notebook. The methods that you use to solve a problem, the corrections that you make in your approach, the means by which you test the validity of your solutions, and your ability to communicate ideas are just as important as getting the correct answer. Proper spelling is essential for clear written communication.

About technology: Many of the problems in this book require the use of technology (graphing calculators or computer software) in order to solve them. Moreover, you are encouraged to use technology to explore, and to formulate and test conjectures. Keep the following guidelines in mind: write before you calculate, so that you will have a clear record of what you have done; store intermediate answers in your calculator for later use in your solution; pay attention to the degree of accuracy requested; refer to your calculator's manual when needed; and be prepared to explain your method to your classmates. Also, if you are asked to "graph $y = (2x - 3)/(x + 1)$", for instance, the expectation is that, although you might use your calculator to generate a picture of the curve, you should sketch that picture in your notebook or on the board, with correctly scaled axes.

Shady Side Academy
Introductory Math Guide for Students

Homework

First, we expect you to attempt every problem. More than merely writing the problem number in your notebook, write an equation or draw a picture or write a definition; in other words, indicate in some way that you have thought about and tried the problem. As stated in the Student Handbook: "Homework for Forms III and IV normally is limited to 45 minutes of homework per night per subject on days when that class meets." We expect you to spend 45 minutes on mathematics homework to prepare for each class meeting.

Going to the Board

It is very important to go to the board to put up homework problems. Usually, every homework problem is put on the board at the beginning of class, presented, and then discussed in class. By doing this, you will develop your written and oral presentation skills.

Plagiarism

You can get help from almost anywhere, but make sure that you cite your help, and that all work shown or turned in is your own, even if someone else showed you how to do it. Never copy work from others. Teachers do occasionally give problems/quizzes/tests to be completed at home. You may not receive help on these assessments, unless instructed to by your teacher; it is imperative that all the work is yours. More information about plagiarism can be found on page vii in your binder.

Math Extra-Help

Getting help is an integral part of staying on top of the math program here at Shady Side Academy. It can be rather frustrating to be lost and stuck on a problem. Teachers, peer tutors, study groups, the internet, your resource book and classmates are all helpful sources.

Teachers and Meetings

The very first place to turn for help should be your teacher. Teachers at SSA are always eager to help you succeed. The Math Department office is located on the 3rd floor of Rowe Hall. Individual meetings can be arranged with teachers during study halls, free periods, or after school. You can always ask or email any teacher in the department for help. Getting help from your teacher is the first and most reliable source to turn to for extra help.

SSA Student Quotes

"This program really helped me learn and understand the concepts of Algebra II. It helped us as a group because we covered materials together. We all said our own ideas and accepted when they were wrong. It gave each individual confidence in their understanding of the material. Some days we did not check over every problem like I would have liked to, but this allowed me to be a frequent visitor in the math office. It was a different approach that ran very smoothly in this class."
--Betsy Vuchinich, '12

"I loved Math II this year because the curriculum was completely different from anything I've previously encountered in math. We didn't use a book for the majority of the year, instead we focused on more complicated word problems and worked together in small groups to solve these difficult problems. This forced us to think through the problems and think about "Why?" more so than "How?" and this was a much different look for a math class. Working with your peers in a setting that promoted group work was refreshing, and I enjoyed it very much. I hope the Math Department continues to use this curriculum."
--Jonathan Laufe, '12

"I came into the year unsure of what to think about this approach to mathematics. I had criticism and positive words about the packet, and I didn't know what to expect. Though sometimes I was confused, in the end, everything worked out."
--Erin Gorse, '12

"The curriculum definitely took some getting used to but once you figure it out, it has a balance of being challenging and easy at the same time."
--Elijah Williams, '13

"I thought the word problems were unnecessary at first, then I found my mind starting to expand."
--Guy Philips, '13

"The curriculum for Math II under Ms. Whitney was not easy, but the use of a packet full of word problems that challenged our minds to apply concepts previously learned really expanded our knowledge much easier than traditional out of the book teaching. The packet introduced to me a new way of learning that I was not familiar with, but even if students are struggling to understand concepts of problems then teachers make themselves available to work with you very often. You will not get by easily in this course by daydreaming, but this hands on experience in the classroom of interacting with your classmates and teacher will show how much easier learning is because you stress previously learned material and open windows to other, more complex problems."
--Christopher Bush, '13

“Math I is a great way to learn and if I had to describe it in one word it would be ‘Utopian’: The classroom environment motivates me to do better and it teaches you to either accept your method or to abandon it for a better one. The fact that the teachers act as moderators in the classroom makes it a better way to learn because it really gets you to think. I loved Math One and look forward to doing more Exeter problems in Math II.”
--Adam D'Angelo, ‘14

“I think Math II will teach you a lot about not only math. Even coming out of a year of "the binder", Math I, I found that I actually learned math a lot differently this year than last; this required me to be malleable with how I approached things and studied. "Doing math differently", for lack of a better word, was something that confronted me this year, and it challenged things I already did in a healthy way: communicating via email, asking questions, organizing things differently, learning how to take notes in new ways, and meeting new people. A beneficial thing I recommend is visiting the math office even once a week after school to meet with your teacher(or any teacher) and go over homework, do practice problems, and chat. There are a lot of cool people in that office, and the more you communicate with them, the more you will enjoy and feel confident with math. Overall, you will probably grow a lot as a person throughout taking Math II, and learn many valuable life skills; be excited for that.”
--Felicia Reuter, ‘17

- “ I have found with the Math II binder that it is much easier to approach the problems with other people. Doing homework with friends mimics an actual class and confusion is more easily avoided. It is also helpful to bounce ideas off each other and you might even find a new way to solve a problem!
- If you are struggling with one type of problem (i.e. proofs) go back to the basics and get worksheets or practice problems from a teacher. You will find that once you have mastered the foundation of the problem you will more easily be able to attempt the harder stuff.
- Always have the full answer to the problems written down by the end of class. Or using a smartphone, take a picture of the board. This will ensure a solid reference for studying.
- Take advantage of your resources! Go to your teacher outside of class, or any other teacher in the math department, they are more than willing to help. It will pay off to put in the extra effort.
- Lean into discomfort. It is okay to not have a full solution to a problem, explain what you know and trust that your class and teacher are there to support you and help you find an answer without judgement.
- Stay organized! Often in class teachers will reference problems from previous pages to make connections or to draw conclusions so it is most helpful to organize your binder in a way that makes the most sense to you. Also, tests and quizzes are pulled from a number of different pages so find a system of organization that can help you succeed in your studies.”

--Caroline Benec, ‘17

SHADY SIDE ACADEMY SENIOR SCHOOL
DEPARTMENT OF MATHEMATICS

Policy on Plagiarism and Cheating

At the beginning of each course, each teacher in the Mathematics Department will explain to the class what is expected with regard to the daily completion of homework, the taking of in-class tests, make-up tests, and take-home tests. Students will be told whether or not they may use books and/or other people when completing in-class or out-of class assignments/tests. The consequences listed below will take effect if a teacher suspects that a student is in violation of the instructions given for a particular assignment or test.

PLAGIARISM

Plagiarism is the act of representing something as one's own without crediting the source. This may be manifest in the mathematics classroom in the form of copying assignments, fabricating data, asking for or giving answers on a test, and using a "cheat sheet" on an exam.

CHEATING

If, during an in-class test, the teacher in that room considers that a student has violated the teacher's instructions for the test, the teacher will instruct the student that there is a suspicion of cheating and the teacher will initiate the consequences below. If a student is taking a make-up test out of class and any teacher considers that the student is, or has been, cheating the teacher will bring the issue to the notice of the Department Chair, and initiate the consequences below. Sharing the content of a particular test with an individual who has not taken the test is considered by the department to be cheating by both parties.

CONSEQUENCES

When a teacher suspects plagiarism or academic dishonesty, the teacher and Department Chair will speak with the student. The Department Chair, in conjunction with the Dean of Student Life, will determine whether plagiarism or academic dishonesty has occurred. If plagiarism or academic dishonesty is determined, the Dean of Student Life and the Department Chair make the decision about the appropriate response to the situation, which will likely include referral to the Discipline Committee. The Department Chair will contact the family to discuss the infraction and consequence. If a Discipline Committee referral is made, the Dean of Student Life will follow up with the family as well.

In any case of plagiarism or cheating, the student concerned will likely receive a failing grade for that piece of work, as well as any other appropriate steps deemed necessary by the Department Chair and the Dean of Academic Life.

Math II Guided Notes

The following pages are a place for you to organize the concepts, topics, formulas and ideas you learn this year. You can use them in any way you wish. It is suggested that when you come upon an important finding or result in class or on your own, that you write it in these notes so that it is easily accessible when it comes time to study for an exam or review material. These notes are not a substitute for taking notes in other ways, and the Mathematics Department encourages you to use a notebook to have a record of your work, corrections and any notes you get in class. We hope this is useful to you, and we welcome any feedback.

--SSA Senior School Math Department

Triangles

Special right triangles:

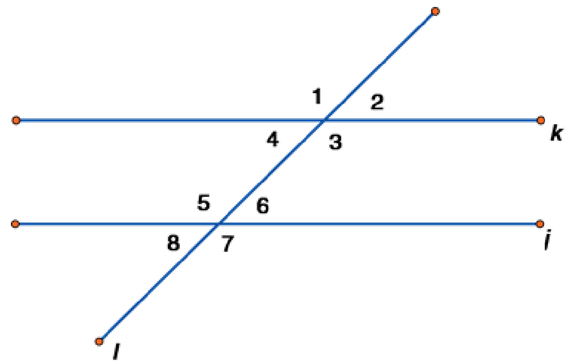
Type	Ratio/Notes/Drawing
45-45-90	
30-60-90	

Draw an example of an altitude, a median and an angle bisector. Are these ever the same?

Angles

Interior angles of a polygon:

Exterior angles of a polygon:



Proofs

List all of the triangle congruence postulates and draw an example of each.

Circles

A circle is ____

The equation of a circle with radius r centered at (h,k) is ____________________________________

Draw an example of an inscribed angle and a central angle and how they relate to their intercepted arcs.

Geometry

	Parallelogram	**Rectangle**	**Square**	**Rhombus**	**Trapezoid**	**Kite**
Sketch						
Diagonals bisect each other						
Diagonals are congruent						
Diagonals are perpendicular						
One diagonal is an angle bisector						
Both diagonals are angle bisectors						
Both pairs of opposite sides are congruent						
All sides congruent						
Both pairs of opposite sides parallel						
Exactly one pair of opposite sides is parallel						
All angles congruent						
Both pairs of opposite angles congruent						
All adjacent angles are supplementary						

Theorems

The main geometric theorems we have learned this year and a description (in my own words) and/or sketch of the theorem are:

Theorem	**Notes**

Trapezoids

You find the length of the midsegment by:

You find the length of a segment parallel to the parallel sides and a certain fraction of the way down by:

You find the lengths of segments the parallel lines within the trapezoid are cut into by the diagonals by:

Distance and Equidistance

- You find the distance between two points by:

- You find the shortest distance from a point to a line by:

- You find the shortest distance between two lines by:

- You find a point equidistant from two points by:

- You find the set of all points equidistant from two points by:

- You find a point that is a certain fraction or distance from two fixed points by:

Vectors:

Topics	Notes
Finding a vector from one point to another	
Finding the slope/direction of a vector $\langle a, b\rangle$	
Finding a vector going in the opposite direction of $\langle a, b\rangle$	
Finding vector going perpendicularly to $\langle a, b\rangle$	
Finding the length of a vector $\langle a, b\rangle$	
Finding a unit vector going in the same direction as $\langle a, b\rangle$	
Changing the length of the vector $\langle a, b\rangle$ OR making it a specific length	
Finding the dot product of two vectors $\langle a, b\rangle$ and $\langle m, n\rangle$	
What does it mean if the dot product of two vectors is zero?	
Adding and subtracting vectors $\boldsymbol{u} = \langle a, b\rangle$ and $\boldsymbol{v} = \langle c, d\rangle$	
What is the geometric interpretation of $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$?	

Math II

UNLESS OTHERWISE STATED, EXPRESS ALL ROUNDED DECIMAL VALUES TO THE NEAREST THOUSANDTH (3 DECIMAL PLACES).

1. (I.79.5) Plot a point near the upper right corner of a sheet of graph paper. Move your pencil 15 graph-paper units (squares) to the left and 20 units down, then plot another point. Use your ruler to measure the distance between the points. Because the squares on your graph paper are probably larger or smaller than the squares on your classmates' graph paper, it would not be meaningful to compare ruler measurements with anyone else in class. You should therefore finish by converting your measurement to graph-paper units.

2. (I.79.6) (Continuation) Square your answer (in graph-paper units), and compare the result with the calculation $15^2 + 20^2$.

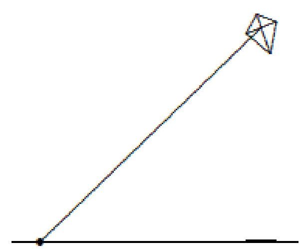
3. (I.79.7) (Continuation) Repeat the entire process, starting with a point near the upper left corner, and use the instructions "20 squares to the right and 21 squares down." You should find that the numbers in this problem again fit the equation. These are instances of the *Pythagorean Theorem*, which is a statement about right-angled triangles. Write a clear statement of this useful result. You will need to refer to the longest side of a right triangle, which is called the *hypotenuse*.

4. (I.80.3) Starting at school, you and a friend ride your bikes in different directions—you ride 4 blocks north and your friend rides 3 blocks west. At the end of this adventure, how far apart are you and your friend?

5. (I.80.4) From the library, you ride your bike east at a rate of 10 mph for half an hour while your friend rides south at a rate of 15 mph for 20 minutes. How far apart are you? How is this problem similar to the preceding problem? How do the problems differ?

6. (I.80.6) Imagine a circle of rope, which has twelve evenly spaced knots tied in it. Suppose that this rope has been pulled into a taut, triangular shape, with stakes anchoring the rope at knots numbered 1, 4, and 8. Make a conjecture about the angles of the triangle.

7. (I.81.4) While flying a kite at the beach, you notice that you are 100 yards from the kite's shadow, which is directly beneath the kite. You also know that you have let out 150 yards of string. How high is the kite?



Problem 7

8. (I.81.5) Starting from home, Jamie haphazardly walks 2 blocks north, 3 blocks east, 1 block north, 3 blocks east, 1 block north, 5 blocks east, and 1 block north. How far is Jamie from home if each block is 150 meters long?

9. (I.81.6) The sides of Fran's square are 5 cm longer than the sides of Tate's square. Fran's square has 225 sq cm more area. What is the area of Tate's square?

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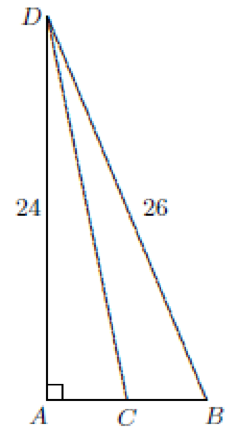
10. (I.82.4) A football field is a rectangle, 300 feet long (from goal to goal) and 160 feet wide (from sideline to sideline). To the nearest foot, how far is it from one corner of the field (on one of the goal lines) to the furthest corner of the field (on the other goal line)?



Problem 11

11. (I.81.7) In the figure at right, angle BAD is a right angle, and C is the *midpoint* of *line segment* AB (denoted $\overline{AB}$). Given the dimensions marked in the figure, find the length of CD.

12. It is sometimes useful to express a number in its *simplest radical form*. One process for simplifying $\sqrt{99}$ looks like this: $\sqrt{99} = \sqrt{3 \cdot 3 \cdot 11} = \sqrt{3 \cdot 3} \cdot \sqrt{11} = 3\sqrt{11}$. In other words, factor the number underneath the square root to see any factors that occur twice. A factor that occurs twice under the square root can be written once outside of the square root symbol (e.g. $\sqrt{3 \cdot 3} = 3$). The radical is fully simplified when there are no pairs of the same factor underneath the square root. Express each of the following in simplest radical form:
    a. $\sqrt{8}$ b. $7\sqrt{8}$ c. $\sqrt{72}$ d. $7\sqrt{72}$

13. (I.83.11) The expression $4x+3x$ can be combined into one term, but $4x+3y$ cannot. Explain. Can $4\sqrt{5} + 3\sqrt{5}$ be combined into one term? Can $\sqrt{2}+\sqrt{2}$ be combined into one term? Can $\sqrt{2}+\sqrt{3}$ be combined into one term? At first glance, it may seem that $\sqrt{2}+\sqrt{8}$ cannot be combined into one term. Take a close look at $\sqrt{8}$ and show that $\sqrt{2}+\sqrt{8}$ can in fact be combined.

14. (I.84.3) Because $\sqrt{8}$ can be rewritten as $2\sqrt{2}$, the expression $\sqrt{8}+5\sqrt{2}$ can be combined into a single term $7\sqrt{2}$. Combine each of the following into one term, without using a calculator:
    a. $\sqrt{12}+\sqrt{27}$ b. $\sqrt{63}-\sqrt{28}$ c. $\sqrt{6}+\sqrt{54}+\sqrt{150}$ d. $2\sqrt{20}-3\sqrt{45}$

15. (I.84.5) Solve each of the following for x. Express your answers in *exact form*.
    a. $x\sqrt{2}=\sqrt{18}$ b. $x\sqrt{6}=-\sqrt{30}$ c. $\sqrt{2x}=5$ d. $2\sqrt{5x}=\sqrt{30}$

16. (I.84.6) Show that $\sqrt{a+b}$ does not equal $\sqrt{a}+\sqrt{b}$ for all values of a and b by finding at least two sets of values for a and b that make the statement $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$ false.

17. (I.84.8) Given that $\sqrt{72}+\sqrt{50}-\sqrt{18}=\sqrt{h}$, find h without using a calculator.

18. (I.85.3) A sign going down a hill on Route 910 says "8% grade. Trucks use lower gear." The hill is a quarter of a mile long. How many vertical feet will a truck descend while going from the top of the hill to the bottom?

19. (I.85.6) Given that $\sqrt{k}=\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$, find the value of k without using a calculator.

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20. (I.86.2) Find $\sqrt{4+\frac{1}{16}}$ on your calculator. Is the result equivalent to $\sqrt{4}+\sqrt{\frac{1}{16}}$? Explain.

21. (I.86.10) What is the distance from the point (4, 2) to the point (−3, −2)? Be prepared to explain your method.

22. (I.87.1) Calculate the following distances, and briefly explain your method.
    a. from (2, 1) to (10, 10)
    b. from (-2, 3) to (7, -5)
    c. from (0, 0) to (9, 8)
    d. from (4, -3) to (-4, 6)

23. (I.87.7) The distance from (0, 0) to (8, 6) is exactly 10.
    a. Find coordinates for all the lattice points that are exactly 10 units from (0, 0). A *lattice point* is a point whose coordinates are integers.
    b. Find coordinates for all the lattice points that are exactly 10 units from (−2, 3).

24. A *circle* is the set of all points (x, y) that are a given distance, *r* (the radius of the circle), from a fixed point (h, k) (the center of the circle).
    a. Draw a diagram of the circle centered at the origin whose radius is 5. Plot a point P anywhere on the circle in Quadrant I with the coordinates (x, y). Use the distance formula to write an expression for the distance between (x, y) and the origin. Notice that this distance is the same as the radius of the circle, 5, so set your distance expression equal to 5.
    b. Square both sides of your equation from above to remove the square root. The resulting equation is an equation of the circle centered at $(0,0)$ with a radius of 5.
    c. Draw a diagram of the circle centered at $(1,3)$ whose radius is 7. Use the process described in parts a. and b. to write an equation satisfied by x and y for a point (x, y) on this circle.
    d. Draw a diagram of the circle centered at $(-2, -4)$ whose radius is 2. Write an equation satisfied by x and y for a point (x, y) on this circle.
    e. Write an equation satisfied by x and y for a point (x, y) on the circle whose center is (h, k) and radius is r.

25. (I.87.8) Find the distance between points (1,– 5) and (4, – 1). Now, generalize this procedure for computing the distance between any two points *(a, b)* and *(c, d)*, where *a, b, c* and *d* represent real numbers.

26. (I.87.4) Pat and Kim are having an algebra argument. Pat is sure that $\sqrt{x^2}$ is equivalent to x, but Kim thinks otherwise. How would you resolve this disagreement?

27. (II.57.11) Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is 13.

28. (I.88.3) Find the distance from
    a. $P = (3,1)$ to $Q = (x, 1)$
    b. $P = (3,1)$ to $Q = (x, y)$

29. (I.88.4) Complete the following statement without using any *variable* names: Given two points in a coordinate plane, you find the distance between them by _____________________.

30. Suppose that both legs of a right triangle are the same length. Such triangles are known as *45-45-90 triangles* and are a type of *special right triangle.*
    a. If the legs are both 8 cm long, how long is the hypotenuse (in simplest radical form)?
    b. How long would the hypotenuse be if both legs were k long, in terms of k?
    c. Write down the ratio of the lengths of the two legs to the length of the hypotenuse for any 45-45-90 triangle in the form *leg : leg : hypotenuse*.

31. Usually the two legs of a right triangle are not the same length. Suppose the hypotenuse of a right triangle is twice as long as the smaller leg. These triangles are called *30-60-90 triangles* and are also a type of *special right triangle.*
    a. If the smaller leg is 4 inches long, how long is the hypotenuse? In simplest radical form, what is the length of the longer leg?
    b. If the shorter leg has a length of m, what are the lengths of the hypotenuse and the longer leg in terms of m?
    c. Write down the ratio of the length of the shorter leg to the length of the longer leg to the length of the hypotenuse for any 30-60-90 triangle in the form *short leg : long leg : hypotenuse*.

32. (II.58.3) Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1, x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

33. Find the missing lengths for each special right triangle.
    a. A 45-45-90 triangle with a leg whose length is 7 feet.
    b. A 45-45-90 triangle whose hypotenuse is $5\sqrt{2}$ kilometers.
    c. A 30-60-90 triangle whose long leg is $12\sqrt{3}$ meters.
    d. A 30-60-90 triangle whose hypotenuse is 10 yards.

34. (I.88.7) Can you find integer lengths for the legs of a right triangle whose hypotenuse has length $\sqrt{5}$? What about $\sqrt{7}$? Explain your reasoning.

35. (II.13.9) The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle—try it. Hint: Find $(l + w)^2$.

36. (I.88.8) Find as many points as you can that are exactly 25 units from (0, 0). What is the *configuration* (the pattern formed when the points are plotted in the coordinate plane) of all such points? How many of them are lattice points?

37. (I.88.11) A triangle has $K = (3,1)$, $L = (-5,-3)$, and $M = (-8,3)$ for its vertices. Draw triangle KLM. Verify that the lengths of the sides of triangle *KLM* fit the Pythagorean equation.

38. (I.88.13) How far is the point (5, 5) from the origin? Find two other first-quadrant lattice points that are exactly the same distance from the origin as (5, 5) is.

39. (I.89.6) What is the y-intercept of the line $ax + by = c$? What is the x-intercept?

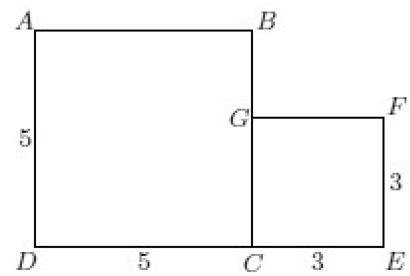
Math II

40. (I.89.1) At noon one day, AJ decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 7 miles east and 24 miles north of its point of departure. What was AJ's position at two o'clock? How far had AJ traveled? What was AJ's speed?

41. (I.89.2) (Continuation) Assume that the gas tank initially held 5 gallons of fuel, and that the boat gets 32.5 miles to the gallon. How far did AJ get before running out of fuel? When did this happen? How did AJ describe the boat's position to the Coast Guard when radioing for help?

42. (I.90.7) Give an example of a line that is parallel to $2x + 5y = 15$. Describe your line by means of an equation. Which form for your equation is most convenient? Now find an equation for a line that is *equidistant* from your line and the line $2x + 5y = 40$.

43. (II.1.1) A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square as follows. (See diagram to the right.)
    a. In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*.
    b. Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.
    c. Find the lengths of the sides of the third square.



Problem 43

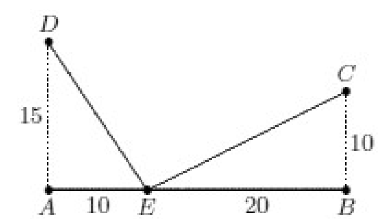
44. (II.1.2) (Continuation) Change the sizes of the squares to *AD* = 8 and *EF* = 4, and redraw the diagram. Where should point P be marked this time? Form the third square again.

45. (II.1.3) (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample—two specific squares that *cannot* be converted to a single square.

46. (II.1.5) Let *A* = (0, 0), *B* = (7, 1), *C* = (12, 6), and *D* = (5, 5). Plot these points and connect the dots to form the *quadrilateral ABCD*. As a reminder, if a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Verify that all four sides have the same length. Such a figure is called *equilateral*.

47. (II.1.8) In the diagram to the right, *AEB* is straight and angles *A* and *B* are right. Calculate the total distance *DE* + *EC*.

48. (II.1.9) (Continuation) If *AE* = 15 and *EB* = 15 instead, would *DE* + *EC* be the same?



Problems 47 and 48

49. (II.1.6) The main use of the Pythagorean Theorem is to find distances. Originally (6th century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

50. (II.2.1) Two different points on the line *y* = 2 are each exactly 13 units from the point (7, 14). Draw a picture of this situation, and then find the *coordinates* of these points.

Math II

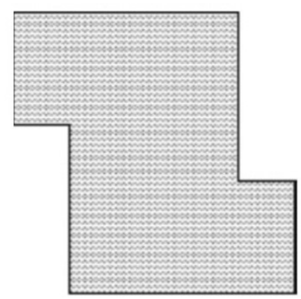
51. (II.57.11) Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is *r*.

52. (II.2.2) Give an example of a point that is the same distance from (3, 0) as it is from (7, 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

53. (II.2.3) Verify that the hexagon formed by $A = (0, 0)$, $B = (2, 1)$, $C = (3, 3)$, $D = (2, 5)$, $E = (0, 4)$, and $F = (-1, 2)$ is equilateral. Is it also *equiangular*?

54. (II.2.4) Draw a 20-by-20 square *ABCD*. Mark *P* on *AB* so that $AP = 8$, *Q* on *BC* so that $BQ = 5$, *R* on *CD* so that $CR = 8$, and *S* on *DA* so that $DS = 5$. Find the lengths of the sides of quadrilateral *PQRS*. Is there anything special about this quadrilateral? Explain.

55. (II.2.5) Verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$. It is customary to say that *P* is *equidistant* from *A* and *B*. Find three more points that are equidistant from *A* and *B*. By the way, to "find" a point means to find its coordinates. Can points equidistant from *A* and *B* be found in every *quadrant*?



Problem 56

56. (II.3.1) Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown to the right into two congruent parts. How many different ways of doing this can you find?

57. (II.2.8) If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition?

58. (II.2.9) Find both points on the line $y = 3$ that are 10 units from $(2, -3)$.

59. (II.2.10) Let p and q be two numbers. On a number line, where is $\frac{1}{2}(p+q)$ in relation to p and q? What is a good name for this point? Experiment with various values of p and q to see this relationship.

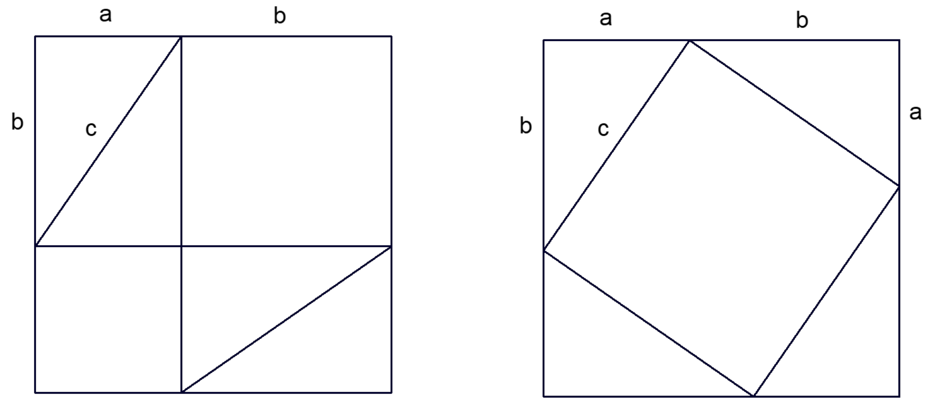
60. (II.3.6) Find two points on the *y*-axis that are 9 units from (7, 5).

61. (II.4.11) What is the relation between the lines described by the equations $-20x + 12y = 36$ and $-35x + 21y = 63$? Find a third equation in the form $ax + by = 90$ that fits this pattern.

62. (II.2.7) Inside a 5-by-5 square, it is possible to place four 3-4-5 triangles so that they do not overlap. Show how. Then explain why you can be sure that it is impossible to squeeze in a fifth triangle of the same size.

63. (II.3.2) Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles *ABC* and *TUV* are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

64. (II.2.6) The two-part diagram below, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.

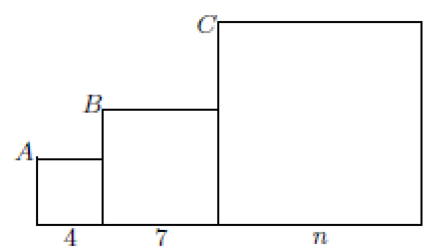


Problem 64

65. (II.3.3) A triangle that has two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose vertices is (3, 5). If you can, find a triangle that does not have any horizontal or vertical sides.

66. (II.3.5) Let $A = (1, 5)$ and $B = (3, -1)$. Verify that $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

67. (II.3.7) Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart? Is it possible to form a square whose area is 18 by connecting four lattice points? Explain.

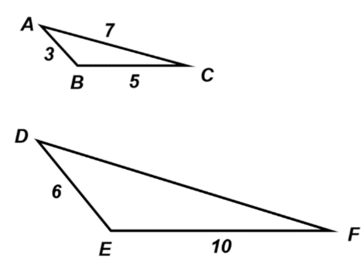


Problem 69

68. (II.5.9) A slope can be considered to be a *rate*. Explain this interpretation.

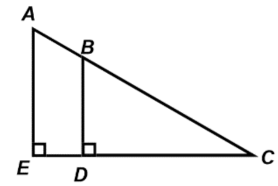
69. (II.6.1) Three squares are placed next to each other as shown in the diagram to the right. The vertices A, B, and C are collinear. Find the dimension n.

70. When geometric figures have the same shape and their *corresponding* side lengths are in proportion, the figures are called *similar*. In the diagram to the right, ΔABC is similar ΔDEF. When we divide the length of one side of ΔABC by the length of the corresponding side of ΔDEF, we get the same value, regardless of the set of corresponding sides. For example, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{1}{2}$. Find the length of $\overline{DF}$.



Problem 70

71. In the previous problem, we learned that triangles with proportional sides are similar. Another situation in which two triangles are similar is when their corresponding angles are congruent. In the figure to the right, ΔACE contains ΔBCD.
    a. Verify that the corresponding angles in ΔACE and ΔBCD are congruent and therefore that ΔACE is similar to ΔBCD.
    b. Because ΔACE is similar to ΔBCD, we know the corresponding sides of the two triangles are proportional. Write a proportion containing the lengths $AE, BD, DC,$ and EC.
    c. If $AE = 10$, $BD = 8$, and $DC = 16$, find EC.



Problem 71

72. (II.6.3) A five-foot freshman casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

73. (II.6.4) (Continuation) How far from the streetlight should the freshman stand, in order to cast a shadow that is exactly as long as the freshman is tall?

Some terminology: When two angles fit together to form a straight angle (a 180° angle), they are called *supplementary angles*, and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary angles*, and either angle is the *complement* of the other. Two lines whose intersection forms a right angle are said to be *perpendicular*.

74. (II.3.8) The three angles of a triangle fit together to form a straight angle. In one form or another, this statement is a fundamental *postulate* of *Euclidean geometry*—accepted as true, without proof. What can be said about the two non-right angles in a right triangle?

75. (II.3.9) Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where a and b are positive numbers. Prove that angle PQR is right, by introducing two congruent right triangles into your diagram. Verify that the slope of $\overline{QP}$ is the negative reciprocal of the slope of $\overline{QR}$.

76. (II.4.3) The point on segment AB that is equidistant from A and B is called the *midpoint* of AB. For each of the following, find coordinates for the midpoint of AB:
    a. $A = (-1, 5)$ and $B = (5, -7)$
    b. $A = (m, n)$ and $B = (k, l)$

77. (II.4.4) Write a formula for the distance from $A = (-1, 5)$ to $P = (x, y)$, and another formula for the distance from $P = (x, y)$ to $B = (7, 1)$. Set your two distance expressions equal to obtain an equation. This equation says that P is equidistant from A and B. Simplify your equation to linear form. This line is called the *perpendicular bisector of AB*. Verify this by calculating two slopes and one midpoint.

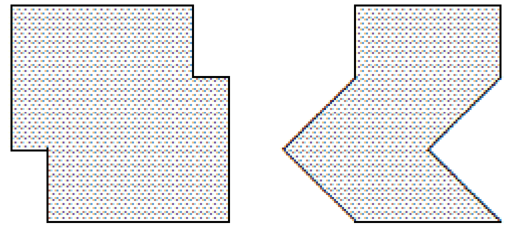
78. (II.5.11) Given the points $A = (-2, 7)$ and $B = (4, 3)$, find two points P that are on the perpendicular bisector of $\overline{AB}$. In each case, what can be said about the triangle PAB?

79. (II.5.12) Explain the difference between a line that has no slope and a line whose slope is zero.

80. (II.6.9) Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are *collinear*.

Math II

81. (II.6.10) Find as many ways as you can to dissect each figure to the right into two congruent parts.

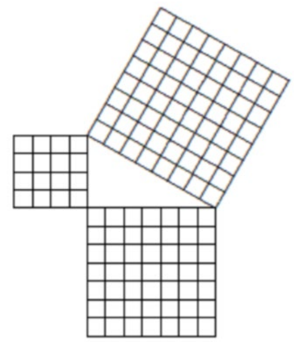


Problem 81

82. (II.7.6) Suppose that numbers a, b, and c fit the equation $a^2 + b^2 = c^2$, with $a = b$. Express c in terms of a. Draw a good picture of such a triangle. What can be said about its angles?

83. (II.6.11) Let $A = (4, 2)$, $B = (11, 6)$, $C = (7, 13)$, and $D = (0, 9)$. Show that $ABCD$ is a square. Calculate the slopes of the sides of the square, and notice any significant patterns.

84. (II.7.2) Is there anything wrong with the figure shown at right?



Problem 84

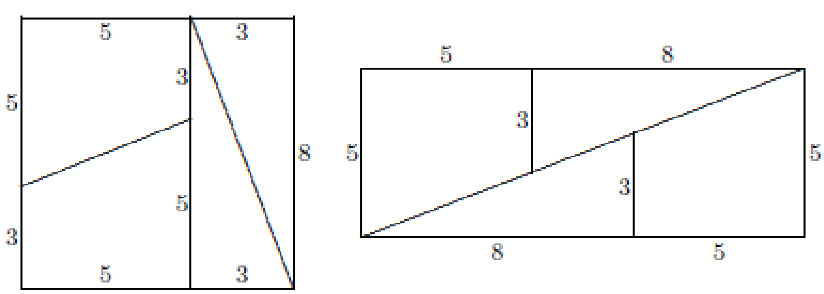
85. (II.7.4) At noon one day, Corey decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 6 miles east and 8 miles north of its point of departure. What was Corey's position at two o'clock? How far had Corey traveled? What was Corey's speed?

86. (II.7.5) (Continuation) Assume that the fuel tank initially held 12 gallons and that the boat gets 4 miles to the gallon. How far did Corey get before running out of fuel? When did this happen? When radioing the Coast Guard for help, how should Corey describe the boat's position?

87. (II.10.1) In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. How far is it from home plate to second base?

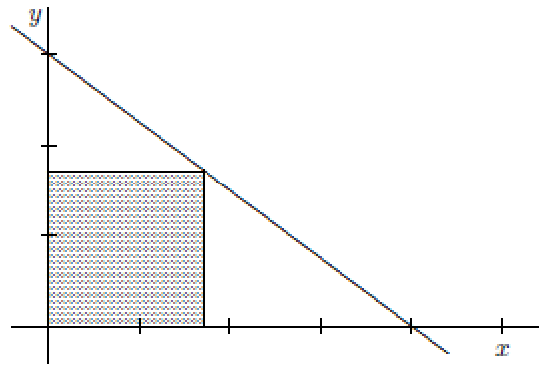
88. (II.10.7) An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

89. (II.10.9) Compare the two figures shown below. Is there anything wrong with what you see?



Problem 89

90. (II.13.4) The diagram at right shows the graph of $3x+4y=12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find
    a. the x-and y-intercepts of the line;
    b. the length of the side of the square.



Problem 90

91. (II.13.5) (Continuation) Draw a rectangle that is twice as wide as it is tall, and that fits snugly into the triangular region formed by the line $3x+4y=12$ and the positive coordinate axes, with one corner at the origin and the opposite corner on the line. Find the dimensions of this rectangle.

92. (II.13.6) Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. What is special about these points?

93. (II.13.7) Sidney calculated three distances and reported them as $PQ = 29$, $QR = 23$, and $PR = 54$. What do you think of Sidney's data, and why?

94. (II.14.3) Given the points $K = (-2, 1)$ and $M = (3, 4)$, find coordinates for a point J that makes angle JKM a right angle.

95. (II.14.4) When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles (called *vertical angles*) in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer?

96. (II.14.7) Find a point on the line $y = 2x-3$ that is 5 units from the x-axis.

97. (II.14.13) Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of k that makes the line AB a. parallel to PQ; b. perpendicular to PQ.

98. (II.14.14) Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the opposite sides of quadrilateral $ABCD$ are parallel. Such a quadrilateral is called a *parallelogram*.

99. (II.15.10) Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$. Find x-and y-values that make ABP a right angle.

100. (II.15.11) (Continuation) Describe the configuration of all such points P.

101. (II.20.4) A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
    a. Find an equation for the line that contains the median drawn from A to BC.
    b. Find an equation for the line that contains the median drawn from B to AC.
    c. Find coordinates for G, the intersection of the medians from A and B. Do this by solving the system of equations from parts (a) and (b).
    d. Let M be the midpoint of AB. Determine whether or not M, G, and C are collinear.

Math II

102. (II.19.7) How large a square can be put inside a right triangle whose legs are 5 cm and 12 cm?

103. (II.19.1) An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.
    a. Draw the altitude from C to side AB. Find the length of this altitude.
    b. Find the area of ΔABC.
    c. Find the slope of side BC and use this information to draw the altitude from A to side BC.
    d. Find an equation for the line containing the altitude from A to BC. Find an equation for the line containing side BC.
    e. Find coordinates for the point where the altitude from A meets side BC.
    f. Find the length of the altitude from A to side BC.
    g. Find the length of side BC and multiply it by your answer to part (f). Divide that answer by 2. Your answer should equal the area of the triangle. Why?

104. Fold down a corner of a rectangular sheet of paper as shown to the right. Then, fold the next corner so that the edges touch. Measure the angle formed by the folded lines. Repeat with another sheet of paper, folding the corner at a different angle. What do you notice? Explain why the angles formed are congruent.

Problem 104

105. (II.21.11) Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments AB and AC have the same length. Use a protractor to measure angles ABC and ACB. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length. Prove your assertion, which might be called the *Isosceles-Triangle Theorem*.

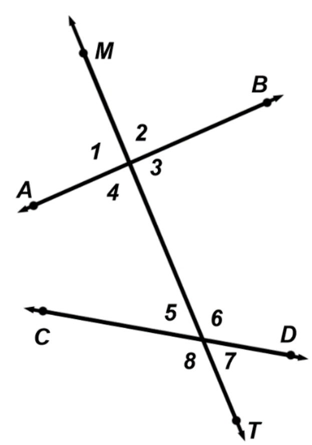
106. Let $P = (1,7)$, $Q = (3, 1)$, and $R = (8,6)$. Prove that angles RPQ and PQR have the same size.

107. Alex starts making a triangular wind chime out of a 12-cm long steel rod by bending the rod 4 cm from one end.
    a. Describe one place Alex could make the second bend to form the triangle. Explain how you know a triangle will be formed.
    b. Describe all the places Alex could make a second bend to form a triangle.

108. (II.27.10) Suppose that triangle PAB is isosceles, with $AP = PB$, and that C is on side PB, between P and B. Show that $CB < AC$.

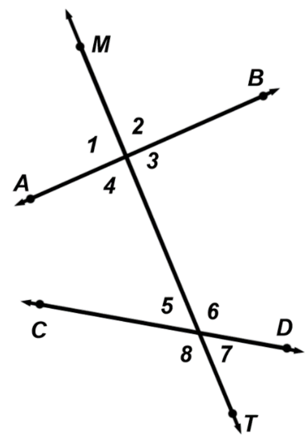
109. (II.71.12) Find an equation for the circle of radius 5 whose center is at $(3, -1)$.

110. The diagram at right shows lines AB and CD intersected by line MT, which is called a *transversal*. This configuration forms two intersections, each of which has four angles. There is special terminology to describe pairs of angles formed when a transversal intersects two lines. Name all pairs of vertical angles in the diagram.



Problems 110 - 113

111. (Continuation) If the angles are on different sides of the transversal and between the other two lines, they are called *alternate interior.* $\angle 4$ and $\angle 6$ are an example of *alternate interior* angles. Name the other pair of alternate interior angles.



Problems 110 - 113

112. (Continuation) $\angle 2$ and $\angle 8$ are called *alternate exterior angles* because they are on opposite sides of the transversal and are outside the other two lines. Name the other pair of alternate exterior angles.

113. (Continuation) $\angle 2$ and $\angle 6$ are called *corresponding angles* because they are in the same (corresponding) positions in at their respective intersections. Refer to the diagram and name the angles that
   a. correspond to $\angle 8$ b. correspond to $\angle 7$

114. (II.31.2) Mark points $A = (1, 7)$ and $B = (6, 4)$ on your graph paper. Use your protractor to draw two lines of positive slope that make 40-degree angles with line *AB*—one through *A* and one through *B*. What can you say about these two lines, and how can you be sure?

115. (II.31.3) In general, if two lines are crossed by a transversal, what can be said about those two lines if
   a. one pair of corresponding angles is equal? b. one pair of alternate interior angles is equal?

116. (II.31.4) If it is known that one pair of alternate interior angles is equal, what can be said about
   a. the other pair of alternate interior angles?
   b. either pair of alternate exterior angles?
   c. any pair of corresponding angles?
   d. either pair of non-alternate interior angles?

117. (II.31.5) You probably know that the sum of the angles of a triangle is a straight angle. One way to confirm this is to draw a line touching one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

118. (II.31.6) Suppose that for ΔABC and ΔPQR, $\angle A \cong \angle P$ and $\angle C \cong \angle R$. The symbol "$\cong$" means "is congruent to". What can be said about the third angles? Is it necessarily true that $\Delta ABC \cong \Delta PQR$?

119. (II.31.7) Suppose that *ABCD* is a square, and that *CDP* is an equilateral triangle, with *P outside* the square. What is the size of angle *PAD*?

120. (II.32.2) Triangle *ABC* is isosceles, with *AB* congruent to *AC*. *Extend* segment *BA* to a point *T* (in other words, *A* should be between *B* and *T*). Prove that angle *TAC* must be twice the size of angle *ABC*. Angle *TAC* is called one of the *exterior angles* of triangle *ABC*.

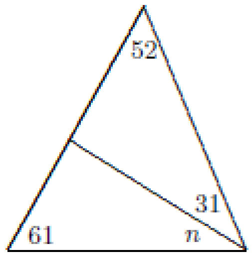
121. (II.32.3) If *ABC* is any triangle, and *TAC* is one of its exterior angles, then what can be said about the size of angle *TAC*, in relation to the other angles of the figure?

Math II

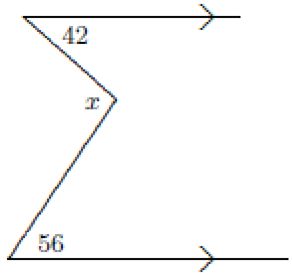
122. (II.32.5) Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?

123. (II.32.6) In the diagrams below, use the given numerical information to find the sizes of the angles marked with variables. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.

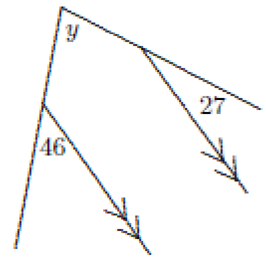
a.



b.

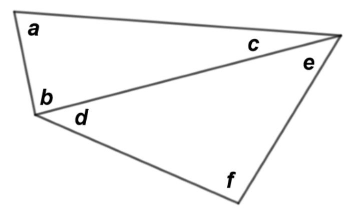


c.



124. (II.32.8) Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What can be said about the angles of such a figure?

125. Draw a quadrilateral that is not any of the types of quadrilaterals we've learned about. Draw one of the quadrilateral's diagonals so that you've made two triangles. Label the *interior angles* of one of the triangles *a, b,* and *c*, and label the interior angles of the other quadrilateral *d, e,* and *f.* (Your diagram should look something like the diagram to the right.) Use your diagram to prove that the sum of the interior angles of any quadrilateral is 360 degrees.



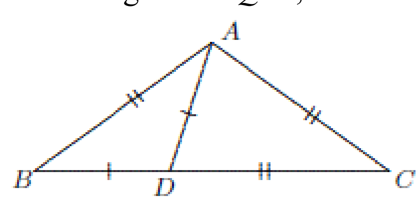
Problem 125

126. (Continuation) Use the method from the last problem to find the sum of the interior angles of a pentagon? a hexagon? a 57-sided polygon? an *n-gon*?

127. (II.73.10) Write an equation for the circle that is centered at $(-4, 5)$ and *tangent* to the x-axis. Find the area and circumference of this circle.

128. (II.33.2) Given parallelogram *PQRS*, let *T* be the intersection of the bisectors of angles *P* and *Q*. An *angle bisector* is a segment, ray, or line that passes through the vertex of an angle and splits the angle into two smaller, congruent angles. Without knowing the sizes of the angles of *PQRS*, calculate the size of angle *PTQ*.

129. (II.33.5) In the figure to the right, it is given that *BDC* is straight, $BD = DA$, and $AB = AC = DC$. Find the size of angle *C*.



Problem 129

130. (II.33.6) Mark the point *P* inside square *ABCD* that makes triangle *CDP* equilateral. Calculate the size of angle *PAD*.

131. (II.33.7) The *converse* of a statement of the form "If A, then B" is the statement "If B, then A."
    a. Write the converse of the statement "If point P is equidistant from the coordinate axes, then point P is on the line $y = x$."
    b. Give an example of a true statement whose converse is false.
    c. Give an example of a true statement whose converse is also true.

132. (II.33.8) If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? If you think that the converse is true, then prove it; if not, explain why not.

133. (II.34.1) In *regular* pentagon *ABCDE*, draw diagonal *AC*. What are the sizes of the angles of triangle *ABC*? Prove that segments *AC* and *DE* are parallel.

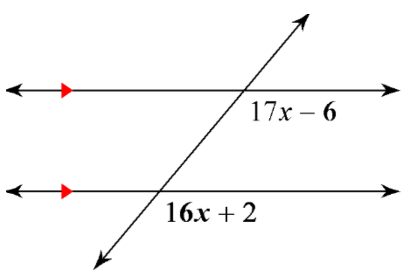
134. Given $A = (-3,10)$ and $B = (9,-2)$. Find the equation, in point-slope form, of the perpendicular bisector of segment AB.

135. (II.34.3) The sides of an equilateral triangle are 12 cm long. What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with its other two sides? How long is the altitude of this triangle?

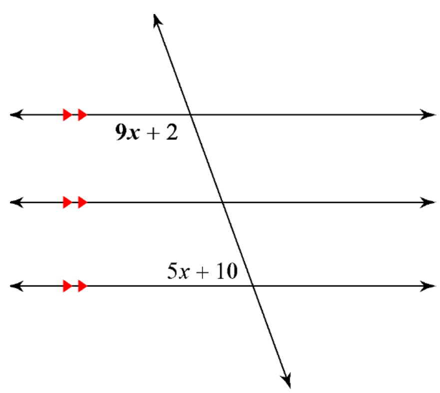
136. (II.34.6) In triangle *ABC*, it is given that angle *A* is 59 degrees and angle *B* is 53 degrees. The altitude from *B* to line *AC* is extended until it intersects the line through *A* that is parallel to segment *BC*; they meet at *K*. Calculate the size of angle *AKB*.

137. Solve for x in each diagram below.

a.



b.



138. (II.35.12) A right triangle has a 24-cm perimeter, and its hypotenuse is twice as long as its shorter *leg*. To the nearest tenth of a cm, find the lengths of all three sides of this triangle.

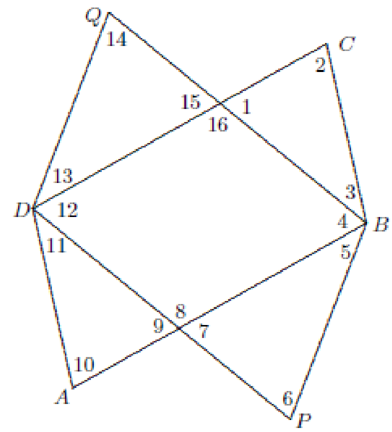
139. (II.34.13) Tate walks along the boundary of a four-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these four numbers?

140. (II.35.9) Jackie walks along the boundary of a five-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

141. (II.35.10) Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

142. (II.35.11) The preceding three questions illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

143. (II.37.2) In the figure to the right, it is given that *ABCD* and *PBQD* are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1?

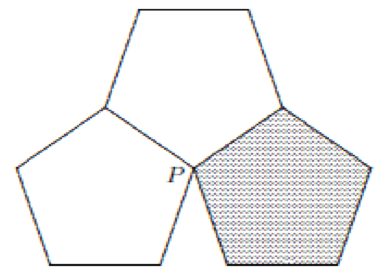


Problem 143

144. (II.39.3) There are four special types of lines associated with triangles: Medians, perpendicular bisectors, altitudes, and angle bisectors.
    a. Which of these lines *must* go through the vertices of the triangle?
    b. Is it possible for a median to also be an altitude? Explain.
    c. Is it possible for an altitude to also be an angle bisector? Explain.

145. (II.38.1) The diagram to the right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.

146. (II.38.2) (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the value of *n* that makes the three polygons fit perfectly.



Problems 145, 146

147. (II.40.8) Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

148. (II.42.6) How many diagonals can be drawn inside a pentagon? a hexagon? a *decagon*? A twenty-sided polygon? an *n*-sided polygon?

149. (II.44.7) Given regular hexagon *BAGELS*, show that *SEA* is an equilateral triangle.

150. (II.48.3) Hexagon *ABCDEF* is regular. Prove that segments *AE* and *ED* are perpendicular.

151. (II.48.4) Suppose that *PQRS* is a rhombus, with $PQ = 12$ and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

152. (II.53.9) Show that the altitude drawn to the hypotenuse of any right triangle divides the triangle into two triangles that have the same angles as the original.

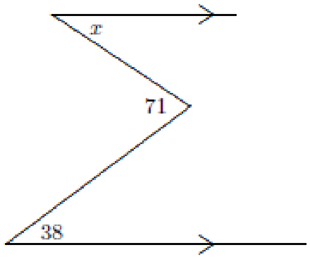
153. (II.32.4) Given triangle *ABC*, with $AB = AC$, extend segment *AB* to a point *P* so that $BP = BC$. In the resulting triangle *APC*, show that angle *ACP* is exactly three times the size of angle *APC*. (By the way, notice that extending segment *AB* does *not* mean the same thing as extending segment *BA*.)

154. (II.23.10) Give an example of an *equiangular* polygon that is not *equilateral*.

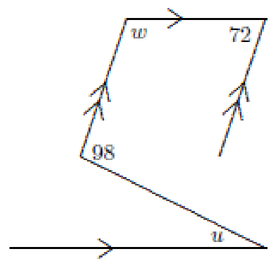
155. In general, one side of a triangle has to be shorter than the sum of the lengths of the other two sides. Explain why this is the case. If one side of a triangle equals the sum of the other two side lengths, this is known as a *degenerate triangle*.) Draw a diagram of what a degenerate triangle looks like. This result comes from the *triangle inequality*. Look this phrase up in the reference section, and use it to answer the following questions.
    a. Can a triangle have side lengths that are 7 cm, 11 cm, and 14 cm in length?
    b. If two sides of a triangle are 7 cm and 11 cm, what is the longest the third side could be? What is the shortest the third side could be?

156. (II.33.3) In the figures below, find the sizes of the angles indicated by variables.

a.



b.



157. (II.22.4) Triangle ABC is *isosceles*, with $AB = BC$, and angle BAC is 56 degrees. Find the remaining two angles of this triangle.

158. (II.22.5) Triangle ABC is isosceles, with $AB = BC$, and angle ABC is 56 degrees. Find the remaining two angles of this triangle. This is *not* the same problem as above.

159. (I.88.5) Both legs of a right triangle are 10 cm long. In simplest radical form, how long is the hypotenuse? How long would the hypotenuse be if both legs were x cm long?

160. (I.88.6) In ΔABC, $\angle C$ is a right angle, and $AB = 2BC$. Find the measures of $\angle B$ and $\angle A$.

161. (II.36.12) A regular, n-sided polygon has 18-degree exterior angles. Find the integer n.

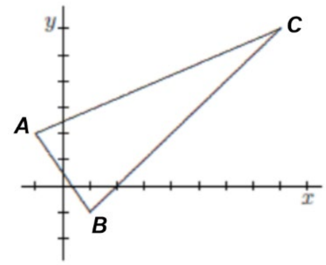
162. (I.3.6) The area of the surface of a sphere is described by the formula $S = 4\pi r^2$, where r is the radius of the sphere. The Earth has a radius of 3960 miles and dry land forms approximately 29.2% of the Earth's surface. What is the area of the dry land on Earth? What is the surface area of the Earth's water?

163. (II.15.4) Given $A = (-1, 5)$, $B = (x, 2)$, and $C = (4, -6)$, find the value of x that makes the path from A to C through B as short as possible.

164. (II.8.4) The perimeter of an isosceles right triangle is 24 cm. How long are its sides?

165. (SAT problem) If the length of the largest side of a right triangle is 10 cm and one of the angles is 60°, what is the length of the smallest side?

166. (II.4.7) The sides of the triangle at right are formed by the graphs of $3x + 2y = 1, y = x - 2,$ and $-4x + 9y = 22$. Verify the coordinates for each point of intersection. Is the triangle isosceles? How do you know?



Problem 166

167. (II.15.9) Let $A = (0, 0)$, $B = (4, 2)$, and $C = (1, 3)$. Find the size of angle *CAB*. Justify your answer.

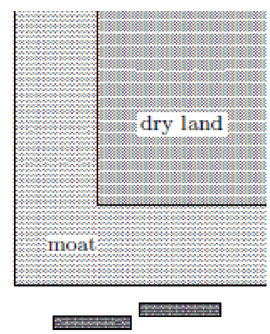
168. (II.41.12) The diagonals of a square have length 10. How long are the sides of the square?

169. (II.29.14) What do you call an a. *equiangular quadrilateral*? b. *equilateral quadrilateral*?

170. (II.40.7) Polygon *MNPQRSTUV* is a regular polygon. Draw a picture of this polygon. As a reminder, if a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. How large is each of the interior angles? If *MN* and *QP* are extended to meet at *A*, then how large is angle *PAN*?

171. (II.62.10) The vertices of a square with sides parallel to the coordinate axes lie on the circle of radius 5 whose center is at the origin. Find coordinates for the four vertices of this square.

172. (II.23.7) A castle is surrounded by a rectangular moat, which is of uniform width 12 feet. A corner is shown in the top view at right. The problem is to get across the moat to the dry land on the other side, without being able to use the drawbridge. All you have to work with are two rectangular planks (nothing else!), whose lengths are 11 feet and 11 feet, 9 inches. Find a way to arrange the planks so that you can get across the moat.



Problem 172

173. (II.41.5) The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

174. (II.36.13) Let $A = (0, 0)$, $B = (7, 2)$, $C = (8, 4)$, $D = (3, 7)$, and $E = (-1, 5)$. Cameron walks the polygonal path *ABCDEA*, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

175. (II.7.9) Draw the following segments on the same graph. What do these segments have in common?
a. from $(3, -1)$ to $(10, 3)$ b. from $(1.3, 0.8)$ to $(8.3, 4.8)$ c. from $(\pi, \sqrt{2})$ to $(7 + \pi,\ 4 + \sqrt{2})$

176. (II.7.10) (Continuation) The segments in the previous problem represent the same *vector* because they have the same length and the same direction. Each represents the vector $\langle 7,4 \rangle$. The *components* of the vector are the numbers 7 and 4, indicating a horizontal shift right 7 and a vertical shift up 4.
   a. Find another example of two points that represents this vector, indicating the starting and ending point as is done in the previous problem. The starting point is called the *tail* of the vector, and the ending point is called the *head*.
   b. Which of the following sets of points represents $\langle 7,4 \rangle$? from $(-2, -3)$ to $(5, -1)$; from $(-3, -2)$ to $(11, 6)$; from $(10, 5)$ to $(3, 1)$; from $(-7, -4)$ to $(0, 0)$.

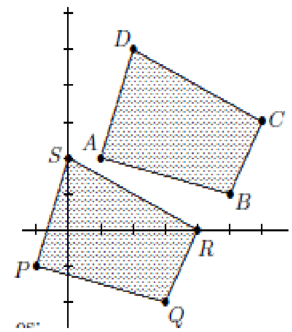
Math II

177. (II.11.1) Instead of saying that Cary moves *3 units left and 2 units up*, you can say that Cary's position is *displaced* by the vector $\langle -3, 2\rangle$. Identify the following displacement vectors for each scenario.
   a. Stacey starts at (2, 3) at 1 PM, and has moved to (5, 9) by 6 am.
   b. At noon, Eugene is at (3, 4); two hours earlier, Eugene was at (6, 2).
   c. During a single hour, a small airplane flew 40 miles north and 100 miles west.

178. (II.8.8) A triangle has vertices $A = (1, 2)$, $B = (3, -5)$, and $C = (6, 1)$. Triangle $A'B'C'$ is obtained by *sliding* triangle ABC 5 units to the right (in the positive x-direction, in other words) and 3 units up (in the positive y-direction). It is also customary to say that vector $\langle 5, 3\rangle$ has been used to *translate* triangle ABC. What are the coordinates of A', B', and C'? By the way, "A prime" is the usual way of reading A'.

179. (II.8.9) (Continuation) When vector $\langle h, k\rangle$ is used to translate triangle ABC, it is found that the *image* of vertex A is $(-3, 7)$. What is the vector $\langle h, k\rangle$? What are the images of vertices B and C? Label these B'' and C''.

180. (II.9.3) Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$, $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral $ABCD$ is related to quadrilateral $PQRS$.



Problem 180

181. (II.9.4) Let $K = (3, 8)$, $L = (7, 5)$, and $M = (4, 1)$. Find coordinates for the vertices of the triangle that is obtained by using the vector $\langle 2, -5\rangle$ to slide triangle KLM. What is the distance each vertex slides? This distance is known as the *magnitude*, or length, of the vector.

182. (II.16.3) If M is the *midpoint* of segment AB, how are the vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$ related?

183. (II.9.7) Let $A = (-5, 0)$, $B = (5, 0)$, and $C = (2, 6)$; let $K = (5, -2)$, $L = (13, 4)$, and $M = (7, 7)$. Verify that the length of each side of triangle ABC matches the length of a side of triangle KLM. Because of this data, it is natural to regard the triangles as being in some sense equivalent. It is customary to call the triangles *congruent*. The basis used for this judgment is called the *side-side-side* criterion. We write $\Delta ABC \cong \Delta KLM$ because of the side-side-side criterion.

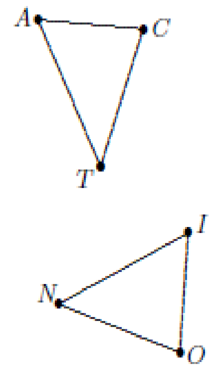
184. (II.11.6) (Continuation) Given two figures, *corresponding parts* are parts of the figures that can be matched to each other. In the triangle ABC, which angle corresponds to angle M? Which side corresponds to KL? It is always true that the corresponding parts of congruent figures are congruent. When talking about congruent triangles, we say that the ***C****orresponding* ***P****arts of* ***C****ongruent* ***T****riangles are* ***C****ongruent*. We abbreviate this fact as *CPCTC*. List all pairs of congruent parts for ΔABC and ΔKLM.

185. (II.49.1) The sides of a polygon are cyclically extended to form *rays*, creating one exterior angle at each vertex. Viewed from a great distance, what theorem does this figure illustrate?

Math II

186. (II.10.6) Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.

187. (II.16.6) In the diagram to the right, $\overline{AC} \cong \overline{IO}, \overline{CT} \cong \overline{ON}$, and $\overline{TA} \cong \overline{NI}$. Justify that this is enough information to prove that the two triangles are congruent. Write a congruence statement for the two triangles so that corresponding vertices are in corresponding positions in your statement.



Problems 187, 188

188. (II.16.7) (Continuation) How many ways are there of arranging the six letters of your congruence statement from the previous problem to express the two-triangle congruence?

189. (II.10.8) Using a ruler and protractor, draw a triangle that has an 8-cm side and a 6-cm side, which make a 30-degree angle. This is a *side-angle-side* description (SAS). Cut out the figure so that you can compare triangles with your classmates. Will your triangles be congruent?

190. (II.11.4) With the aid of a ruler and protractor, draw a triangle that has an 8-cm side, a 6-cm side, and a 45-degree angle that is not formed by the two given sides. This is a *side-side-angle* description (SSA). Cut out the figure so that you can compare triangles with your classmates. Do you expect your triangles to be congruent?

191. (II.12.5) With the aid of a ruler and protractor, draw and cut out three non-congruent triangles, each of which has a 40-degree angle, a 60-degree angle, and an 8-cm side. One of your triangles should have an *angle-side-angle* description (ASA), while the other two have *angle-angle-side* descriptions (AAS). What happens when you compare your triangles with those of your classmates?

192. (II.27.9) If the parts of two triangles are matched so that two angles of one triangle are congruent to the corresponding angles of the other, and so that a side of one triangle is congruent to the corresponding side of the other, then the triangles must be congruent. Justify this *angle-angle-corresponding side* (AAS) criterion for congruence. Would AAS be a valid test for congruence if the word *corresponding* were left out of the definition? Explain.

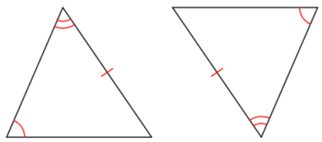
193. (II.27.3) With the aid of a ruler and protractor, draw a triangle that has an 8-cm side, a 6-cm side, and a 90-degree angle that is not formed by the two given sides. Cut out and compare your triangle with your classmates' triangles. Note that your triangle is a SSA construction; however, if the angle part of such a construction is a *right* angle, the criterion *is* reliable for proving congruent triangles.

194. (Continuation) What makes HL different from other SSA cases? Why does HL work for proving triangles are congruent?

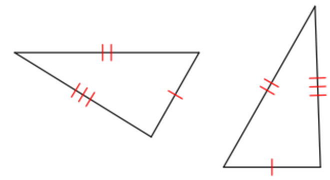
195. (II.16.8) What can be concluded about triangle ABC if it is given that
a. $\Delta ABC \cong \Delta ACB$?   b. $\Delta ABC \cong \Delta BCA$?

196. For each set of triangles below, determine whether or not there is enough information to conclude that the two triangles are congruent. If the triangles are congruent, state how you know (SSS, SAS, ASA, AAS, or HL).

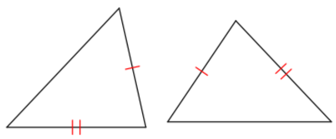
a.



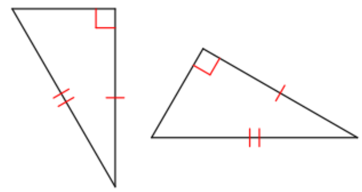
b.



c.



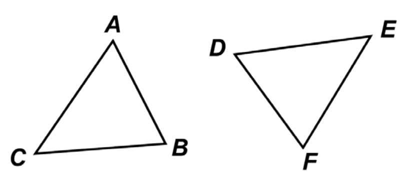
d.



197. Whenever a mathematician comes up with a new result, they must write a proof in order to justify to their colleagues that their findings are logically sound. A *two-column proof* is a type of proof that is often the first type of proof students learn because it helps in organizing the logic of an argument. A two-column proof consists of a sequence of statements in logical order, with every statement having a reason written next to it. This allows the proof writer to make sure that everything they are using to prove their claim has valid support. In any proof, the first statements describe what information is already known. Such statements are referred to as *given* because they are given to be true. From the given statements, it us up to the proof writer to use mathematical knowledge to build a logically sound argument that proves their claim. The claim that is being argued is always the last statement in a proof.

Look over the two-column proof below. Mark the given information on the provided diagram, and fill in the missing statements and reasons.

Given: $\overline{AB} \cong \overline{FD}, \angle A \cong \angle F$, and $\angle B \cong \angle D$
Prove: $\Delta ABC \cong \Delta FDE$



Statements	**Reasons**
1. $\overline{AB} \cong \overline{FD}$	1. Given
2. $\angle A \cong \angle F$	2. ?????
3. ???????	3. Given
4. $\Delta ABC \cong \Delta FDE$	4. ??????

198. (II.12.6) A triangle has six principal parts - three sides and three angles. If the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent? There are multiple answers.

199. (II.11.5) Plot points $K = (0, 0)$, $L = (7, -1)$, $M = (9, 3)$, $P = (6, 7)$, $Q = (10, 5)$, and $R = (1, 2)$. Show that the triangles KLM and RPQ are congruent. Write a two-column proof outlining your logic.

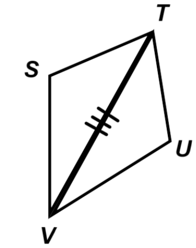
200. (II.9.8) (Continuation) Show also that neither triangle is a vector translation of the other. Describe how one triangle has been transformed into the other.

Math II

201. When proving that two triangles are congruent, sometimes a provided diagram contains information that is not marked or mentioned as given. One example is a diagram containing vertical angles. Another example is when the triangles share a side. In the latter case, we are able to say that the shared side is congruent to itself by the *reflexive property of congruence*. Each diagram below is of two triangles with a shared side or with vertical angles in common. For each diagram, mark the vertical angles or the shared side, and write a two-column proof to show that the triangles are congruent.

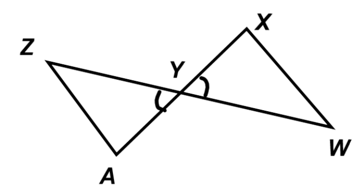
Reflexive Property:

$\overline{VT} \cong \overline{VT}$



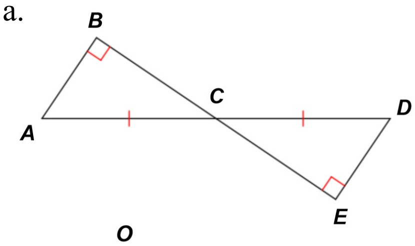
Vertical Angles Theorem:

$\angle ZYA \cong \angle WYX$

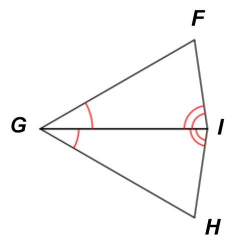


Problem 201

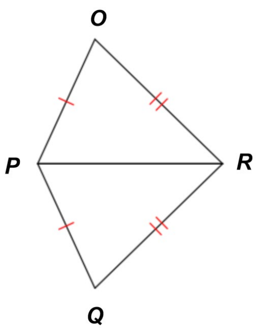
a.



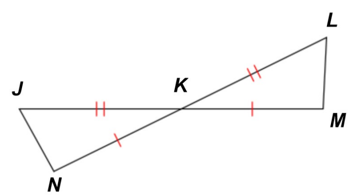
b.



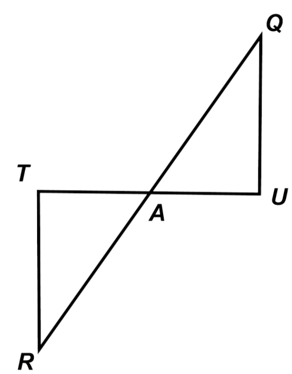
c.



d.



202. In the diagram to the right, $\angle T$ and $\angle U$ are right angles and $\overline{RA} \cong \overline{QA}$. Use CPCTC to prove that $\overline{RT} \cong \overline{QU}$. Write your proof in a two-column format.

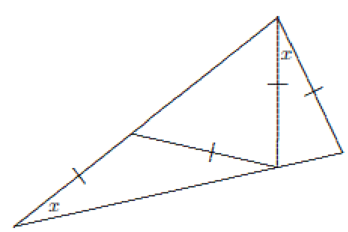


Problem 202

203. (II.13.1) Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles DAB and DCB are the same size. Can anything be said about the angles ABC and ADC?

204. (II.17.8) Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$, and $E = (1, -3)$. Show that angle CAB is the same size as angle EAD.

205. (II.49.9) In the figure at right, there are two x-degree angles, and four of the segments are congruent as marked. Find x.

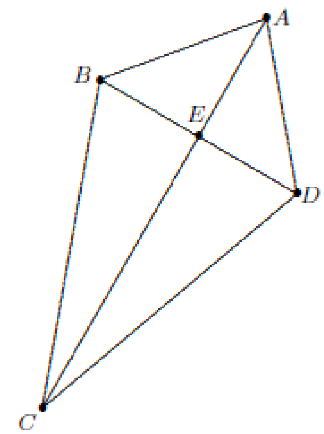


Problem 205

206. (II.19.3) Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles ABC and PQR have the same size.

207. (II.19.4) (Continuation) Let D be the point on segment AB that is exactly 3 units from B, and let T be the point on segment PQ that is exactly 3 units from Q. (Do your best to estimate the location of points B and T on your diagram.) What evidence can you give for the congruence of triangles BCD and QRT?

Here are two examples of proofs that do not use coordinates. Both proofs show how specific *given* information can be used to logically deduce *new* information. Each example concerns a *kite ABCD*, for which $AB = AD$ and $BC = DC$ is the given information. The first proof, which consists of simple text, shows that diagonal AC creates angles BAC and DAC of the same size.



Problems 208, 209, 210

Proof 1: Because $AB = AD$ and $BC = DC$, and because the segment AC is shared by the triangles ABC and ADC, it follows from the SSS criterion that these triangles are congruent. Thus it is safe to assume that all the corresponding parts of these triangles are congruent as well (often abbreviated to *CPCTC*, as in proof 2 below.) In particular, angles BAC and DAC are the same size. ■

Now let E mark the intersection of diagonals AC and BD. The second proof, which is an example of a two-column proof, is written symbolically in outline form. It shows that the diagonals intersect perpendicularly. This proof builds on the first proof, which thus reappears as the first five lines.

Proof 2:

Statements	Reasons
1. $AB = AD$	1. Given
2. $BC = DC$	2. Given
3. $AC = AC$	3. Shared side
4. $\Delta ABC \cong \Delta ADC$	4. SSS
5. $\angle BAC \cong \angle DAC$	5. CPCTC
6. E = intersection of AC and BD	6. By construction *(use this reason when you add something to the diagram)*
7. $AB = AD$	7. Given
8. $\angle BAE \cong \angle DAE$	8. Preceding CPCTC
9. $AE = AE$	9. Shared side
10. $\Delta ABE \cong \Delta ADE$	10. SAS
11. $\angle BEA \cong \angle DEA$	11. CPCTC
12. $\angle BEA$ and $\angle DEA$ supplementary	12. E is on BD
13. $\angle BEA$ is right	13. Definition of right angle

208. (II.18.1) In the fourth line, why would it have been wrong to write $\Delta ABC \cong \Delta ACD$?

209. (II.18.2) Refer to the kite data above and prove that angles ABC and ADC are also the same size.

210. (II.19.11) Refer to the data above and prove that one of the diagonals of a kite is bisected by the other.

Math II

211. (II.23.1) If triangle *ABC* is isosceles, with $AB = AC$, then the medians drawn from vertices *B* and *C* must have the same length. Write a *two-column proof* of this result.

212. (II.34.4) If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

213. (II.34.10) If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. Prove that this is so. What about the converse statement?

214. (II.34.14) How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion?

215. (II.38.11) Suppose that quadrilateral *ABCD* has the property that *AB* and *CD* are congruent and parallel. Is this enough information to prove that *ABCD* is a parallelogram? Explain.

216. (II.19.9) If a quadrilateral is equilateral, its diagonals are perpendicular. True or false? Prove your claim.

217. (II.39.4) The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

218. (II.48.4) Suppose that *PQRS* is a rhombus, with $PQ = 12$ and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

219. (II.19.10) The diagonals *AC* and *BD* of quadrilateral *ABCD* intersect at *O*. Given the information $AO = BO$ and $CO = DO$, what can you deduce about the lengths of the sides of the quadrilateral? Prove your response.

220. (II.23.8) Find *k* so that the vectors $\langle 4, -3\rangle$ and $\langle k, -6\rangle$
   a. point in the same direction;
   b. are perpendicular

221. (II.21.10) Prove that one of the diagonals of a kite bisects two of the angles of the kite. What about the other diagonal—must it also be an *angle bisector*? Explain your response.

222. (II.41.7) Triangle *ABC* has $AB = AC$. The bisector of angle *B* meets *AC* at *D*. Extend side *BC* to *E* so that $CE = CD$. Triangle *BDE* should look isosceles. Is it? Explain.

223. (II.14.9) A line goes through the points (2, 5) and (6, – 1). Let *P* be the point on this line that is closest to the origin. Calculate the coordinates of *P*.

224. (II.20.6) In quadrilateral *ABCD*, it is given that $AB = CD$ and $BC = DA$. Prove that angles *ACD* and *CAB* are the same size. As a reminder, if a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus *AC* should be one of the diagonals of *ABCD*.

225. (II.24.7) Find a vector that is perpendicular to the line $3x - 4y = 6$.

Math II

226. (II.21.8) Let $A = (1, 4)$, $B = (8, 0)$, and $C = (7, 8)$. Find the area of triangle ABC.

227. (II.11.10) Let $A = (3, 2)$ and $B = (7, -10)$. What is the displacement vector that moves point A onto point B? What vector moves B onto A?

228. (II.11.11) Show that the segment from (a, b) to (c, d) has the same length as the segment from $(a + h, b + k)$ to $(c + h, d + k)$.

229. (II.21.12) If the diagonals of a quadrilateral bisect each other, then any two nonadjacent sides of the figure must have the same length. Prove that this is so.

230. (II.29.10) The *sum* of two vectors $\langle a, b\rangle$ and $\langle p, q\rangle$ is defined as $\langle a+p, b+q\rangle$. Find the components of the vector $\langle 2,3\rangle + \langle -7,5\rangle$. Graph $\langle 2,3\rangle$, $\langle -7,5\rangle$, and their sum on the same set of axes.

231. (III.9.7) The *dot product* of vectors $\boldsymbol{u} = \langle a, b\rangle$ and $\boldsymbol{v} = \langle m, n\rangle$ is the number $\boldsymbol{u} \cdot \boldsymbol{v} = am + bn$. In general, the dot product of two vectors is the sum of all the products of corresponding components. For example, $\langle 2, -5\rangle \cdot \langle 1, 9\rangle = 2(1) + (-5)(9) = -43$. Let $\boldsymbol{u} = \langle -2,3\rangle, \boldsymbol{v} = \langle 0,4\rangle$, and $\boldsymbol{w} = \langle -5,1\rangle$. Calculate the following.
a. $\boldsymbol{u} \cdot \boldsymbol{v}$ b. $\boldsymbol{v} \cdot \boldsymbol{w}$ c. $\boldsymbol{w} \cdot \boldsymbol{v}$ d. $\boldsymbol{w} \cdot \boldsymbol{w}$ e. $\boldsymbol{u} \cdot (\boldsymbol{v} + \boldsymbol{w})$ f. $\boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \cdot \boldsymbol{w}$

232. Verify that $\langle 2, 4\rangle$ and $\langle -6,3\rangle$ are perpendicular. Now find the dot product of the two vectors. Do you think this is a coincidence? Why or why not?

233. (II.17.5) Given $A = (6, 1)$, $B = (1, 3)$, and $C = (4, 3)$, find a *lattice point* P that makes $\overrightarrow{CP}$ perpendicular to $\overrightarrow{AB}$.

234. (II.17.6) (Continuation) Describe the set of points P for which $\overrightarrow{AB}$ and $\overrightarrow{CP}$ are perpendicular.

235. (II.24.6) Given the points $A = (0, 0)$, $B = (7, 1)$, and $D = (3, 4)$, find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ABDC$ instead?

236. (II.22.9) Find the area of the triangle whose vertices are $A = (-2, 3)$, $B = (6, 7)$, and $C = (0, 6)$.

237. (II.22.6) Show that two vectors $\langle a, b\rangle$ and $\langle c, d\rangle$ are perpendicular if, and only if, $ac + bd = 0$. Recall that the number $ac + bd$ is called the *dot product* of the vectors s $\langle a, b\rangle$ and $\langle c, d\rangle$.

238. (II.11.9) Choose a point P on the line $2x + 3y = 7$ and draw the vector $\langle 2,3\rangle$ with its tail at P and its head at Q. Confirm that the vector is perpendicular to the line. What is the distance from Q to the line? Repeat the preceding, with a different choice for point P.

239. (II.12.3) *Some terminology*: When the components of the vector $\langle 5, -7\rangle$ are multiplied by a given number *t*, the result may be written either as $\langle 5t, -7t\rangle$ or as $t\langle 5, -7\rangle$. This is called the *scalar multiple* of vector $\langle 5, -7\rangle$ by the *scalar t*. Find components for the following scalar multiples:
   a. $\langle 12, -3\rangle$ by scalar 5
   b. $\langle \sqrt{5}, \sqrt{10}\, \rangle$ by scalar $\sqrt{5}$
   c. $\left\langle -\frac{3}{4}, \frac{2}{3} \right\rangle$ by scalar $-\frac{1}{2} + \frac{2}{6}$
   d. $\langle p, q\rangle$ by scalar $\frac{p}{q}$

240. (II.12.4) Find the magnitude (length) for each of the following vectors.
   a. $\langle 3,4\rangle$ b. $2019\langle 3,4\rangle$ c. $\frac{2019}{5}\langle 3,4\rangle$ d. $t\langle 3,4\rangle$ e. $t\langle a, b\rangle$

241. (II.12.2) Given the vector $\langle -5, 12\rangle$, find the following vectors.
   a. same direction, twice as long
   b. same direction, length 1
   c. opposite direction, length 10
   d. opposite direction, length *c*

242. (II.24.8) Measurements are made on quadrilaterals *ABCD* and *PQRS*, and it is found that angles *A*, *B*, and *C* are the same size as angles *P*, *Q*, and *R*, respectively, and that sides *AB* and *BC* are the same length as *PQ* and *QR*, respectively. Is this enough evidence to conclude that the quadrilaterals *ABCD* and *PQRS* are congruent? Explain.

243. (II.31.10) Write an equation that says that vectors $\langle a, b\rangle$ and $\langle m, n\rangle$ are perpendicular.

244. (II.25.8) In triangle *ABC*, it is given that *CA* = *CB*. Points *P* and *Q* are marked on segments *CA* and *CB*, respectively, so that angles *CBP* and *CAQ* are the same size. Prove that *CP* = *CQ*.

245. (II.25.9) (Continuation) Segments *BP* and *AQ* intersect at *K*. Explain why you can be sure that quadrilateral *CPKQ* is a kite. You might want to consider triangles *AKP* and *BKQ*.

246. (II.13.10) After drawing the line $y = 2x - 1$ and marking the point *A* = (−2, 7), Kendall is trying to decide which point on the line is closest to *A*. The point *P* = (3, 5) looks promising. Confirm that *P* really is on the line $y = 2x - 1$. Is *P* the point on the line that is the closest to *A*? Show why or why not.

247. (II.14.8) Find a point on the line $2x + y = 8$ that is *equidistant* from the coordinate axes. How many such points are there?

248. (II.15.8) Find a vector that translates the line $2x - 3y = 18$ onto the line $2x - 3y = 24$. (There is more than one correct answer.) Can you find the distance that separates these lines? Recall that unless otherwise specified, "distance" means the shortest distance.

249. (II.20.3) Let $A = (-2, 3)$, $B = (6, 7)$, and $C = (-1, 6)$.
   a. What is a *perpendicular bisector*?
   b. Find an equation for the perpendicular bisector of AB.
   c. Find an equation for the perpendicular bisector of BC.
   d. Find coordinates for a point K that is equidistant from A, B, and C. By the way, if you draw a triangle connecting points A, B and C, then K is the *circumcenter* of that triangle.

250. (II.25.6) Plot all points that are 3 units from the x-axis. Describe the configuration.

251. (II.25.7) Plot all points that are 3 units from the x-axis *and* 3 units from $(5, 4)$. How many did you find?

252. (II.26.7) Simplify the equation $\sqrt{(x-3)^2 + (y-5)^2} = \sqrt{(x-7)^2 + (y+1)^2}$. Interpret your result.

253. (II.20.8) A *direction vector* for a line is any vector that joins two points on that line. Find a direction vector for $2x + 5y = 8$. It is not certain that you and your classmates will get exactly the same answer. How should your answers be related, however?

254. (II.20.9) (Continuation) Show that $\langle b, -a \rangle$ is a direction vector for the line $ax + by = c$.

255. (II.20.10) (Continuation) Show that any direction vector for the line $ax + by = c$ must be perpendicular to $\langle a, b \rangle$.

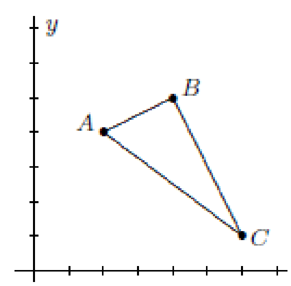
256. (I.50.2) Find values for a and b that make $ax + by = 14$ parallel to $12 - 3y = 4x$. Is there more than one answer? If so, how are the different values for a and b related?

257. (II.23.9) The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why, then find the distance that separates these lines. Recall that unless otherwise specified, "distance" means the shortest distance.

258. (II.28.12) Given: $(5, 2)$ is reflected onto $(-1, 4)$ across a line. What is the equation of this *line of reflection*?

259. (II.9.9) Let $A = (2, 4)$, $B = (4, 5)$, and $C = (6, 1)$. Triangle ABC is shown at right. Draw two new triangles as follows:
   a. ΔPQR has $P = (11, 1)$, $Q = (10, -1)$, and $R = (6, 1)$.
   b. ΔKLM has $K = (8, 10)$, $L = (7, 8)$, and $M = (11, 6)$.

   These triangles are not obtained from ABC by applying vector translations. Instead, each of the appropriate transformations is described by the suggestive names *reflection* or *rotation*. Decide which is which, with justification.

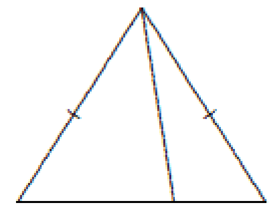


Problem 259

260. (II.25.10) A polygon that is both equilateral and equiangular is called regular. Prove that all diagonals of a regular *pentagon* (five sides) have the same length.

261. (II.16.9) Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle KLM is congruent to triangle XZY. Describe a *transformation* that transforms KLM onto XZY. Where does this transformation send the point $(-5, 0)$?

262. (II.26.12) Use the diagram to the right to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.



Problem 262

263. (II.17.9) Let $A = (-2, 4)$ and $B = (7, -2)$. Find the point Q on the line $y = 2$ that makes the total distance $AQ + BQ$ as small as possible.

264. (II.17.10) (Continuation) Let $A = (-2, 4)$ and $B = (7, 6)$. Find the point P on the line $y = 2$ that makes the total distance $AP + BP$ as small as possible.

265. (II.26.13) The diagonals of a kite are 6 cm and 12 cm long. Is it possible for the lengths of the sides of this kite to be in a 2-to-1 ratio?

266. (II.28.3) Find the area of the triangle having sides 10, 10, and 5.

267. (II.27.12) A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

268. (II.28.13) Draw a parallelogram whose adjacent edges are determined by vectors $\langle 2,5 \rangle$ and $\langle 8,0 \rangle$, placed so that they have a common initial point. This is called placing vectors *tail-to-tail*. Find the area of the parallelogram.

269. (II.28.13) (Continuation) Draw a parallelogram whose adjacent edges are determined by vectors $\langle 2,5 \rangle$ and $\langle 7,-1 \rangle$, placed tail-to-tail. Find the area of the parallelogram. What is the length of the altitude of the parallelogram?

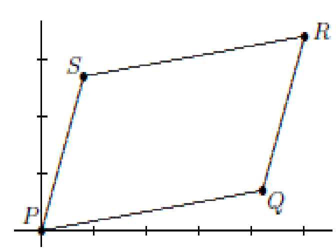
270. (II.26.5) Find coordinates for a point that is three times as far from the origin as $(2, 3)$ is. Describe the configuration of all such points.

271. (II.47.9) Suppose that $ABCD$ is a parallelogram, in which $AB = 2BC$. Let M be the midpoint of segment AB. Prove that segments CM and DM bisect angles BCD and CDA, respectively. What is the size of angle CMD? Justify your response.

272. (II.25.11) Find coordinates for the point equidistant from $(-1, 5)$, $(8, 2)$, and $(6, -2)$. Recall that this point is called the circumcenter.

273. (II.29.9) The diagonals of quadrilateral $ABCD$ intersect perpendicularly at O. What can be said about quadrilateral $ABCD$?

274. (II.25.2) The figure at right shows a parallelogram $PQRS$, three of whose vertices are $P = (0, 0)$, $Q = (a, b)$, and $S = (c, d)$.
    a. Find the coordinates of R.
    b. Find the area of $PQRS$, and simplify your formula.



Problem 274

Math II

275. (II.25.5) Find points on the line $3x + 5y = 15$ that are equidistant from the coordinate axes.

276. (II.30.6) In quadrilateral $ABCD$, it is given that $\overrightarrow{AB} = \overrightarrow{DC}$. What kind of a quadrilateral is $ABCD$? What can be said about the vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$?

277. (II.29.6) Find the area of a triangle formed by placing the vectors $\langle 3,6 \rangle$ and $\langle 7,1 \rangle$ tail-to-tail.

278. (II.23.3) Given points $A = (0, 0)$ and $B = (-2, 7)$, find coordinates for C and D so that $ABCD$ is a square.

279. (II.33.5) Given parallelogram $PQRS$, let T be the intersection of the bisectors of angles P and Q. Without knowing the sizes of the angles of $PQRS$, calculate the size of angle PTQ.

280. (II.39.5) A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is *isosceles*. Make a diagram of an isosceles trapezoid whose sides have lengths 7 in, 10 in, 19 in, and 10 in. Find the *altitude* of this trapezoid (the distance that separates the parallel sides). Now find the area of the trapezoid.

281. (II.34.11) Suppose that one of the *medians* of a triangle happens to be exactly half the length of the side to which it is drawn. Prove that the triangle must be a right triangle.

282. (II.34.12) (Continuation) Prove that the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices of the triangle. What is this point called? How does this statement relate to the preceding?

283. (II.26.1) Let $E = (2, 7)$ and $F = (10, 1)$. On the line through E and F, there are two points that are 3 units from E. Find coordinates for both of them.

284. (II.37.1) Draw a triangle ABC, and let AM and BN be two of its medians, which intersect at G. Extend AM to the point P that makes $GM = MP$. Prove that $PBGC$ is a parallelogram.

285. (II.37.4) Triangle PQR has a right angle at P. Let M be the midpoint of QR, and let F be the point where the altitude through P meets QR. Given that angle FPM is 18 degrees, find the sizes of angles Q and R.

286. (II.38.6) Suppose that triangle ABC has a right angle at B, that BF is the altitude drawn from B to AC, and that BN is the median drawn from B to AC. Find angles ANB and NBF, given that
    a. angle C is 42 degrees;
    b. angle C is 48 degrees.

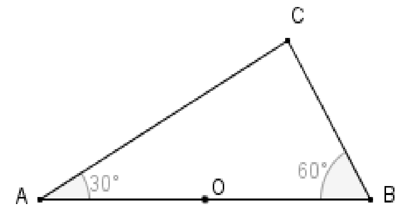
287. (II.39.12) A trapezoid has a 60-degree angle and a 45-degree angle. What are the other angles?

288. (II.39.13) A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles?

289. (II.26.14) Translate the line $5x + 7y = 35$ by vector $\langle 3,10 \rangle$. Find an equation for the new line.

Math II

290. (II.63.5) As shown in the diagram at right, triangle ABC has a 30-degree angle at A and a 60-degree angle at B. Let O be the midpoint of AB.
    a. Draw the *circle* centered at O that goes through A.
    b. Explain why this circle also goes through B and C.
    c. Draw angle BOC and find its measure.



Problems 290 and 291

291. (Continuation) BOC is called a *central angle* of the circle because its vertex is at the center. Angle BOC and *minor arc BC* correspond to the same section of the circle, so we say that minor arc BC has the same *angular size* as angle BOC.
    a. What is the angular size of minor arc BC?
    b. What is the angular size of *major arc BAC*?
    c. What is the angular size of minor arc AC?
    d. What is the angular size of major arc ABC?
    e. How does the actual length of minor arc AC compare to the length of minor arc BC?

292. (From Dr. Sutula) A plane (flat, 2D surface) 6 cm from the center of a sphere intersects the sphere in a circle with diameter 16 cm. Find the radius and the volume of the sphere.

293. (II.40.2) Trapezoid $ABCD$ has parallel sides AB and CD, a right angle at D, and the lengths $AB = 15$, $BC = 10$, and $CD = 7$. Find the length DA.

294. (II.40.6) What can be said about quadrilateral $ABCD$, if it has supplementary adjacent angles?

295. (II.28.11) When translation by vector $\langle 2,5 \rangle$ is followed by translation by vector $\langle 5,7 \rangle$, the net result can be achieved by applying a *single* translation; what is its vector?

296. (II.41.7) If $ABCD$ is a quadrilateral, and BD bisects both angle ABC and angle CDA, then what sort of quadrilateral must $ABCD$ be?

297. (II.40.10) Prove that an isosceles trapezoid must have two pairs of equal adjacent angles.

298. (II.40.11) (Continuation) The *converse* question: If a trapezoid has two pairs of equal adjacent angles, is it necessary that its non-parallel sides have the same length? Explain.

299. (II.39.11) The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other? Explain.

300. (II.41.6) It is given that the sides of an isosceles trapezoid have lengths 3 in, 15 in, 21 in, and 15 in. Make a diagram. Show that the diagonals intersect perpendicularly.

301. (II.29.13) Plot points $A = (1, 2)$, $B = (-4, 1)$, and $C = (-2, 3)$. Calculate components for the vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{AB} + \overrightarrow{AC}$, then translate point A by vector $\overrightarrow{AB} + \overrightarrow{AC}$. Call the new point D. What kind of quadrilateral is $ABDC$? (Note that this is not the same as $ABCD$.)

Math II

302. (II.30.5) (Continuation) Simplify the sum of vectors $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$.

303. (II.41.8) In quadrilateral *ABCD*, angles *ABC* and *CDA* are both bisected by *BD*, and angles *DAB* and *BCD* are both bisected by *AC*. What sort of quadrilateral must *ABCD* be?

304. (II.46.5) Let $P = (-15, 0)$, $Q = (5, 0)$, $R = (8, 21)$, and $S = (0, 15)$. Draw quadrilateral *PQRS* and measure its sides and angles. Pat and Kim are arguing about this shape. Pat claims that *PQRS* must be a parallelogram because it contains a pair of congruent opposite angles and a pair of congruent opposite sides. Kim says, sure, it has those properties but that does not necessarily mean that *PQRS* is a parallelogram. Settle their argument.

305. (III.11.10) Given a vector $\boldsymbol{u} = \langle a, b \rangle$, the notation $|\boldsymbol{u}|$ is often used for its magnitude (length). This notation makes sense because absolute value indicates distance.
    a. Find $|\boldsymbol{u}|$ if $\boldsymbol{u} = \langle -3,4 \rangle$.
    b. Find $|\boldsymbol{u}|$ if $\boldsymbol{u} = \langle 1,1 \rangle$.
    c. Find $|\boldsymbol{u}|$ if $\boldsymbol{u} = \langle a, b \rangle$.
    d. Show that $\boldsymbol{u} \cdot \boldsymbol{u} = |\boldsymbol{u}|^2$. What theorem does this recall?

306. (II.46.3) Suppose that a quadrilateral is measured and found to have a pair of equal nonadjacent sides and a pair of equal nonadjacent angles. Is this enough evidence to conclude that the quadrilateral is a parallelogram? Explain.

307. (II.42.4) Is it possible for the diagonals of a parallelogram to have the same length? How about the diagonals of a trapezoid? How about the diagonals of a non-isosceles trapezoid?

308. (II.32.9) Let *ABCD* be a parallelogram.
    a. Express $\overrightarrow{AC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
    b. Express $\overrightarrow{AC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.
    c. Express $\overrightarrow{BD}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

309. (II.28.10) Let $P = (2,7)$, $B = (6, 11)$, and $M = (5, 2)$. Find a point *D* that makes $\overrightarrow{PB} = \overrightarrow{DM}$. What can you say about quadrilateral *PBMD*?

310. (II.64.8) Suppose that *MP* is a diameter of a circle centered at *O*, and *Q* is any other point on the circle. Draw the line through *O* that is parallel to *MQ*, and let *R* be the point where it meets minor arc *PQ*. Prove that *R* is the midpoint of minor arc *PQ*.

311. (II.49.3) Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

312. (II.30.7) Mark the lattice point $(6, -1)$ on your graph paper. Define vector $\boldsymbol{u}$ (which can be handwritten $\vec{u}$) by moving 5 units to the right and 2 units up. Define vector $\boldsymbol{v}$ by moving 1 unit to the right and 3 units down. Diagram the vectors $\boldsymbol{u} + \boldsymbol{v}$, $\boldsymbol{u} - \boldsymbol{v}$, and $2\boldsymbol{u} - 3\boldsymbol{v}$.

313. (II.30.8) Draw a parallelogram. Choose one of its vertices and let $\boldsymbol{u}$ and $\boldsymbol{v}$ be the vectors defined by the sides that originate at that vertex. Draw $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$. The vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ represent the sides of the parallelogram; what do $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ represent?

314. (III.18.11) For the vectors $\boldsymbol{u} = \langle 3,4\rangle$ and $\boldsymbol{v} = \langle 5,12\rangle$,
   a. Draw these vectors so that the tail of $\boldsymbol{v}$ is at the head of $\boldsymbol{u}$.
   b. Find $\boldsymbol{u} + \boldsymbol{v}$ and draw it so that $\boldsymbol{u}$ and $\boldsymbol{u} + \boldsymbol{v}$ have a common initial point (tail-to-tail).
   c. Calculate $|\boldsymbol{u}|$, $|\boldsymbol{v}|$, and $|\boldsymbol{u} + \boldsymbol{v}|$, and verify that $|\boldsymbol{u} + \boldsymbol{v}| \leq |\boldsymbol{u}| + |\boldsymbol{v}|$.
   d. This result is known as *the Triangle Inequality*. Using the reference section as a guide, describe what the triangle inequality means in your own words.

315. (II.41.3) A parallelogram has two 19-inch sides and two 23-inch sides. What is the range of possible lengths for the diagonals of this parallelogram?

316. (II.36.7) Draw a triangle *ABC* with vertex A at the top, and label vertices B and C clockwise. Let *M* and *N* be the midpoints of sides *AB* and *AC*, respectively, and label these on the triangle.
   a. Express $\overrightarrow{BC}$ in terms of $\overrightarrow{BA}$ and $\overrightarrow{AC}$.
   b. Express $\overrightarrow{MN}$ in terms of $\overrightarrow{BA}$ and $\overrightarrow{AC}$.
   c. Use this information to describe how segments *MN* and *BC* are related to each other.
   d. This result is given by the *Midline Theorem*. In your own words, what does the Midline Theorem say?

317. (II.41.4) Is it possible for a trapezoid to have sides of lengths 3, 7, 5 and 11?

318. (II.66.6) Triangle *ABC* has a 53-degree angle at *A*, and its circumcenter is at *K*.
   a. Draw a good picture of this triangle, and describe the process you used to find *K*. Then draw the circle centered at *K* that goes through *A*, *B*, and *C*.
   b. With your protractor, measure the size of angle *BKC*. What is the angular size of arc *BC*?
   c. Measure the angles *B* and *AKC* of your triangle. What is the angular size of arc *AC*?
   d. Using these measurements, find the angular size of *C*, *AKB* and arc *AB*.
   e. Angles *A*, *B* and *C* are *inscribed angles* because all three points are on the circle. Make a conjecture about arcs intercepted by inscribed angles.

319. (II.46.12) What is the radius of the smallest circle that encloses an equilateral triangle with 12-inch sides? What is the radius of the largest circle that will fit inside the same triangle?

320. Suppose *M* and *N* are the respective midpoints of sides *AB* and *AC* of ΔABC. If *MN* = 4, what is the length of segment *BC*? What can be said about angles *AMN* and *ABC*? Justify.

321. (II.43.9) Draw an acute-angled triangle *ABC*, and mark points *P* and *Q* on sides *AB* and *AC*, respectively, so that *AB* = 3*AP* and *AC* = 3*AQ*. Express $\overrightarrow{PQ}$ and $\overrightarrow{BC}$ in terms of $\overrightarrow{PA}$ and $\overrightarrow{AQ}$. How are segments *PQ* and *BC* related to each other? This result is known as the *Triangle Proportionality Theorem*. Write down the Triangle Proportionality Theorem in your own words.

322. (II.46.10) Let *RICK* be a parallelogram, with *M* the midpoint of *RI*. Draw the line through *R* that is parallel to *MC* so that it meets the extension of *IC* at *P*. Prove that *CP* = *KR*.

Math II

323. (II.60.7) What is the radius of the smallest circle that surrounds a 5-by-12 rectangle?

324. (II.42.11) In triangle *TOM*, let *P* be the midpoint of segment *TO* and let *Q* be the midpoint of segment *TM*. Draw the line through *P* parallel to segment *TM*, and the line through *Q* parallel to segment *TO*; these lines intersect at *J*. What can you say about the location of point *J*?

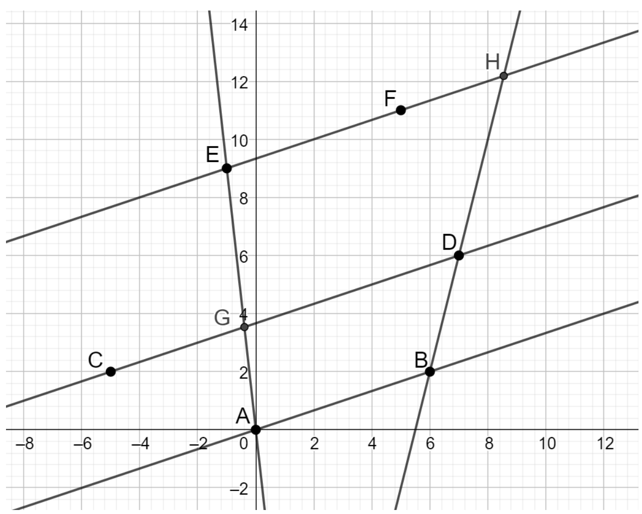
325. (II.38.9) The midpoints of the sides of a triangle are $(3, -1)$, $(4, 3)$, and $(0, 5)$. Find coordinates for the vertices of the triangle.

326. (II.43.8) A line drawn parallel to the side *BC* of triangle *ABC* intersects side *AB* at *P* and side *AC* at *Q*. The measurements $AP = 3.8$ in, $PB = 7.6$ in, and $AQ = 5.6$ in are made. If segment *QC* were now measured, how long would it be?

327. (II.43.4) Find coordinates for a point that is 5 units from the line $3x + 4y = 10$.

328. (III.1.2) In mathematical discussion, a *right prism* is defined to be a solid figure that has two parallel, congruent polygonal bases, and rectangular *lateral faces*. How would you find the volume of such a figure? Explain your method.

329. (II.42.9) Let $A = (0, 0)$, $B = (6, 2)$, $C = (-5, 2)$, $D = (7, 6)$, $E = (-1, 9)$, and $F = (5, 11)$. These points have been plotted below with lines *AB*, *CD*, and *EF*.



Problem 329

a. Verify that lines *AB*, *CD*, and *EF* are parallel.
b. Use your ruler to measure *EG* and *GA*.
c. Use your ruler to measure *HD* and *DB*.
d. Compare the ratios *GA*:*EG* and *DB*:*HD*.
e. This result is known as the *Three Parallels Theorem*. Using your findings from above and the reference as a guide, write down the Three Parallels Theorem in your own words.

Math II

330. (II.44.11) Segments AC and BD intersect at E, so as to make AE twice EC and BE twice ED. Prove that segment AB is twice as long as segment CD, and parallel to it.

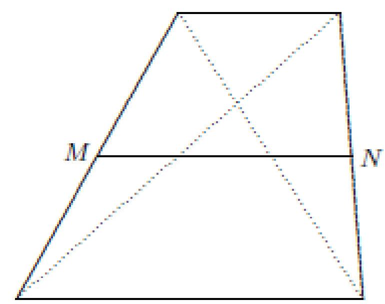
331. (II.50.1) In triangle ABC, points M and N are marked on sides AB and AC, respectively, so that $AM{:}AB = 17{:}100 = AN{:}AC$. Show that segments MN and BC are parallel.

332. (II.50.2) (Continuation) In triangle ABC, points M and N are marked on sides AB and AC, respectively, so that the ratios $AM{:}AB$ and $AN{:}AC$ are both equal to r, where r is some number between 0 and 1. Show that segments MN and BC are parallel. Note that triangles ABC and AMN are similar. By the way, r is known as the *constant of proportionality*.

333. (II.42.10) In triangle ABC, let M be the midpoint of AB and N be the midpoint of AC. Suppose that you measure MN and find it to be 7.3 cm long. How long would BC be, if you measured it? If you were to measure angles AMN and ABC, what would you find?

334. (III.1.7) In the middle of the nineteenth century, octagonal barns and sheds (and even some houses) became popular. How many cubic feet of grain would an octagonal barn hold if it were 12 feet tall and had a regular base with 10-foot edges?

335. (II.47.10) If M and N are the midpoints of the non-parallel sides of a trapezoid, it makes sense to call segment MN the *midline* of the trapezoid. Why? (It is typically called the *midsegment*, and some textbooks call it the *median*). Suppose that the parallel sides of a trapezoid have lengths 7 and 15. What is the length of the midline of the trapezoid? What are the lengths of the three pieces into which the midline is divided by the points where it intersects the diagonals of the trapezoid?



Problem 335

336. (II.44.8) Let $A = (0, 0)$, $B = (0, 21)$, and $C = (28, 0)$. Let F be the point where the bisector of angle BAC meets side BC. Find exact coordinates for F. Notice that F is not the midpoint of BC. Finally, calculate the distances BF and CF. Which of the two is larger and why do you think that is?

337. (II.44.9) (Continuation) Show that $AB{:}BF = AC{:}CF$. This is the *Angle Bisector Theorem.* Express the Angle Bisector Theorem in your own words. Find another equivalent statement of the theorem using these side lengths.

338. (II.44.10) Choose four lattice points for the vertices of a non-isosceles trapezoid $ABCD$, with AB longer than CD and parallel to CD. Extend AD and BC until they meet at E. According to the Triangle Proportionality Theorem, what ratios will be equal? Verify that this is the case.

339. (II.39.1) Suppose that square $PQRS$ has 15-cm sides, and that G and H are on QR and PQ, respectively, so that PH and QG are both 8 cm long. Let T be the point where PG meets SH. Find the size of angle STG, with justification.

340. (II.39.2) (Continuation) Find the lengths of PG and PT.

Math II

341. (II.46.11) Suppose that *ABCD* is a trapezoid, with *AB* parallel to *CD*. Let *M* and *N* be the midpoints of *DA* and *BC*, respectively. What can be said about segment *MN*? Explain.

342. (II.48.14) The diagonals of a non-isosceles trapezoid divide the midline into three segments, whose lengths are 8 cm, 3 cm, and 8 cm. How long are the parallel sides? From this information, is it possible to infer anything about the distance that separates the parallel sides? Explain.

343. (II.66.9) Draw a circle with a 2-inch radius, mark four points randomly (not evenly spaced) on it, and label them consecutively *G*, *E*, *O*, and *M*. Measure angles *GEO* and *GMO*. Could you have predicted the result? Name another pair of angles that would have produced the same result.

344. (II.49.7) In trapezoid *ABCD*, *AB* is parallel to *CD*, $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points *P* and *Q* are marked on *BC* so that $BP = PQ = QC = 3$, and points *R* and *S* are marked on *DA* so that $DR = RS = SA = 5$. Find the lengths *PS* and *QR*.

345. (II.46.7) Given triangle *ABC*, let *F* be the point where segment *BC* meets the bisector of angle *BAC*. Draw the line through *B* that is parallel to segment *AF*, and let *E* be the point where this parallel meets the extension of segment *CA*.
    a. Find the four congruent angles in your diagram.
    b. How are the lengths *EA*, *AC*, *BF*, and *FC* related?
    c. Use the above information to prove the *Angle Bisector Theorem.*

346. (II.65.3) On a circle whose center is *O*, mark points *P* and *A* so that minor arc *PA* is a 46-degree arc. What does this tell you about angle *POA*? Next, extend *PO* until it hits the circle. Call the point where this extension meets the circle *T*. Find angle *AOT*. Use this to find the size of angle *PTA*.

347. (II.65.4) (Continuation) If minor arc *PA* is a *k*-degree arc, what is the size of angle *PTA*? Justify your answer.

348. (II.49.8) *The Varignon quadrilateral*. A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?

349. (II.49.10) The parallel bases of a trapezoid have lengths 12 cm and 18 cm. Find the lengths of the two segments into which the midline of the trapezoid is divided by a diagonal.

350. (II.47.1) In acute triangle *ABC*, the bisector of angle *ABC* meets side *AC* at *D*. Mark points *P* and *Q* on sides *BA* and *BC*, respectively, so that segment *DP* is perpendicular to *BA* and segment *DQ* is perpendicular to *BC*. Prove that triangles *BDP* and *BDQ* are congruent. What about triangles *PAD* and *QCD*?

351. (SAT Problem) If ΔABC is an isosceles triangle with $\angle A \cong \angle B$, a height of 15, and base *AB* of length 16, find the perimeter of the triangle.

352. (II.63.2) Draw a circle and label one of its diameters *AB*. Choose any other point on the circle and call it *C*. What can you say about the size of angle *ACB*? Does it depend on which *C* you chose? Justify your response.

Math II

353. (II.13.8) Find the number that is two thirds of the way **(a)** from -7 to 17; **(b)** from m to n.

354. (II.14.1) The components of vector $\langle 24,7 \rangle$ are 24 and 7. Find the components of a vector that is three fifths as long as $\langle 24,7 \rangle$.

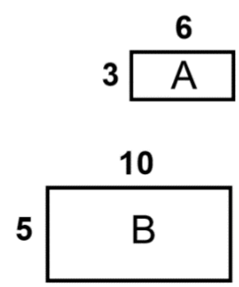
355. (III.2.8) The Great Pyramid at Gizeh was originally 483 feet tall, and it had a square base that was 756 feet on a side. It was built from rectangular stone blocks measuring 7 feet by 7 feet by 14 feet. Such a block weighs seventy tons. Approximately how many tons of stone were used to build the Great Pyramid? The volume of a pyramid is one third the base area times the height.

356. (II.48.5) A triangle, whose sides are 6, 8, and 10, and a circle, whose radius is r, are drawn so that no part of the triangle lies outside the circle. How small can r be?

357. (II.58.10) The vertices of triangle ABC are $A = (-5, -12)$, $B = (5, -12)$, and $C = (5, 12)$. Confirm that the circumcenter of ABC lies at the origin. What is an equation for the *circumcircle*? Find the area of this circumcircle, and also prove that angle ABC is right.

358. (II.66.8) Given that triangle ABC is *similar* to triangle PQR, write the three term proportion that describes how the six sides of these figures are related.

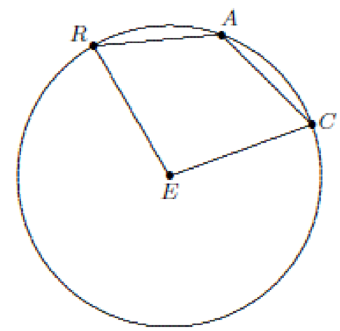
359. (II.69.11) Rectangles A and B are shown in the figure to the right.
   a. What is the ratio of the lengths of the corresponding sides?
   b. What is the ratio of the perimeters of the rectangles?
   c. What is the ratio of the areas of the rectangles?
   d. Summarize your results.



Problem 359

360. (Continuation) If corresponding sides of two similar square pyramids are in a 3:5 ratio, then what is the ratio of their volumes?

361. (II.14.2) Let $A = (-5, 2)$ and $B = (19, 9)$. Find coordinates for the point P between A and B that is three fifths of the way from A to B. Find coordinates for the point Q between A and B that is three fifths of the way from B to A.



Problem 362

362. (II.67.10) The figure at right shows points C, A, and R marked on a circle centered at E, so that *chords* CA and AR have the same length, and so that major arc CR is a 260-degree arc. Find the angles of quadrilateral $CARE$. What is special about the sizes of angles CAR and ACE?

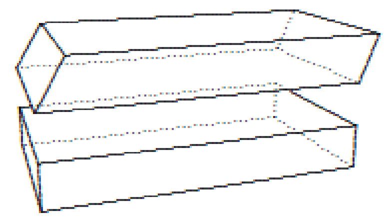
363. (II.67.1) A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on this path. How long is the path?

Math II

364. (II.16.1) The vector that is defined by a directed segment AB is often denoted $\overrightarrow{AB}$. Find components for the following vectors $\overrightarrow{AB}$.
   a. $A = (1, 2)$ and $B = (3, -7)$ b. $A = (2, 3)$ and $B = (2 + 3t, 3 - 4t)$

365. (II.13.12) Let $K = (-2, 1)$ and $M = (3, 4)$. Find coordinates for the two points that divide segment KM into three congruent segments.

366. (III.1.8) Playing cards measure 2.25 inches by 3.5 inches. A full deck of fifty-two cards is 0.75 inches high. What is the volume of a deck of cards? If the cards were uniformly shifted (turning the bottom illustration into the top illustration), would this volume be affected?



Problem 366

367. (II.64.5) If two chords of a circle have the same length, then their minor arcs have the same length, too. True or false? Explain. What about the converse statement? Is it true? Why?

368. (II.66.7) If P and Q are points on a circle, then the center of the circle must be on the perpendicular bisector of chord PQ. Explain. Which point on the chord is closest to the center? Why?

369. (II.68.3) Two circles of radius 10 cm are drawn so that their centers are 12 cm apart. The two points of intersection determine a *common chord*. Find the length of this chord.

370. (II.17.3) Is it possible for a line to go through **(a)** *no* lattice points? **(b)** exactly *one* lattice point? **(c)** exactly *two* lattice points? For each answer, either give an example or else explain the impossibility.

371. (III.35.1) A cylinder of radius 4 and height h is inscribed in a sphere of radius 8. Find h.

372. (II.67.6) A chord 6 cm long is 2 cm from the center of a circle. How long is a chord that is 1 cm from the center of the same circle?

373. (I.88.12) A rectangle has an area of 36 square meters. Its length is $2\sqrt{3}$ meters. In exact form, what is the perimeter of the rectangle?

374. (II.21.5) A rhombus has 25-cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

375. (II.48.8) Diagonals AC and BD of regular pentagon $ABCDE$ intersect at H. Decide whether or not $AHDE$ is a rhombus, and give your reasons.

Math II

Directions: Categorize each of the following true statements (**problems 376 – 392**) as **(a)** a statement whose converse is also in the list, **(b)** a statement that is a definition, **(c)** a statement whose converse is false, **(d)** the Sentry Theorem, **(e)** the Midline Theorem, **(f)** the Three Parallels Theorem, or **(g)** none of the above.

376. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.

377. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.

378. If a quadrilateral is equilateral, then it is a rhombus.

379. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

380. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.

381. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.

382. If a segment joins two of the midpoints of the sides of a triangle, then it is parallel to the third side, and is half the length of the third side.

383. Both pairs of opposite sides of a parallelogram are congruent.

384. The sum of the exterior angles of any polygon—one at each vertex—is 360 degrees.

385. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.

386. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.

387. The diagonals of a parallelogram bisect each other.

388. The bisector of an angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

389. Given three parallel lines, the ratios of the lengths of the *intercepted* segments of all transversals will be the same.

390. Both pairs of opposite angles of a parallelogram are congruent.

391. When a transversal intersects two parallel lines, the alternate interior angles are equal.

392. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

Math II Reference

altitude: In a triangle, an altitude is a segment that joins one of the three vertices to a point on the opposite side, the intersection being perpendicular. In some triangles, it may be necessary to extend the side to meet the altitude. The *length* of this segment is also called an altitude, as is the distance that separates the parallel sides of a trapezoid.

Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS.

angle bisector: A segment, ray, or line that passes through the vertex of an angle and splits the angle into two smaller, congruent angles.

Angle-Bisector Theorem: The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

angular size of an arc: This is the size of the central angle formed by the radii that meet the endpoints of the arc.

arc: The portion of a circle that lies to one side of a chord is called an *arc*.

central angle: An angle formed by two radii of a circle.

chord: A segment that joins two points on a circle is called a *chord* of the circle.

circle: This curve consists of all points that are at a constant distance from a *center*. The common distance is the *radius* of the circle. A segment joining the center to a point on the circle is also called a *radius*.

circumcenter: The center of a *circumcircle.* The perpendicular bisectors of the sides of a triangle are concurrent at the triangle's circumcenter, which is equidistant from the vertices of the triangle.

circumcircle: For every triangle, there is a circle for which the vertices of that triangle form inscribed angles. This inscribed circle is called the triangle's circumcircle.

collinear: Two points that are on the same line are said to be *collinear*.

components of a vector: Describe how to move from one unspecified point to another. They are obtained by *subtracting* coordinates.

configuration: Pattern, shape, etc. Example: *When all the points were plotted, their configuration was a line with the equation* $y = -2x + 3$

Math II Reference

congruent: When the points of one figure can be matched with the points of another figure so that corresponding parts have the same size, then the figures are called *congruent*. Congruent figures are the same size and shape.

conjecture: An unproven statement that seems likely to be true.

converse: The converse of a statement of the form "if [something] then [something else]" is the statement "if [something else] then [something]."

conversions: 1 mile = 5280 feet; 1 foot = 12 inches; 1 inch = 2.54 centimeters; one liter is 1000 milliliters; a milliliter is the same as a cubic centimeter.

convex: A polygon is called *convex* if every segment joining a pair of points within it lies entirely within the polygon.

coordinates: Numbers that indicate the position of an object. In this course, the coordinates used are of the form (x-coordinate, y-coordinate). Such coordinates are called *Cartesian* coordinates, named after the mathematician René Descartes.

corresponding: Describes parts of figures (such as angles or segments) that have been matched by means of a transformation.

CPCTC: *Corresponding Parts of Congruent Triangles are themselves Congruent*.

diagonal: A segment that connects two nonadjacent vertices of a polygon.

diameter: A chord that goes through the center of its circle is called a *diameter*.

direction vector: A vector that describes a line, by pointing from a point on the line to some other point on the line.

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is a consequence of the *Pythagorean Theorem*.

dot product: Given vectors [*a, b*] and [*m, n*], their dot product is the number *am* + *bn*. Given vectors [*a, b, c*] and [*p, q, r*], their dot product is the number *ap* + *bq* + *cr*. In either case, it is the sum of the products of corresponding components. When the value is zero, the vectors are perpendicular, and conversely.

equiangular: A polygon whose angles are the same size.

equidistant: A shortened form of "equally distant". Two points that are equidistant from a third are the same distance from that third point.

equilateral: A polygon whose sides have the same length.

Math II Reference

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line*. A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle*. The Greek mathematician Euclid, who flourished about 2300 years ago, wrote many books, and established a firm logical foundation for geometry.

exact form: A number that is not rounded or approximated.

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle.

function: A function is a rule that describes how the value of one quantity (the dependent variable) is determined uniquely by the value of another quantity (the independent variable). A function can be defined by a formula, a graph, a table, or a text.

head: Vector terminology for the second vertex of a directed segment.

hypotenuse: In a right triangle, the side opposite the right angle. This is the longest side of a right triangle.

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL.

image: The result of applying a transformation to a point P is called the *image of* P, often denoted P'. One occasionally refers to an *image segment* or an *image triangle*.

inscribed angle: An angle formed when two chords meet at a point on the circle. An inscribed angle is *half* the angular size of the arc it intercepts. In particular, an angle that intercepts a semicircle is a *right* angle.

integer: Any whole number, whether it be positive, negative, or zero.

intercepted arc: The portion of a circle that is cut off by the rays of an angle and lies inside that angle.

intercepted segment: A line segment that is cut off from another segment, line, or ray when intersected by two other segments, rays, or lines.

isosceles trapezoid: A trapezoid whose nonparallel sides have the same length.

Isosceles-Triangle Theorem: If a triangle has two sides of equal length, then the angles opposite those sides are also the same size.

Math II Reference

isosceles triangle: A triangle that has two sides of the same length. The word is derived from the Greek *iso* + *skelos* (equal + leg).

kite: A quadrilateral that has two disjoint pairs of congruent adjacent sides. Here, the word *disjoint* means that not all sides of a kite are congruent.

labeling convention: Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed.

lateral face: One of the faces adjacent to the base of a pyramid or prism.

lattice point: A point whose coordinates are both integers. The terminology derives from the lines on a piece of graph paper, which form a lattice.

length of a vector: This is the length of any segment that represents the vector.

line of reflection: See *reflection.*

line segment: A portion of a line named by its endpoints. For example, line segment *AB* (sometimes denoted $\overline{AB}$) refers to a segment whose endpoints are *A* and *B*. Sometimes we say "segment" instead of "line segment".

magnitude: The magnitude of a vector $\boldsymbol{u}$ is its length, denoted by the absolute-value signs $|\boldsymbol{u}|$.

major/minor arc: Two arcs are determined by a given chord. The smaller arc is called *minor*, and the larger arc is called *major*.

median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side.

midline of a trapezoid: This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books.

Midline Theorem: A segment that joins the midpoints of two sides of a triangle is parallel to the third side, and is half as long.

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the coordinates of the endpoints are (a, b) and (c, d), the coordinates of the midpoint are $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

opposite: Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of -17.5, and $\langle 2, -11\rangle$ is the opposite of $\langle -2, 11\rangle$.

parallelogram: A quadrilateral that has two pairs of parallel sides.

Math II Reference

pentagon: A polygon that has five sides.

perpendicular: Coplanar lines that intersect to form a right angle. If m_1 and m_2 are the slopes of two lines in the *xy*-plane, neither line parallel to a coordinate axis, and if $m_1 m_2 = -1$, then the lines are perpendicular.

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment.

point-slope form: The line with slope m that passes through the point (h, k) can be described in point-slope form by either $y - k = m(x - h)$ or $y = m(x - h) + k$.

postulate: A statement that has not been proved but is assumed to be true. Postulates and definitions form a logical base from which to prove other statements. Statements that have been proved are called *theorems*.

prism: A three-dimensional figure that has two congruent and parallel *bases*, and parallelograms for its remaining *lateral faces*. If the lateral faces are all rectangles, the prism is a *right prism*. If the base is a regular polygon, the prism is called *regular*.

pyramid: A three-dimensional figure that is obtained by joining all the points of a polygonal *base* to a *vertex*. Thus all the lateral faces of a pyramid are triangles. If the base polygon is regular, and the lateral edges are all congruent, then the pyramid is called *regular*.

Pythagorean Theorem: The square on the hypotenuse of a right triangle equals the sum of the squares on the legs. If a and b are the lengths of the legs of a right triangle, and if c is the length of the hypotenuse, then these lengths fit the Pythagorean equation $a^2 + b^2 = c^2$. Little is known about the Greek figure Pythagoras, who flourished about 2500 years ago, except that he probably did not discover the theorem that bears his name.

quadrant: The Cartesian coordinate plane (the x-y plane) is divided into four parts called *quadrants*. In Quadrant I, $x > 0$ and $y > 0$. In Quadrant II, $x < 0$, and $y > 0$. In Quadrant III, $x < 0$ and $y < 0$. In Quadrant IV, $x > 0$ and $y < 0$.

quadrilateral: A polygon with four sides

rate: A comparison of how one quantity changes as another quantity changes. For example, the rate 30 mph describes a movement of 30 miles for every hour that elapses.

reflection: A type of transformation that reflects (flips) points across a *straight line.* This line is called the *line of reflection*.

Math II Reference

reflexive property of congruence: This property says that any object is congruent to itself. The reflexive property of congruence is often used in showing that two triangles are congruent when they share a side or overlap so as to share an angle. This property is sometimes simply referred to as "the reflexive property", but it is technically correct to specify the "of congruence". Reflexivity is a property many relations (equality, congruence, similarity, etc.) have. Strict inequality ($<$ or $>$) is an example of a relation that is not reflexive because a number cannot be less than (or greater) than itself.

regular: A polygon that is both equilateral and equiangular.

rhombus: An equilateral quadrilateral.

rotation: A type of transformation that rotates (turns) points around a specified point. This point is called the *center of rotation.* The amount of degrees the points are rotated about the point is called the *angle of rotation.*

scalar: In the context of vectors, this is just another name for a number.

scalar product: Another name for the *dot product*.

scalene: A triangle no two of whose sides are the same length.

segment: See *line segment.*

Sentry Theorem: The sum of the exterior angles (one at each vertex) of any polygon is 360 degrees.

Side-Side-Angle: Insufficient grounds for congruence, unless the congruent angles are right angles; see *Hypotenuse-Leg*.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS.

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity*. Corresponding angles of similar figures must be equal in size.

simplest radical form: Simplifying a square root (or cube root, fourth root, etc.) so that the quantity inside the radical is as small as possible. For example, the simplest radical form of $\sqrt{50}$ is $5\sqrt{2}$.

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slope: The slope of a line is a measure of its steepness. It is computed by the ratio $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$. A line with positive slope rises as the value of x increases. If the slope is negative, the line drops as the value of x increases.

slope-intercept form: The line whose slope is m and whose y-intercept is b can be described in slope-intercept form by $y = mx + b$.

special right triangle: A right triangle whose features make it relatively easy to calculate missing side lengths. A 45-45-90 (isosceles right) triangle and a 30-60-90 triangle are two examples of special right triangles. Triangles whose sides have whole-number lengths (Pythagorean triples) are also a type of special right triangle.

tail: Vector terminology for the first vertex of a directed segment.

tail-to-tail: Vector terminology for directed segments with a common first vertex.

tangent: To touch or overlap at exactly one point. For example, if a line is tangent to a circle, then the circle and line intersect only once.

theorem: A statement that has been proved to be true.

Three-Parallels Theorem: Given three parallel lines, the segments they intercept on one transversal are proportional to the segments they intercept on any transversal.

transformation: A *function* that maps points to points.

translate: To slide a figure by applying a vector to each of its points.

trapezoid: A quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is called *isosceles*.

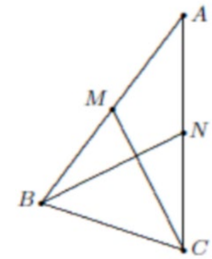
triangle inequality: For any three points P, Q, and R, $PQ \leq PR + RQ$. It says that any side of a triangle is less than the sum of the other two sides. The triangle inequality can also be represented in vector form as $|\boldsymbol{u} + \boldsymbol{v}\ | \leq |\boldsymbol{u}| + |\boldsymbol{v}|$.

Triangle Proportionality Theorem: If a segment that connects two sides of a triangle is parallel to the third side, the segment divides the two sides it connects proportionally.

Math II Reference

two-column proof: A way of outlining a the logical steps of a mathematical proof. The statements are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle *ABC* has two medians of the same length. It is given that $AB = AC$ and that *M* and *N* are the midpoints of sides *AB* and *AC*, respectively. The desired conclusion is that medians *CM* and *BN* have the same length.

Statement	Reason
1. $AB = AC$	1. Given
2. $AM = AN$	2. *M* and *N* are midpoints of congruent sides
3. $\angle MAC = \angle NAB$	3. Reflexive property
4. $\Delta MAC \cong \Delta NAB$	4. SAS
5. $CM = BN$	5. CPCTC



Varignon quadrilateral: Given any quadrilateral, this is the figure formed by connecting the midpoints of consecutive sides. The French mathematician Pierre Varignon (1654-1722) learned calculus when it was a new science, then taught it to others.

vectors: have *magnitude* (size) and *direction*. Visualize vectors as directed segments (arrows). Vectors are described by *components*, just as points are described by coordinates. The vector from point *A* to point *B* is often denoted $\overrightarrow{AB}$ or abbreviated by a boldface letter such as $\boldsymbol{u}$, and its magnitude is often denoted $|\overrightarrow{AB}|$ or $|\boldsymbol{u}|$.

vector triangles: Given vectors $\boldsymbol{u} = \langle a, b\rangle$ and $\boldsymbol{v} = \langle c, d\rangle$, a triangle is determined by drawing $\boldsymbol{u}$ and **v** so that they have a common initial point (*tail-to-tail*). No matter what this initial point is, the triangles determined by $\boldsymbol{u}$ and $\boldsymbol{v}$ are all congruent. All have $\frac{1}{2}|ad - bc|$ as their area.

velocity: A vector obtained by dividing a displacement vector by the elapsed time.

volume of a cylinder: This is the product of the base area and the height, which is the distance between the parallel base planes: $V_{cyl} = \pi r^2 h$.

volume of a sphere: The volume enclosed by a sphere of radius *r* is $V_{sphere} = \frac{4}{3}\pi r^3$ which (as Archimedes showed long ago) is two thirds of the volume enclosed by the circumscribed cylinder.

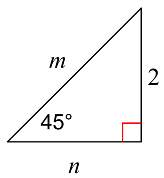
Math II Extra Practice Name________________________________

1. Special Right Triangles

Date________________

Find the missing side lengths. Leave your answers as radicals in simplest form.

1)



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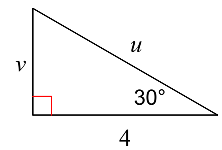

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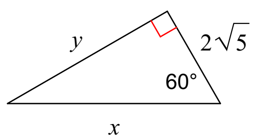

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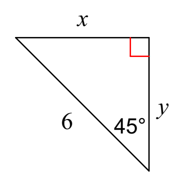

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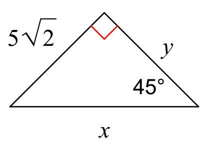

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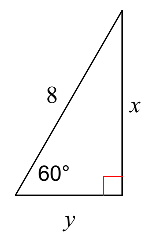

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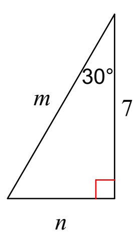

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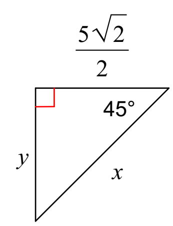

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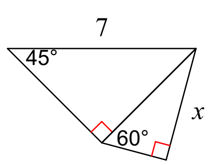

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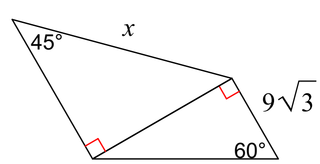

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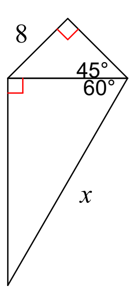

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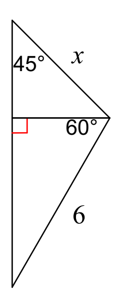


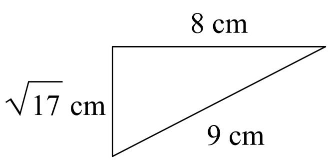
Math II Extra Practice Name________________________________

2. Pythagorean Theorem

Date________________

State if each triangle is a right triangle.

1)



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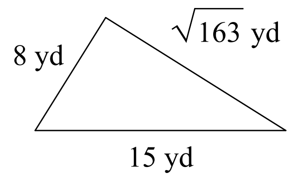


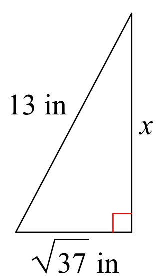
State if the three sides lengths form a right triangle.

3) $4\sqrt{6}$ cm, 12 cm, 14 cm

4) $\sqrt{6}$ m, 3 m, $\sqrt{15}$ m

Find the missing side of each triangle. Leave your answers in simplest radical form.

5)



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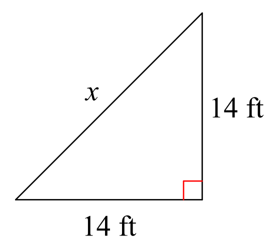

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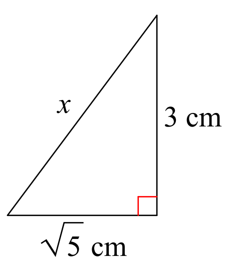

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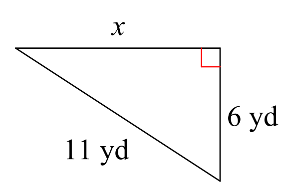


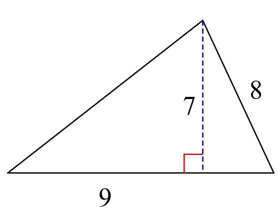
Find the missing side of each right triangle. Side *c* is the hypotenuse. Sides *a* and *b* are the legs. Leave your answers in simplest radical form.

9) $a = 15$ cm, $c = 16$ cm

10) $a = 4$ in, $b = 10$ in

Find the area of each triangle. Round your final answer to the nearest tenth. Answers may vary slightly from those given in back.

11)



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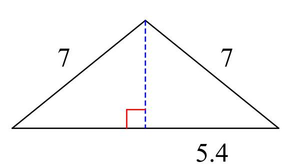

Math II Extra Practice Name______________________________

3. Interior/Exterior Angles of a Polygon

Date________________

Find the interior angle sum for each polygon. Round your answer to the nearest tenth if necessary.

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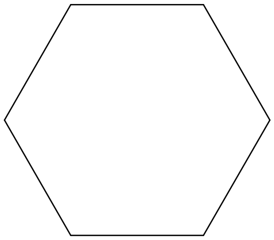

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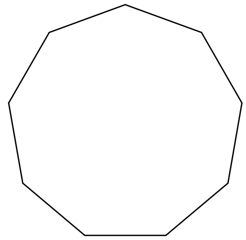
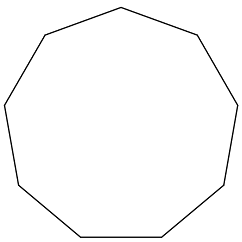


Find the measure of one interior angle in each regular polygon. Round your answer to the nearest tenth if necessary.

3)



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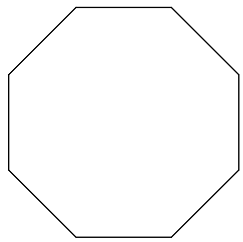

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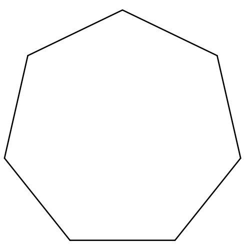

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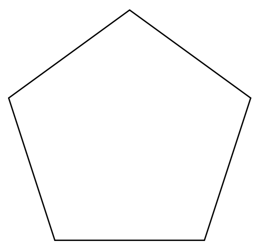

Find the measure of one exterior angle in each regular polygon. Round your answer to the nearest tenth if necessary.

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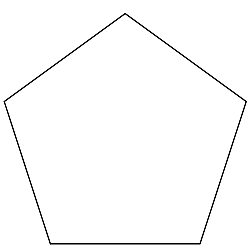

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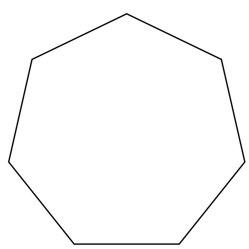

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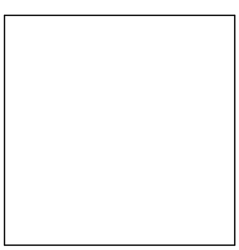

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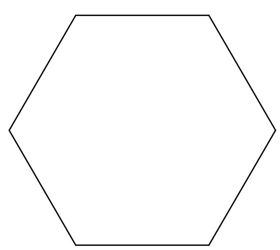


Math II Extra Practice Name________________________________

4. Distance Formula/ Midpoint Formula

Date________________

Find the distance between each pair of points.

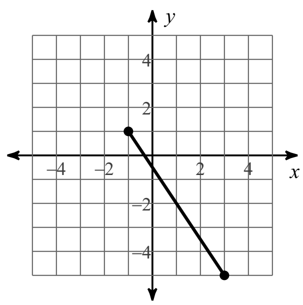
1) $(-7, 6),\ (7, 2)$

2) $(0, -3),\ (-4, -5)$

3) $(2, 2),\ (5, -6)$

4) $(-1, 8),\ (-3, 0)$

5)



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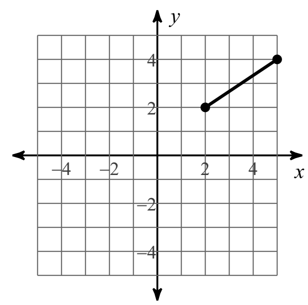

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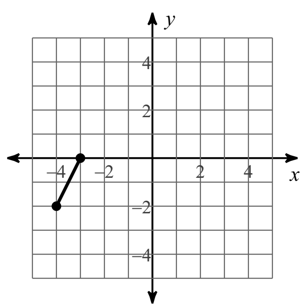

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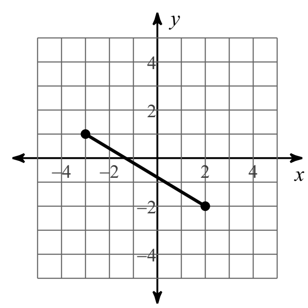


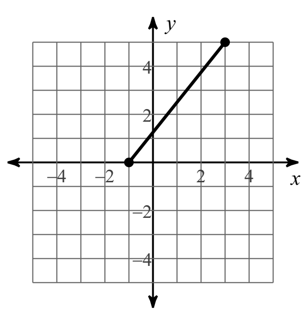
Find the midpoint of the line segment with the given endpoints.

9) $(-10, 2),\ (-8, 5)$

10) $(-5, 6),\ (9, 10)$

Find the midpoint of each line segment.

11)



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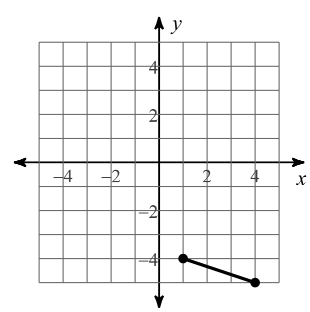


Find the other endpoint of the line segment with the given endpoint and midpoint.

13) Endpoint: $(-1, -5)$, midpoint: $(3, -3)$

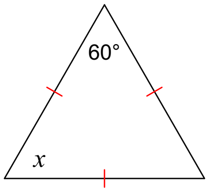
14) Endpoint: $(10, -2)$, midpoint: $(9, -4)$

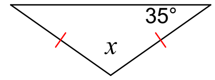
Math II Extra Practice Name________________________________

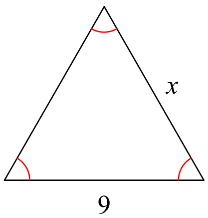
5. Isosceles and Equilateral Triangles

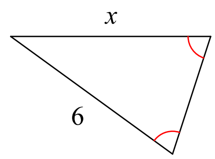
Date________________

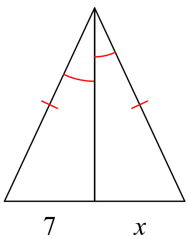
Find the value of *x*.

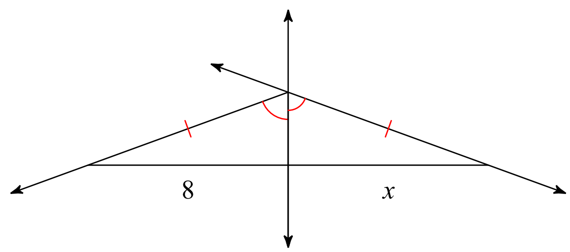
1) 

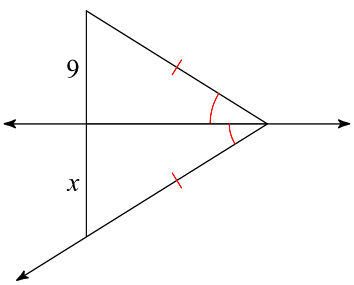
2) 

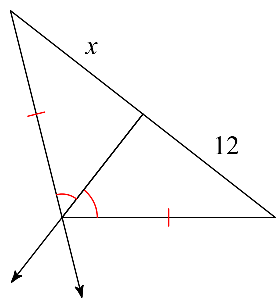
3) 

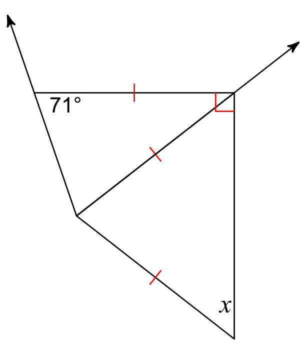
4) 

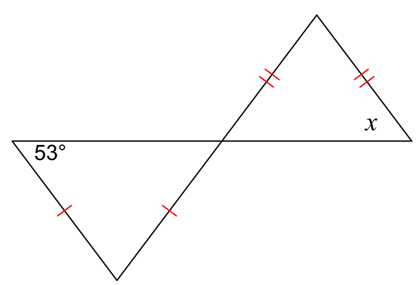
5) 

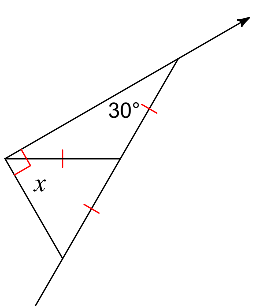
6) 

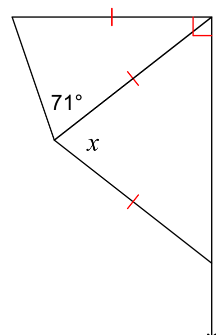
7) 

8) 

9) 

10) 

11) 

12) 

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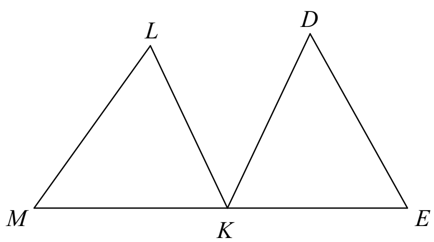
6. Triangle Congruences

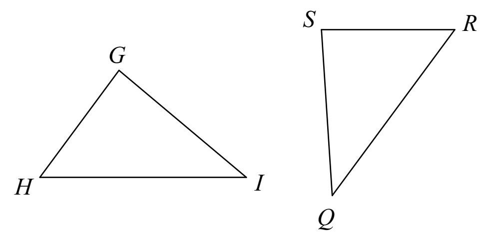
Date________________

Mark the angles and sides of each pair of triangles to indicate that they are congruent.

1) $\triangle MLK \cong \triangle DEK$

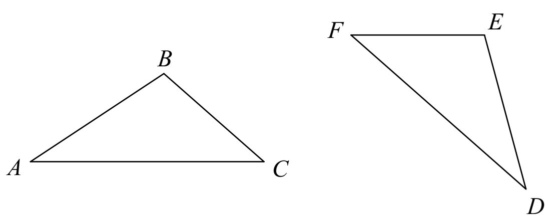


2) $\triangle IHG \cong \triangle QRS$



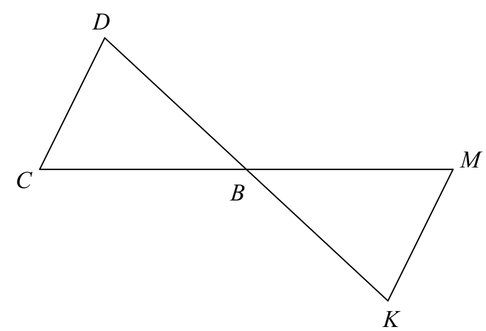
Complete each congruence statement by naming the corresponding angle or side.

3) $\triangle ABC \cong \triangle DEF$



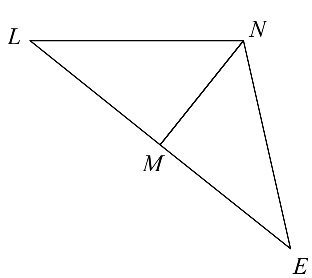
$\angle C \cong ?$

4) $\triangle BCD \cong \triangle BMK$



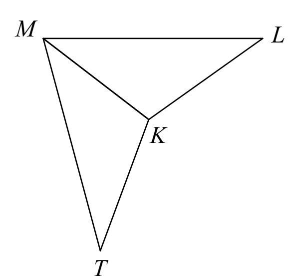
$\overline{DB} \cong ?$

5) $\triangle NML \cong \triangle NME$



$\overline{NM} \cong ?$

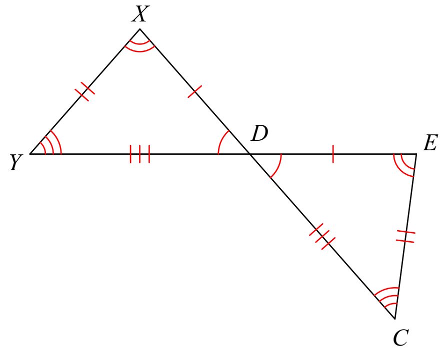
6) $\triangle KML \cong \triangle KMT$



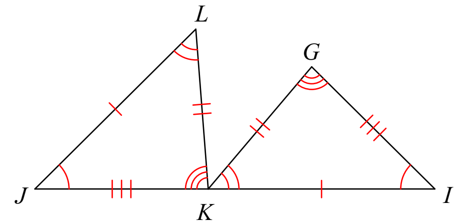
$\angle LKM \cong ?$

Write a statement that indicates that the triangles in each pair are congruent.

7)



8)



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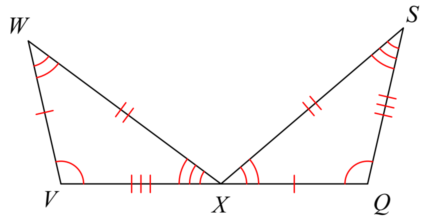
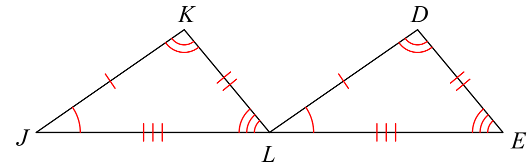
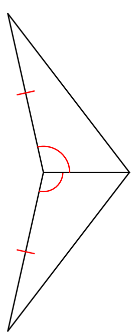


10)

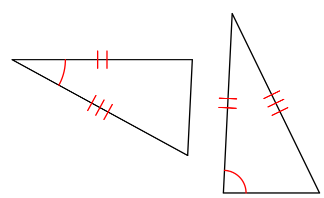


Determine if the two triangles are congruent. If they are, state how you know.

11)



12)



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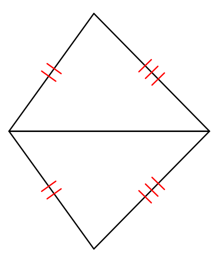

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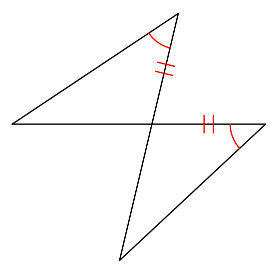
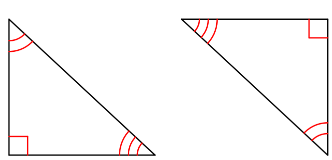


15)



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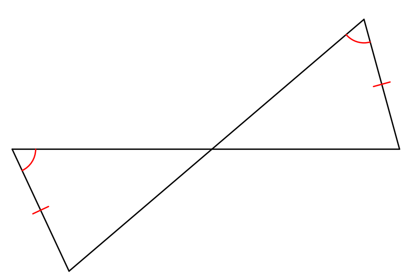
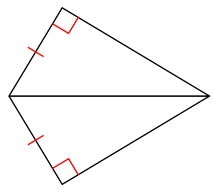
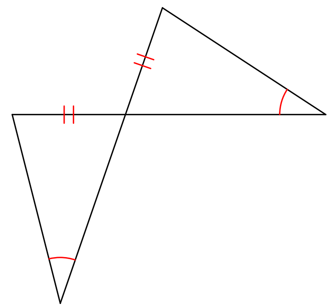


17)

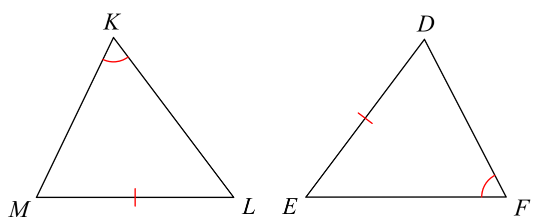


18)

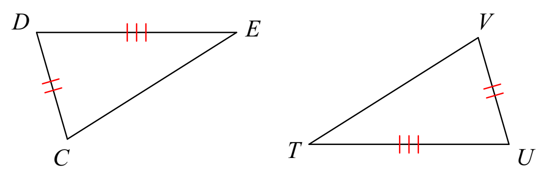


State what additional information is required in order to know that the triangles are congruent for the reason given.

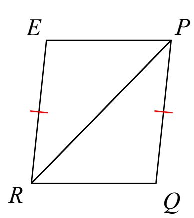
19) AAS



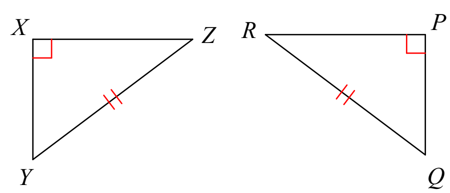
20) SSS



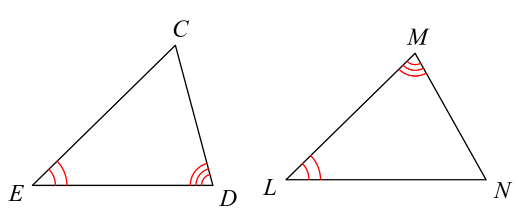
21) SSS



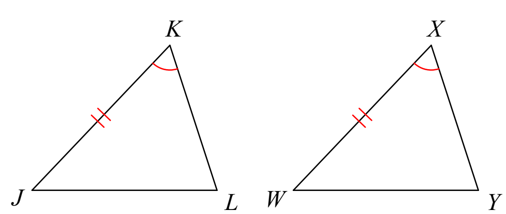
22) HL



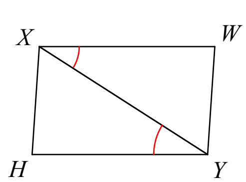
23) AAS



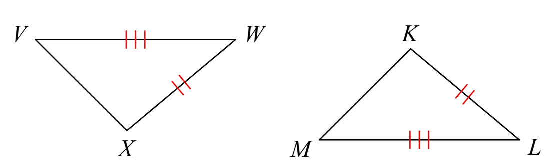
24) SAS



25) ASA



26) SAS



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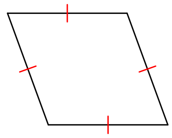
7. Quadrilateral Identification and Area

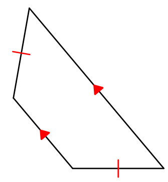
Date________________

State all possible names for each figure.

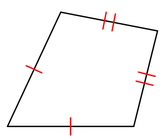
1)



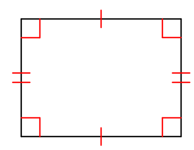
2)



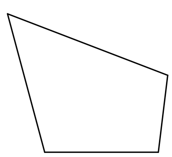
3)



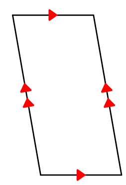
4)



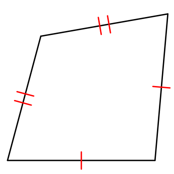
5)



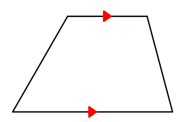
6)



7)

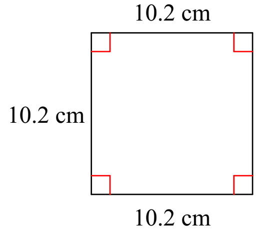


8)

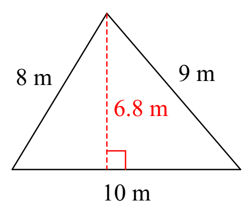


Find the area of each.

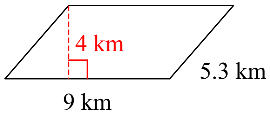
9)



10)



11)



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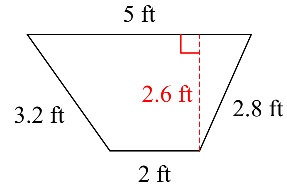
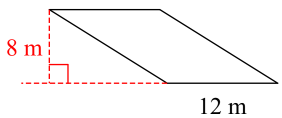
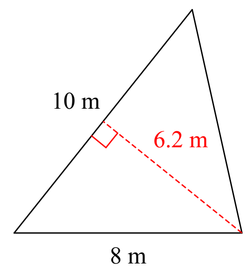


13)

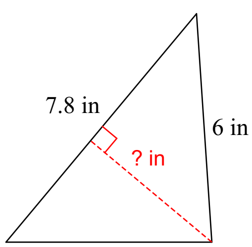


14)



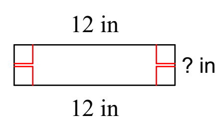
Find the missing measurement. Round your answer to the nearest tenth.

15)



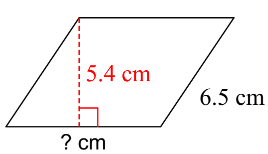
Area = 16 in²

16)



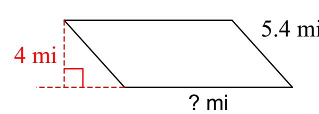
Area = 36 in²

17)



Area = 41.6 cm²

18)



Area = 40 mi²

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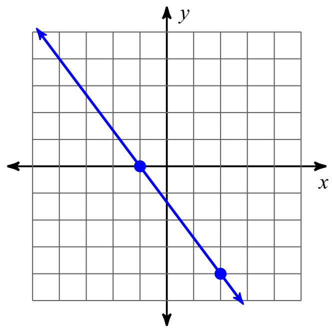
8. Slope

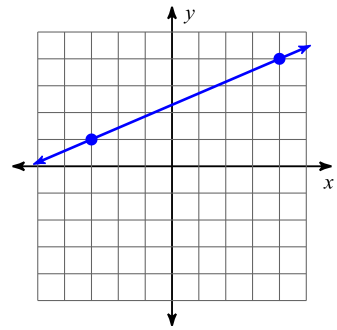
Date________________

Find the slope of each line.

1)



2)



Find the slope of the line through each pair of points.

3) $(8, -11), (1, -17)$

4) $(-12, 2), (-11, -20)$

Find the slope of each line.

5) $6x = 10y - 20$

6) $-y + 3x = 0$

7) $\frac{1}{5}y = 1 + \frac{3}{5}x$

8) $0 = -4x - 2 + 2y$

Find the slope of a line parallel to each given line.

9) $y = 3$

10) $6x = -8y - 8$

11) $0 = 3x + 5 + y$

12) $-x = 1$

Find the slope of a line perpendicular to each given line.

13) $2y = -9x - 10$

14) $2 = x$

15) $1 = y$

16) $0 = y - 3x - 1$

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9. System of Equations

Date________________

Solve each system:

1) $y = -2$
$y = -4x - 18$

2) $y = -7x + 15$
$y = 2x + 6$

3) $-8x + 4y = -12$
$y = 4x - 11$

4) $y = -4x - 12$
$-8x - 2y = 24$

5) $-7x - 5y = 13$
$3x + y = -1$

6) $x + y = -5$
$-6x + 4y = 0$

7) $-4x - 4y = -7$
$-2x - 2y = 1$

8) $-8x - 7y = -3$
$-x + 4y = 24$

9) The senior classes at High School A and High School B planned separate trips to Yellowstone National Park. The senior class at High School A rented and filled 10 vans and 12 buses with 734 students. High School B rented and filled 9 vans and 11 buses with 670 students. Every van had the same number of students in it as did the buses. How many students can a van carry? How many students can a bus carry?

10) Alberto and Aliyah each improved their yards by planting daylilies and ornamental grass. They bought their supplies from the same store. Alberto spent $68 on 8 daylilies and 4 bunches of ornamental grass. Aliyah spent $108 on 14 daylilies and 6 bunches of ornamental grass. What is the cost of one daylily and the cost of one bunch of ornamental grass?

11) Adam's school is selling tickets to a play. On the first day of ticket sales the school sold 12 senior citizen tickets and 6 student tickets for a total of $114. The school took in $87 on the second day by selling 3 senior citizen tickets and 6 student tickets. What is the price each of one senior citizen ticket and one student ticket?

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10. Vectors

Date________________

Sketch a graph of each vector.

1) $\overrightarrow{PQ}$ where $P = (0, 3)$ $Q = (-9, -7)$

2) $\overrightarrow{AB}$ where $A = (6, 5)$ $B = (-3, 8)$

3) $\overrightarrow{CD}$ where $C = (4, 5)$ $D = (-3, -4)$

4) $\overrightarrow{CD}$ where $C = (1, 10)$ $D = (1, -3)$

Write each vector in component form.

5) $\overrightarrow{PQ}$ where $P = (-10, -6)$ $Q = (7, -7)$

6) $\overrightarrow{RS}$ where $R = (0, 4)$ $S = (-4, -8)$

7) $\overrightarrow{RS}$ where $R = (-7, -6)$ $S = (-4, 8)$

8) $\overrightarrow{CD}$ where $C = (-1, 5)$ $D = (-8, -6)$

Find the magnitude for each vector.

9) $\overrightarrow{PQ}$ where $P = (-3, -8)$ $Q = (1, 6)$

10) $\overrightarrow{AB}$ where $A = (8, 6)$ $B = (6, 6)$

11) $\overrightarrow{CD}$ where $C = (9, 9)$ $D = (-2, -10)$

12) $\overrightarrow{AB}$ where $A = (-2, -3)$ $B = (0, -7)$

Find the component form of the specified vector operation.

13) $\mathbf{f} = \langle 0, 8 \rangle$
$\mathbf{g} = \langle -9, -1 \rangle$
Find: $-\mathbf{f} + \mathbf{g}$

14) $\mathbf{u} = \langle -3, -9 \rangle$
$\mathbf{v} = \langle 8, -9 \rangle$
Find: $\mathbf{u} - \mathbf{v}$

15) $\mathbf{f} = \langle -2, -8 \rangle$
$\mathbf{b} = \langle 5, -10 \rangle$
Find: $\mathbf{f} - \mathbf{b}$

16) $\mathbf{a} = \langle 6, -5 \rangle$
$\mathbf{v} = \langle 0, -8 \rangle$
Find: $-\mathbf{a} - \mathbf{v}$

17) $\mathbf{f} = \langle -18, 24 \rangle$
Find: $-7\mathbf{f}$

18) $\mathbf{a} = \langle -5, 11 \rangle$
Find: $8\mathbf{a}$

19) $\mathbf{f} = \langle -21, -\sqrt{43} \rangle$
Find: $\sqrt{2} \cdot \mathbf{f}$

20) $\mathbf{f} = \langle 4, -11 \rangle$
Find: $2\mathbf{f}$

21) $\mathbf{f} = \langle 30, -40 \rangle$
Find the vector opposite **f**

22) $\mathbf{a} = \langle -11, 1 \rangle$
Find the vector opposite **a**

23) $\mathbf{a} = \langle 15, -36 \rangle$
Find the vector opposite **a**

24) $\mathbf{f} = \langle -3, \sqrt{91} \rangle$
Find the vector opposite **f**

25) $\mathbf{u} = \langle -3, 3 \rangle$
$\mathbf{v} = \langle 3, -10 \rangle$
Find: $5\mathbf{u} + 4\mathbf{v}$

26) $\mathbf{a} = \langle -7, 1 \rangle$
$\mathbf{v} = \langle -12, 2 \rangle$
Find: $7\mathbf{a} - 5\mathbf{v}$

27) $\mathbf{u} = \langle -2, 5 \rangle$
$\mathbf{g} = \langle -9, -12 \rangle$
Find: $-4\mathbf{u} - 8\mathbf{g}$

28) $\mathbf{f} = \langle 12, 1 \rangle$
$\mathbf{b} = \langle -11, 2 \rangle$
Find: $-2\mathbf{f} - 3\mathbf{b}$

29) $\mathbf{u} = \langle -18, 24 \rangle$
Unit vector in the direction of **u**

30) $\mathbf{a} = \langle 2\sqrt{3}, 5 \rangle$
Unit vector in the direction of **a**

31) $\mathbf{u} = \langle -11, 7 \rangle$
Unit vector in the opposite direction of **u**

32) $\mathbf{f} = \langle 5, 10 \rangle$
Unit vector in the opposite direction of **f**

Draw a vector diagram to find the sum of each pair of vectors. Then state the magnitude of the resultant. (Ignore the angle degree value in the answers)

33) $\mathbf{t} = \langle 9, 12 \rangle$ $\mathbf{u} = \langle -20, 10 \rangle$

34) $\mathbf{t} = \langle 4, 1 \rangle$ $\mathbf{u} = \langle -5, -12 \rangle$

35) $\mathbf{m} = \langle 5, 12 \rangle$ $\mathbf{n} = \langle -11, 7 \rangle$

36) $\mathbf{a} = \langle 12, 16 \rangle$ $\mathbf{b} = \langle 7, -20 \rangle$

Find the dot product of the given vectors.

37) $\mathbf{u} = \langle 3, 0 \rangle$
$\mathbf{v} = \langle 9, 3 \rangle$

38) $\mathbf{u} = \langle 8, 0 \rangle$
$\mathbf{v} = \langle 0, 7 \rangle$

39) $\mathbf{u} = \langle -2, 2 \rangle$
$\mathbf{v} = \langle -6, -9 \rangle$

40) $\mathbf{u} = \langle -6, 9 \rangle$
$\mathbf{v} = \langle 5, 3 \rangle$

41) $\mathbf{u} = \langle -1, -1 \rangle$
$\mathbf{v} = \langle 8, -8 \rangle$

42) $\mathbf{u} = \langle 4, 2 \rangle$
$\mathbf{v} = \langle 2, 1 \rangle$

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11. Translations, Reflections and Rotations

Date________________

Write a rule to describe each transformation.

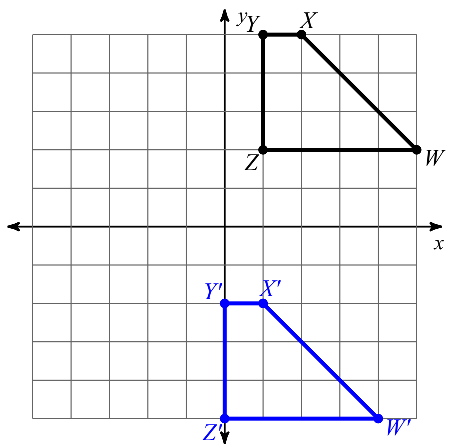
1) $L(1, -4)$, $K(0, -2)$, $J(1, -1)$, $I(5, -4)$
to
$L'(0, 1)$, $K'(-1, 3)$, $J'(0, 4)$, $I'(4, 1)$

2) $Q(-5, -3)$, $R(-3, 1)$, $S(-2, 0)$, $T(-1, -5)$
to
$R'(3, 1)$, $S'(2, 0)$, $T'(1, -5)$, $Q'(5, -3)$

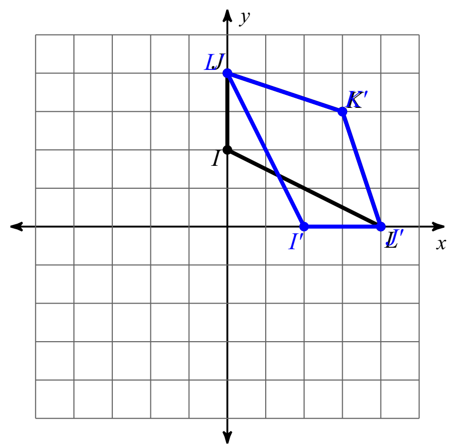
3) $N(-4, 0)$, $M(-3, 5)$, $L(-2, 2)$
to
$N'(-1, -4)$, $M'(0, 1)$, $L'(1, -2)$

4) $L(-1, -4)$, $K(-2, -2)$, $J(0, 2)$, $I(3, -2)$
to
$K'(-2, 2)$, $J'(0, -2)$, $I'(3, 2)$, $L'(-1, 4)$

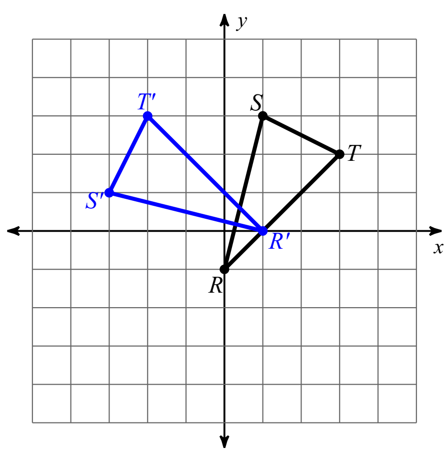
5)



6)



7)



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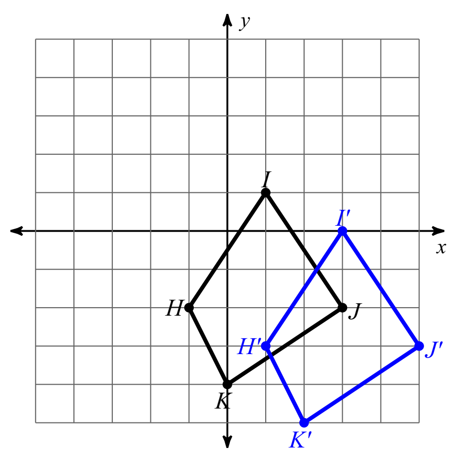


Math II Extra Practice Name________________________________

12. Similar Figures

Date________________

Solve each proportion.

1) $\frac{10}{2} = \frac{b}{6}$

2) $\frac{7}{10} = \frac{3}{x}$

3) $\frac{6}{7} = \frac{7}{m + 4}$

4) $\frac{4}{3} = \frac{m - 6}{4}$

5) $\frac{n - 7}{8} = \frac{8}{9}$

6) $\frac{7}{v - 8} = \frac{9}{2}$

7) $\frac{v}{6} = \frac{v + 3}{3}$

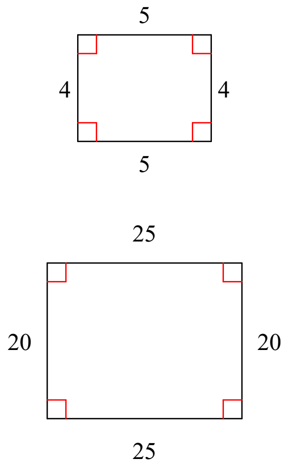
8) $\frac{10}{n} = \frac{2}{n - 4}$

9) $\frac{v}{v + 4} = \frac{8}{2}$

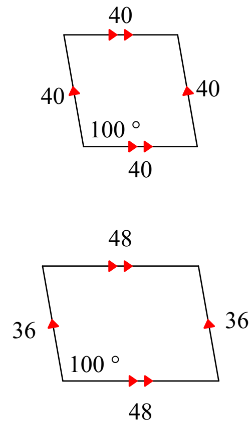
10) $\frac{n - 3}{2n} = \frac{5}{8}$

State if the polygons are similar.

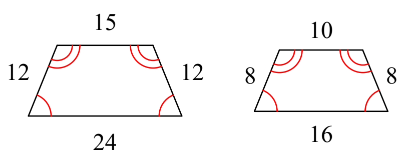
11)



12)

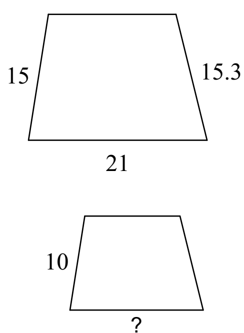


13)

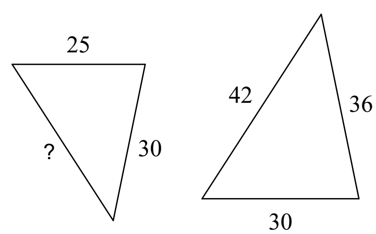


The polygons in each pair are similar. Find the missing side length.

14)

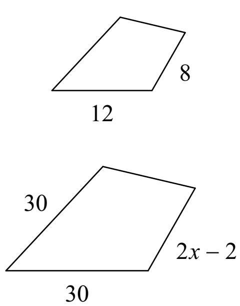


15)

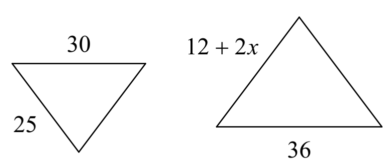


Solve for x. The polygons in each pair are similar.

16)



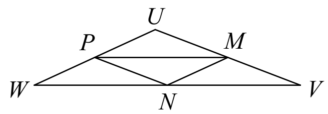
17)



13. Theorems: Midline, Triangle Proportionality, Three Parallels, Angle Bisector

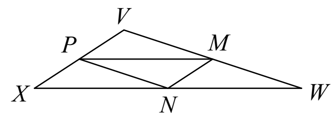
In each triangle, M, N, and P are the midpoints of the sides. Name a segment parallel to the one given.

1)



___ $\parallel \overline{UV}$

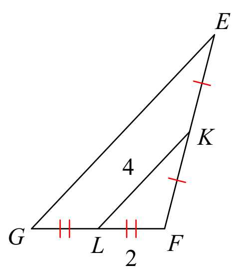
2)



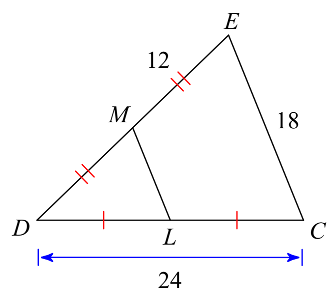
___ $\parallel \overline{VW}$

Find the missing length indicated.

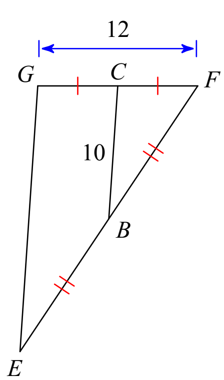
3) Find EG



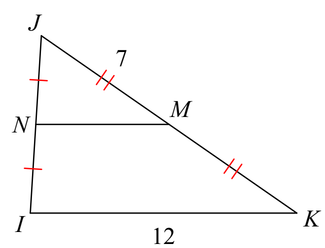
4) Find LM



5) Find GE

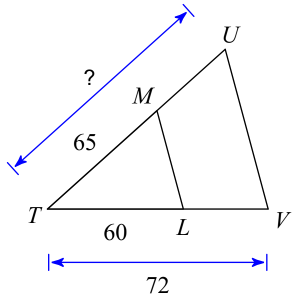


6) Find NM

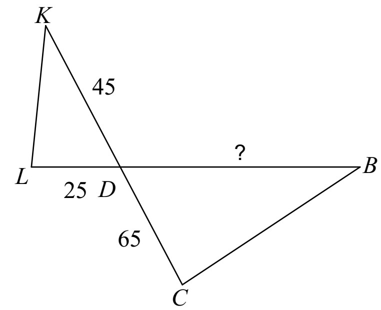


Find the missing length. The triangles in each pair are similar.

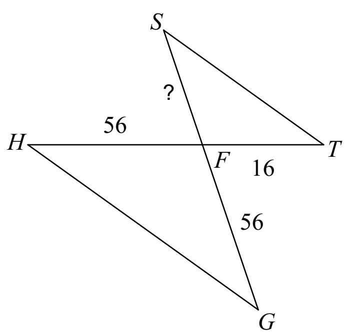
7)



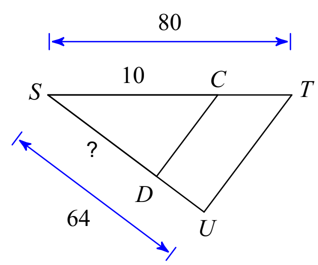
8) $\triangle DCB \sim \triangle DLK$



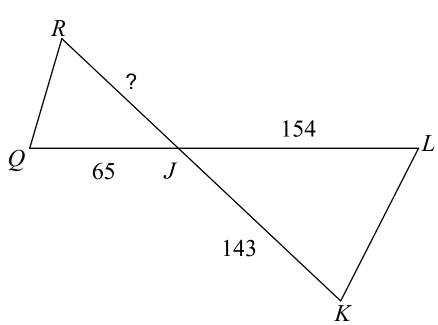
9) $\triangle FGH \sim \triangle FST$



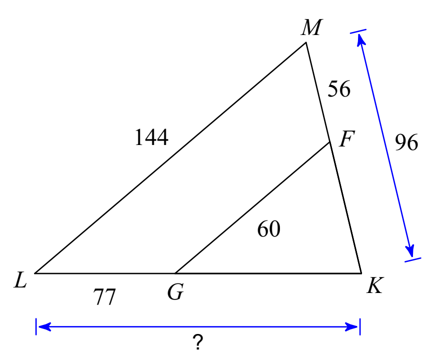
10)



11) $\triangle JKL \sim \triangle JQR$

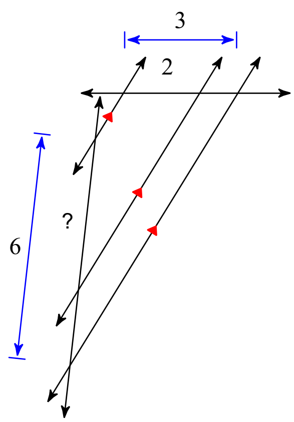


12)



Find the missing length indicated.

13)



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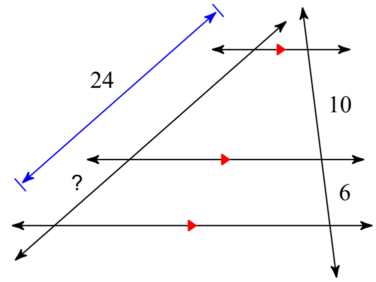

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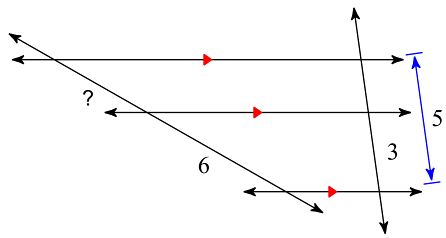

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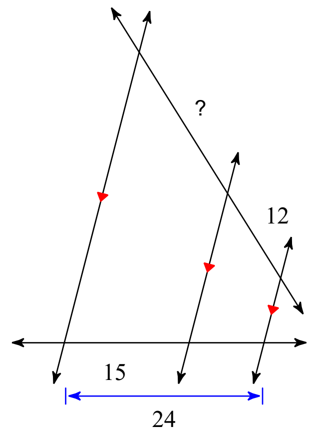

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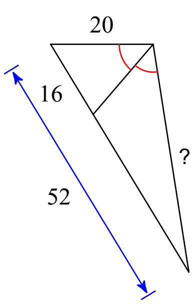

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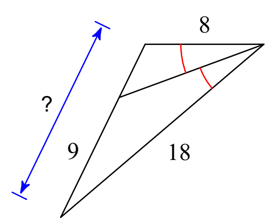

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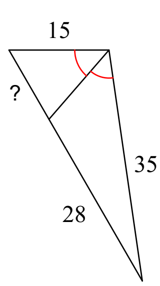
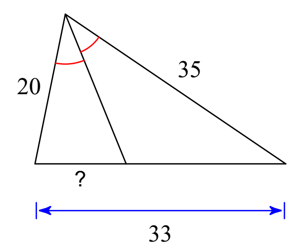


20)



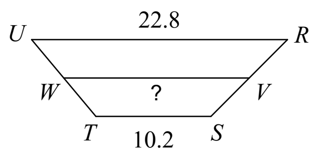
Math II Extra Practice Name________________________

14. Trapezoids

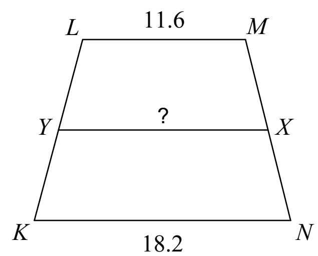
Date________________

Find the length of the midsegment of each trapezoid.

1)

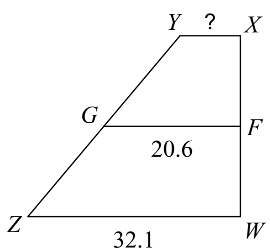


2)

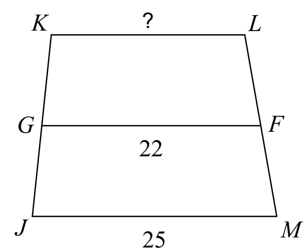


Find the length of the base indicated for each trapezoid.

3)

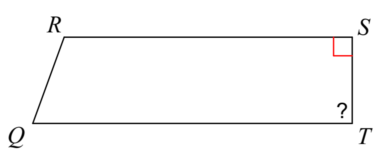


4)

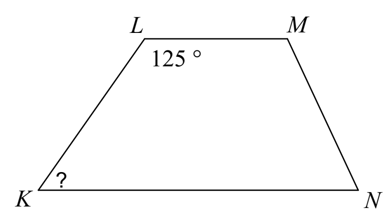


Find the measurement of the angle indicated for each trapezoid.

5)

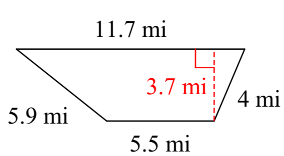


6)

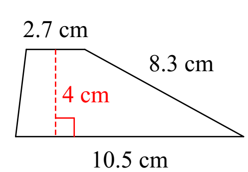


Find the area of each.

7)

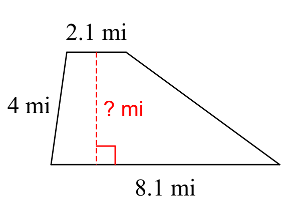


8)



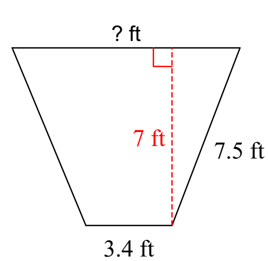
Find the missing measurement. Round your answer to the nearest tenth.

9)



Area = 20.4 mi²

10)



Area = 43.4 ft²

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15. Circles

Identify the center and radius of each.

1) $x^2 + y^2 = 256$

2) $x^2 + y^2 = 144$

3) $(x - 15)^2 + (y + 13)^2 = 1$

4) $(x + 12)^2 + (y - 7)^2 = 1$

5) $(x + 8)^2 + (y - \sqrt{91})^2 = 31$

6) $(x - 3)^2 + (y + 1)^2 = 196$

Use the information provided to write the equation of each circle.

7) Center: $(15, 7)$
Radius: 1

8) Center: $(-5, -11)$
Radius: $3\sqrt{7}$

9) Center: $(14, -5)$
Area: 16π

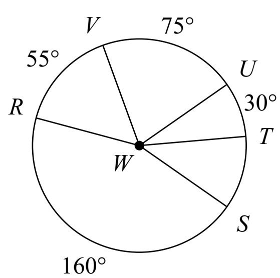
10) Center: $(3, -8)$
Area: 49π

11) Center lies in the second quadrant
Tangent to $x = -15$, $x = -17$, and the x-axis

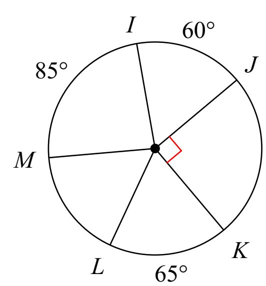
12) Center lies in the second quadrant
Tangent to $x = -1$, $y = 9$, and $y = 5$

Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

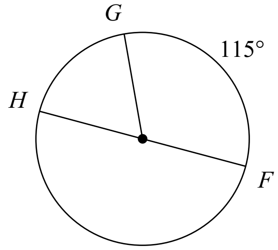
13) $m\angle RWT$



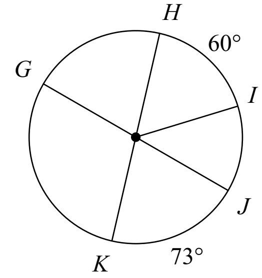
14) $m\overset{\frown}{IJL}$



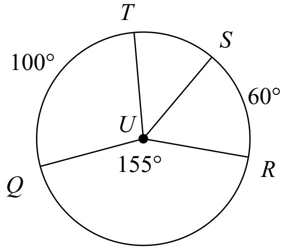
15) $m\overset{\frown}{GFH}$



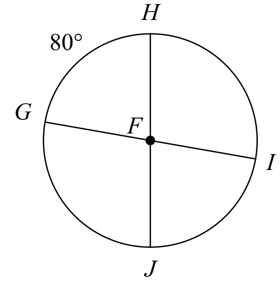
16) $m\overset{\frown}{HJG}$



17) $m\angle TUR$

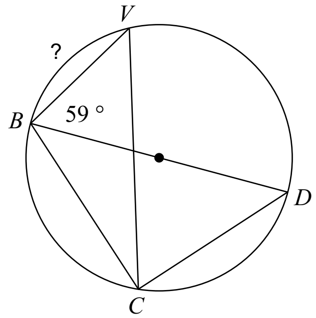


18) $m\angle IFJ$

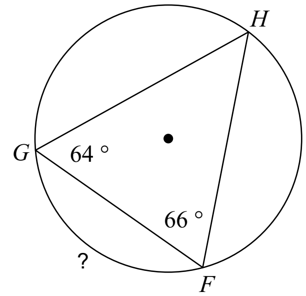


Find the measure of the arc or angle indicated.

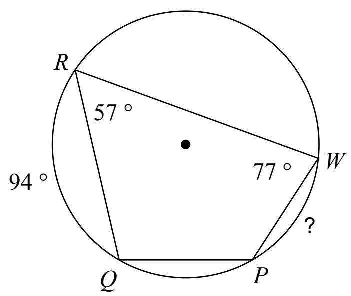
19)



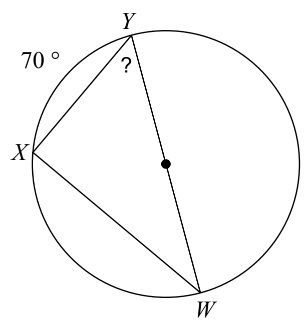
20)



21)

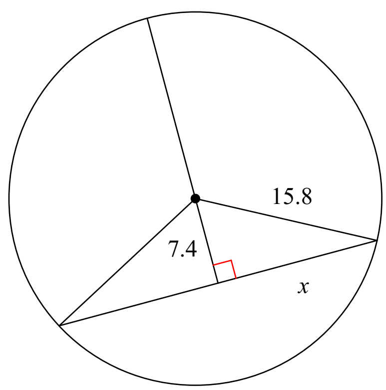


22)



Find the length of the segment indicated. Round your answer to the nearest tenth if necessary.

23)



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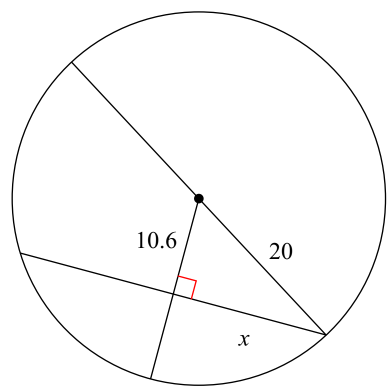

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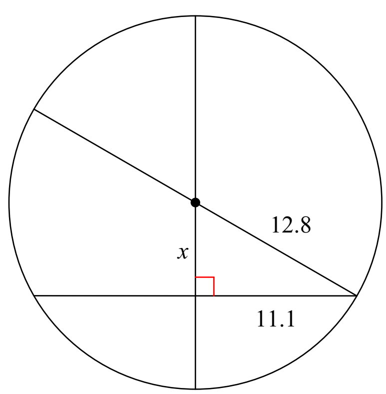
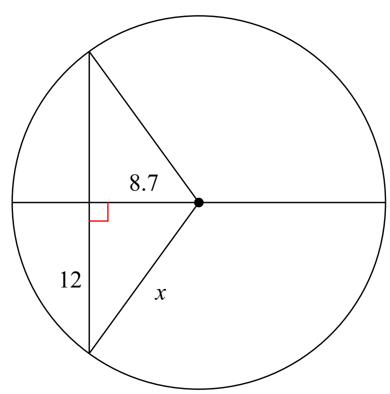


26)



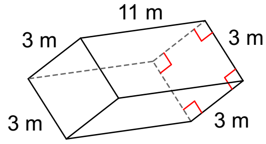
Math II Extra Practice Name________________________________

16. Volume

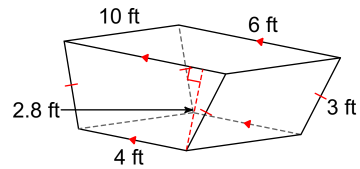
Date________________

Find the volume of each figure. Round your answers to the nearest hundredth, if necessary.

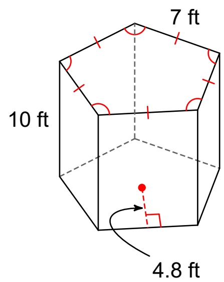
1)



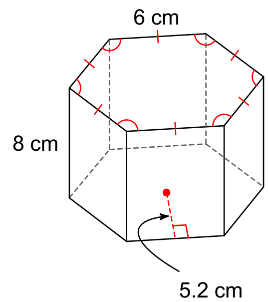
2)



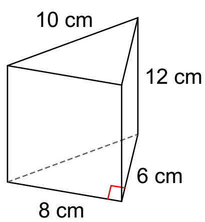
3)



4)



5)



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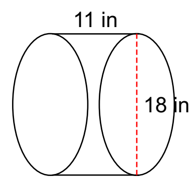

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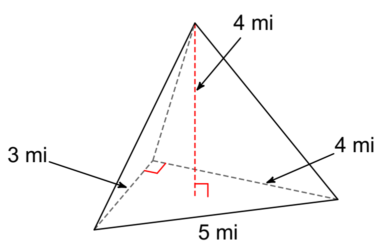

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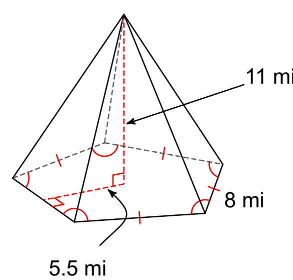
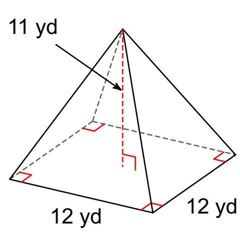
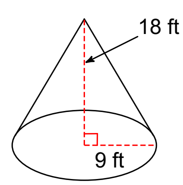


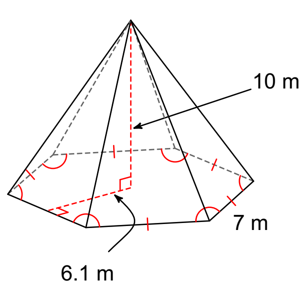
9)



10)



11)



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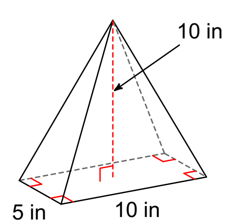

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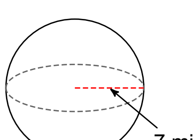


Extra Practice Answers

Answers to 1. Special Right Triangles

1) $m = 2\sqrt{2},\ n = 2$
2) $u = \dfrac{8\sqrt{3}}{3},\ v = \dfrac{4\sqrt{3}}{3}$
3) $x = 4\sqrt{5},\ y = 2\sqrt{15}$
4) $x = 3\sqrt{2},\ y = 3\sqrt{2}$
5) $x = 10,\ y = 5\sqrt{2}$
6) $x = 4\sqrt{3},\ y = 4$
7) $m = \dfrac{14\sqrt{3}}{3},\ n = \dfrac{7\sqrt{3}}{3}$
8) $x = 5,\ y = \dfrac{5\sqrt{2}}{2}$
9) $\dfrac{7\sqrt{6}}{4}$
10) $27\sqrt{2}$
11) $16\sqrt{2}$
12) $3\sqrt{2}$

Answers to 2. Pythagorean Theorem

1) Yes
2) No
3) No
4) Yes
5) $2\sqrt{33}$ in
6) $14\sqrt{2}$ ft
7) $\sqrt{14}$ cm
8) $\sqrt{85}$ yd
9) $\sqrt{31}$ cm
10) $2\sqrt{29}$ in
11) 45.2
12) 24.3

Answers to 3. Interior/Exterior Angles of a Polygon

1) $720°$
2) $1260°$
3) $140°$
4) $135°$
5) $128.6°$
6) $108°$
7) $72°$
8) $51.4°$
9) $90°$
10) $60°$

Answers to 4. Distance Formula/ Midpoint Formula

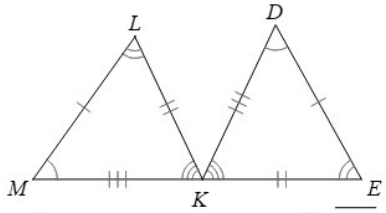
1) $2\sqrt{53}$
2) $2\sqrt{5}$
3) $\sqrt{73}$
4) $2\sqrt{17}$
5) $2\sqrt{13}$
6) $\sqrt{13}$
7) $\sqrt{5}$
8) $\sqrt{34}$
9) $\left(-9, 3\frac{1}{2}\right)$
10) $(2, 8)$
11) $\left(1, 2\frac{1}{2}\right)$
12) $\left(2\frac{1}{2}, -4\frac{1}{2}\right)$
13) $(7, -1)$
14) $(8, -6)$

Answers to 5. Isosceles and Equilateral Triangles

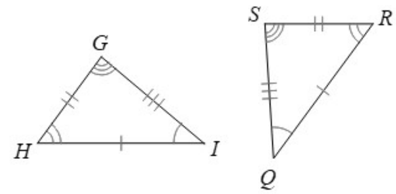
1) $60°$
2) $110°$
3) 9
4) 6
5) 7
6) 8
7) 9
8) 12
9) $52°$
10) $53°$
11) $60°$
12) $76°$

Answers to 6. Triangle Congruences

1)



2)



3) $\angle F$
4) $\overline{KB}$
5) $\overline{NM}$
6) $\angle TKM$
7) $\triangle DEC \cong \triangle DXY$
8) $\triangle JLK \cong \triangle IKG$
9) $\triangle VWX \cong \triangle QXS$
10) $\triangle JKL \cong \triangle LDE$
11) SAS
12) Not enough information
13) SSS
14) ASA
15) Not enough information
16) AAS
17) HL
18) AAS
19) $\angle L \cong \angle E$ or $\angle M \cong \angle D$
20) $\overline{EC} \cong \overline{TV}$
21) $\overline{QR} \cong \overline{EP}$
22) $\overline{XY} \cong \overline{PQ}$ or $\overline{ZX} \cong \overline{RP}$
23) $\overline{DC} \cong \overline{MN}$ or $\overline{CE} \cong \overline{NL}$
24) $\overline{KL} \cong \overline{XY}$
25) $\angle WYX \cong \angle HXY$
26) $\angle W \cong \angle L$

Answers to 7. Quadrilateral Identification and Area

1) quadrilateral, parallelogram, rhombus
2) quadrilateral, trapezoid, isosceles trapezoid
3) quadrilateral, kite
4) quadrilateral, parallelogram, rectangle
5) quadrilateral
6) quadrilateral, parallelogram
7) quadrilateral, kite
8) quadrilateral, trapezoid
9) 104.04 cm^2
10) 34 m^2
11) 36 km^2
12) 9.1 ft^2
13) 96 m^2
14) 31 m^2
15) 4.1 in
16) 3 in
17) 7.7 cm
18) 10 mi

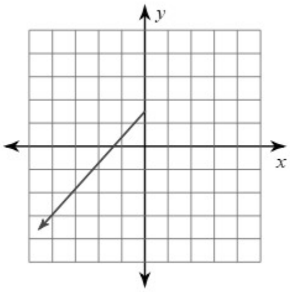
Answers to 8. Slope

1) $-\frac{4}{3}$
2) $\frac{3}{7}$
3) $\frac{6}{7}$
4) -22
5) $\frac{3}{5}$
6) 3
7) 3
8) 2
9) 0
10) $-\frac{3}{4}$
11) -3
12) Undefined
13) $\frac{2}{9}$
14) 0
15) Undefined
16) $-\frac{1}{3}$

Answers to 9. System of Equations

1) $(-4, -2)$
2) $(1, 8)$
3) $(4, 5)$
4) Infinite number of solutions
5) $(1, -4)$
6) $(-2, -3)$
7) No solution
8) $(-4, 5)$
9) Van: 17, Bus: 47
10) daylily: \$3, bunch of ornamental grass: \$11
11) senior citizen ticket: \$3, student ticket: \$13

Answers to 10. Vectors

1)


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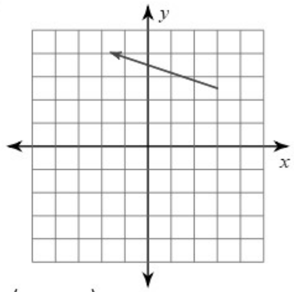
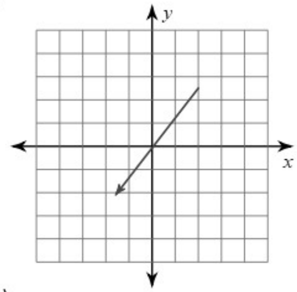
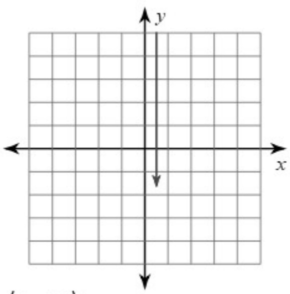


3)


4)


5) $\langle 17, -1\rangle$

6) $\langle -4, -12\rangle$

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7) $\langle 3, 14\rangle$

8) $\langle -7, -11\rangle$

9) $2\sqrt{53} \approx 14.56$

10) 2

11) $\sqrt{482} \approx 21.954$

12) $2\sqrt{5} \approx 4.472$

13) $\langle -9, -9\rangle$

14) $\langle -11, 0\rangle$

15) $\langle -7, 2\rangle$

16) $\langle -6, 13\rangle$

17) $\langle 126, -168\rangle$

18) $\langle -40, 88\rangle$

19) $\langle -21\sqrt{2}, -\sqrt{86}\rangle$

20) $\langle 8, -22\rangle$

21) $\langle -30, 40\rangle$

22) $\langle 11, -1\rangle$

23) $\langle -15, 36\rangle$

24) $\langle 3, -\sqrt{91}\rangle$

25) $\langle -3, -25\rangle$

26) $\langle 11, -3\rangle$

27) $\langle 80, 76\rangle$

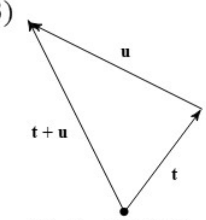
28) $\langle 9, -8\rangle$

29) $\left\langle -\frac{3}{5}, \frac{4}{5}\right\rangle$

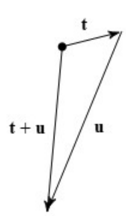
30) $\left\langle \frac{2\sqrt{111}}{37}, \frac{5\sqrt{37}}{37}\right\rangle$

31) $\left\langle \frac{11\sqrt{170}}{170}, -\frac{7\sqrt{170}}{170}\right\rangle$

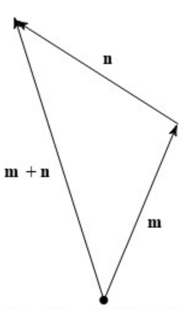
32) $\left\langle -\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right\rangle$

33)


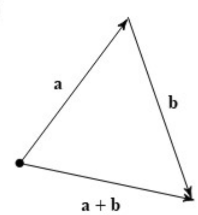
24.6; 116.57°

34)


11.05; 264.81°

35)


19.92; 107.53°

36)


19.42; 348.11°

37) 27

38) 0

39) −6

40) −3

41) 0

42) 10

Answers to 11. Translations, Reflections and Rotations

1) translation: 1 unit left and 5 units up
2) reflection across the y-axis
3) translation: 3 units right and 4 units down
4) reflection across the x-axis
5) translation: 1 unit left and 7 units down
6) reflection across $y = x$
7) rotation 270° clockwise about the origin
8) translation: 2 units right and 1 unit down

Answers to 12. Similar Figures

1) $\{30\}$
2) $\left\{\frac{30}{7}\right\}$
3) $\left\{\frac{25}{6}\right\}$
4) $\left\{\frac{34}{3}\right\}$
5) $\left\{\frac{127}{9}\right\}$
6) $\left\{\frac{86}{9}\right\}$
7) $\{-6\}$
8) $\{5\}$
9) $\left\{-\frac{16}{3}\right\}$
10) $\{-12\}$
11) similar
12) not similar
13) similar
14) 14
15) 35
16) 11
17) 9

Answers to 13. Theorems: Midline, Triangle Proportionality, Three Parallels, Angle Bisector

1) $\overline{NP}$
2) $\overline{NP}$
3) 8
4) 9
5) 20
6) 6
7) 78
8) 117
9) 16
10) 8
11) 70
12) 132
13) 4
14) 9
15) 4
16) 20
17) 45
18) 13
19) 12
20) 12

Answers to 14. Trapezoids

1) 16.5
2) 14.9
3) 9.1
4) 19
5) 90°
6) 55°
7) 31.82 mi^2
8) 26.4 cm^2
9) 4 mi
10) 9 ft

Answers to 15. Circles

1) Center: $(0, 0)$ Radius: 16
2) Center: $(0, 0)$ Radius: 12
3) Center: $(15, -13)$ Radius: 1
4) Center: $(-12, 7)$ Radius: 1
5) Center: $(-8, \sqrt{91})$ Radius: $\sqrt{31}$
6) Center: $(3, -1)$ Radius: 14
7) $(x - 15)^2 + (y - 7)^2 = 1$
8) $(x + 5)^2 + (y + 11)^2 = 63$
9) $(x - 14)^2 + (y + 5)^2 = 16$
10) $(x - 3)^2 + (y + 8)^2 = 49$
11) $(x + 16)^2 + (y - 1)^2 = 1$
12) $(x + 3)^2 + (y - 7)^2 = 4$
13) 160°
14) 215°
15) 295°
16) 287°
17) 105°
18) 80°
19) 62°
20) 100°
21) 54°
22) 55°
23) 14
24) 17
25) 6.4
26) 14.8

Answers to 16. Volume

1) 99 m^3
2) 140 ft^3
3) 840 ft^3
4) 748.8 cm^3
5) 288 cm^3
6) 2799.16 in^3
7) 8 mi^3
8) 403.33 mi^3
9) 528 yd^3
10) 1526.81 ft^3
11) 427 m^3
12) 166.67 in^3
13) 1436.76 mi^3